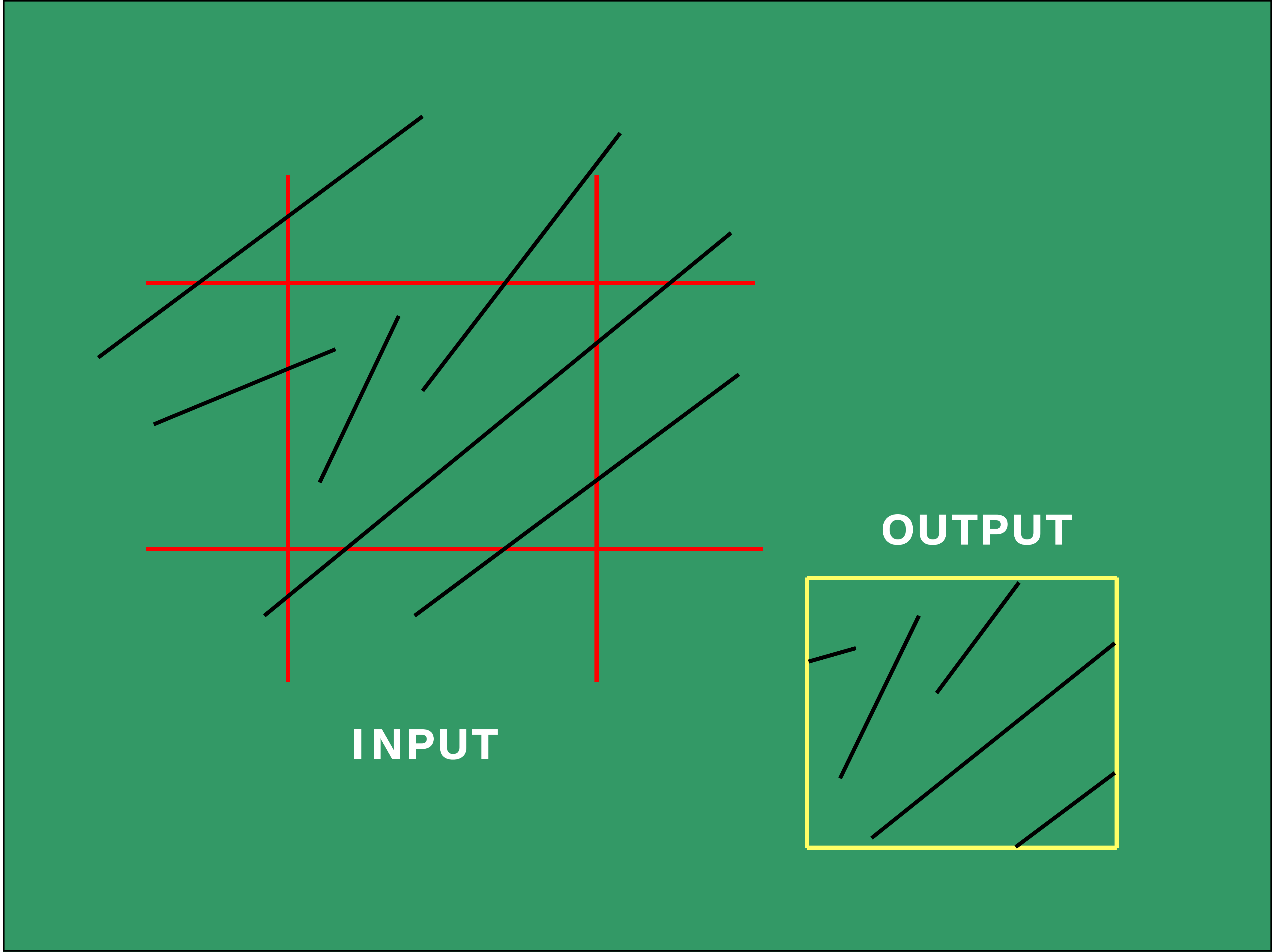
Clipping:

LINES

and

POLYGONS


OUTPUT
INPUT

Solving Simultaneous equations using parametric form of a line:

$$P(t) = (1-t)P_0 + tP_1$$

$$where,\ P(0) = P_0;\ P(1) = P_1$$

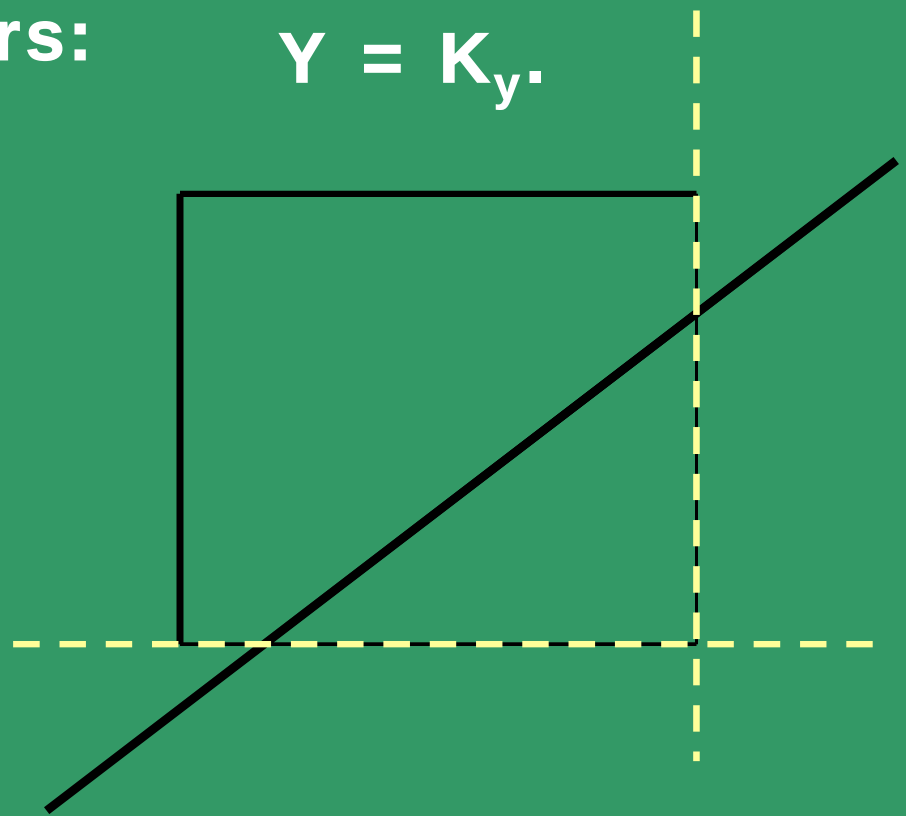
Solve with respective pairs:

$$t_{lx} = \frac{K_x - X_0}{X_1 - X_0}$$

$$t_{ly} = \frac{K_y - Y_0}{Y_1 - Y_0}$$

Vertical Line:
$X = K_x$;

Horizontal Line:
$Y = K_y$.



In general, solve for two sets of simultaneous equations for the parameters:

t_{edge} *and* t_{line}

Check if they fall within range [0 - 1].

i.e. Rewrite

$$P(t) = P_0 + t(P_1 - P_0)$$

and Solve:

$$t_1(P_1 - P_0) - t_2(P_1' - P_0') = P_0' - P_0$$

Cyrus-Beck

Line Clipping

CYRUS-BECK formulation

$$P(t) = P_0 + t(P_1 - P_0)$$

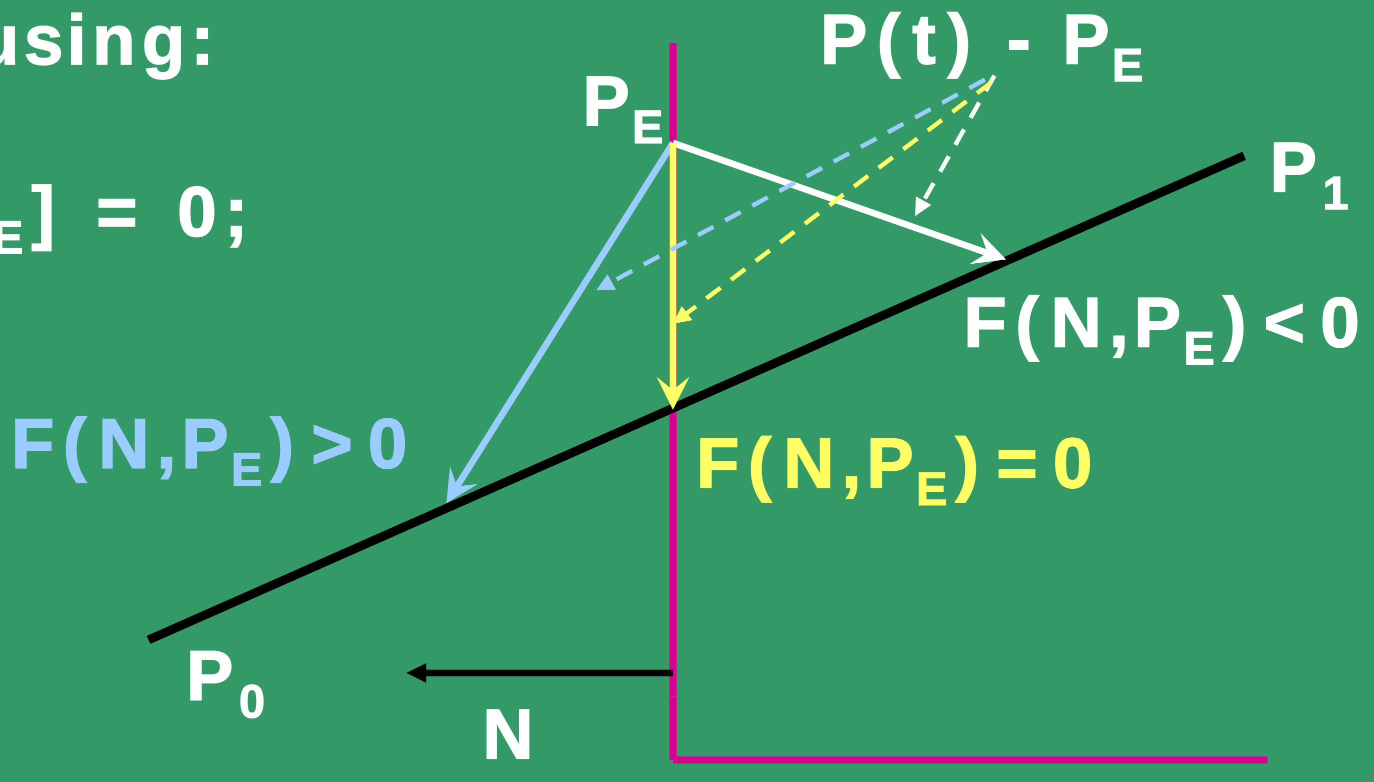
$$where,\ P(0) = P_0;\ P(1) = P_1$$

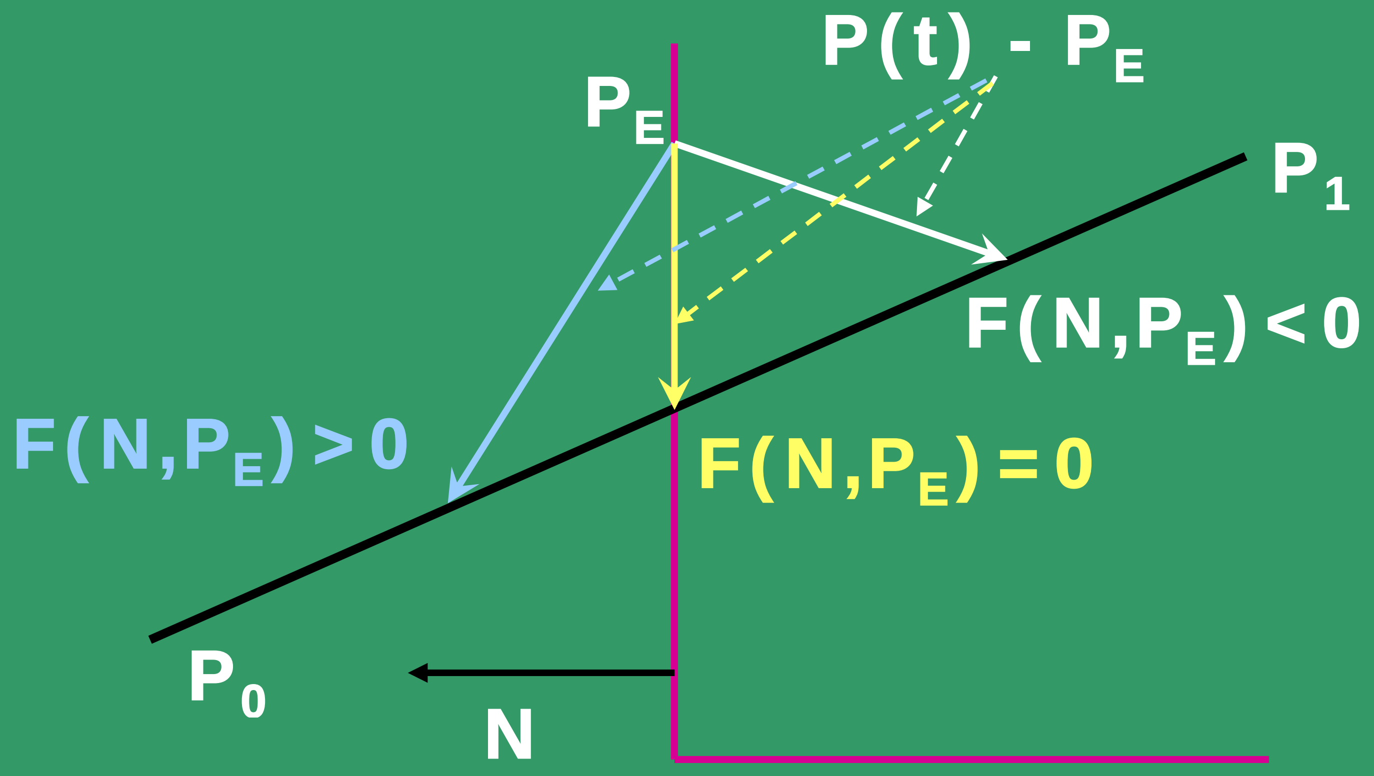
Define,

$F(N, P_E) = N.[P(t) - P_E]$

Solve for **t** using:

$N.[P(t) - P_E] = 0;$





Solve for t using:

$N.[P(t) - P_E] = 0;$

$N.[P_0 + (P_1 - P_0)t - P_E] = 0;$

Substitute, $D = P_1 - P_0;$

To Obtain: $t = \frac{N.[P_0 - P_E]}{-N.D}$

To ensure valid value of t, denominator must be non-zero.

Assuming, that D, N < > 0, check if:
N.D < > 0.   i.e. edge and line are not parallel.

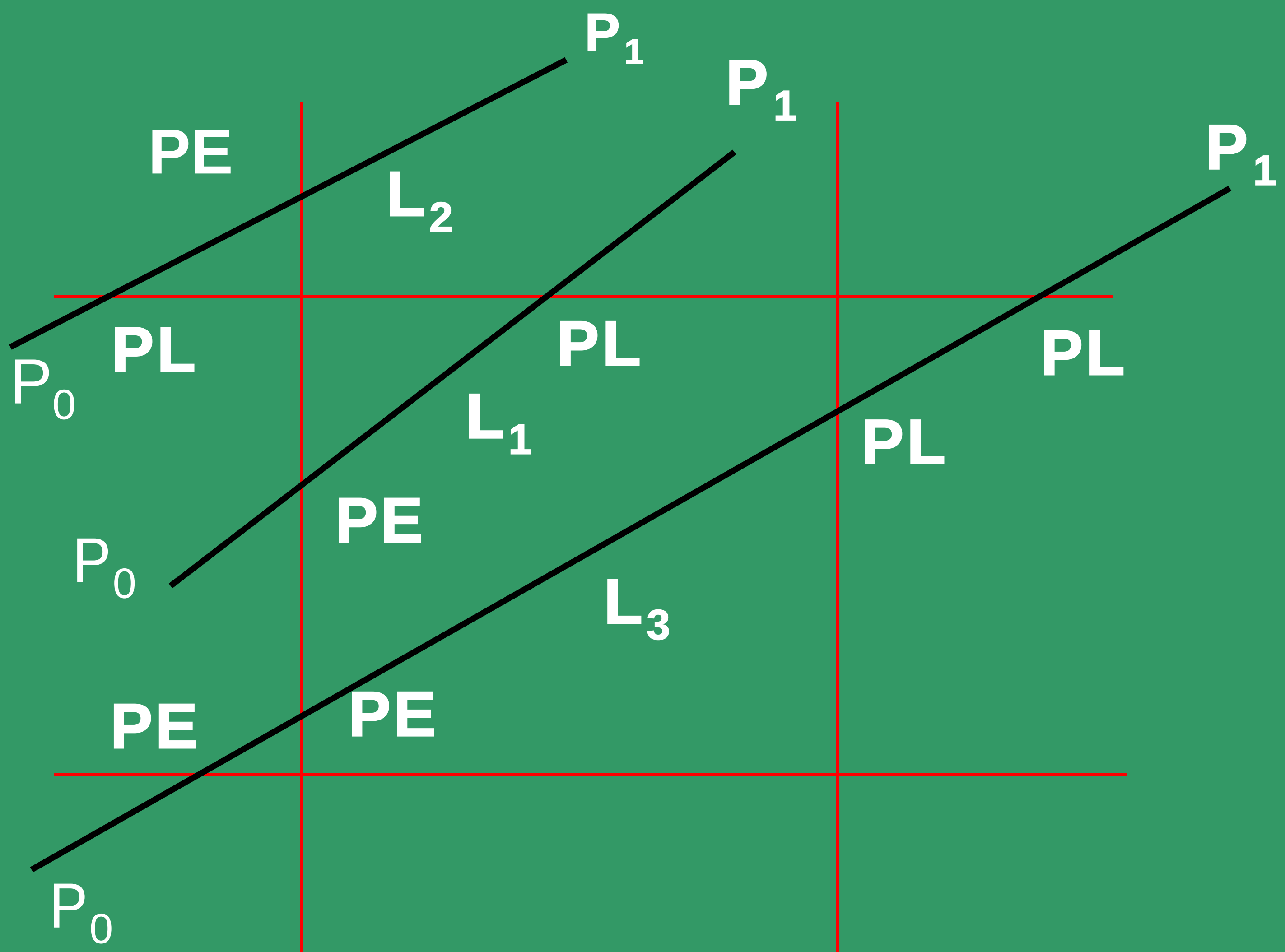
If they are parallel ?

Use the above expression of t to obtain all the four intersections:

- Select a point on each of the four edges of the clip rectangle.
- Obtain four values of t.
- Find valid intersections

How to implement the last step ?

Consider this example



Steps:

- If any value of t is outside the range [0 – 1] reject it.
- Else, sort with increasing values of t.

This solves L_1, but not lines L_2 and L_3.

Criteria to choose intersection points, PE or PL:

Move from point P_0 to P_1;

If you are entering edge's inside half-plane, then that intersection point is marked PE;

else, if you are leaving it is marked as PL.

Check the angle of D and N vectors, for each edge separately.

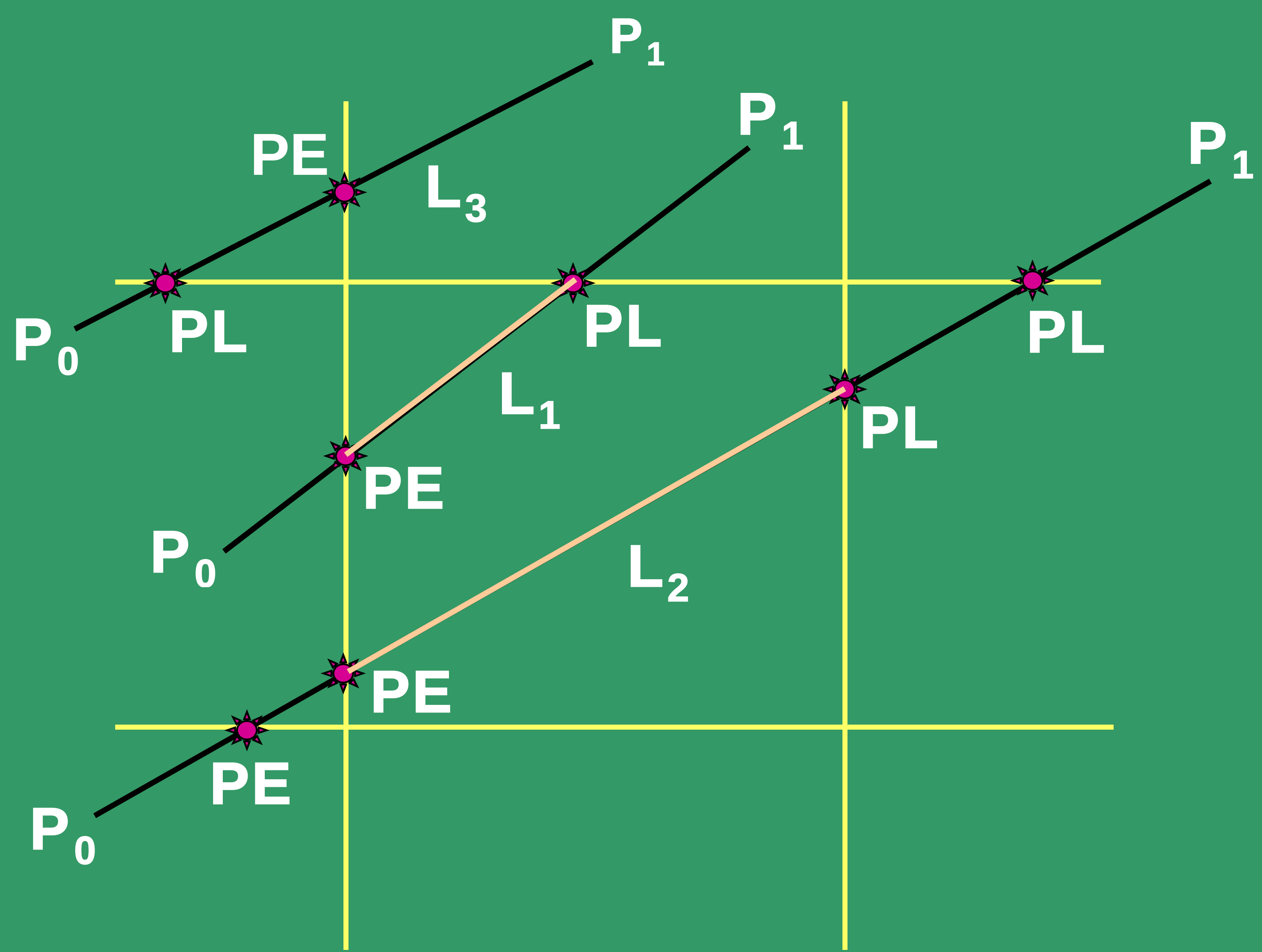
If angle between D and N is:

>90 deg., N.D < 0, mark the point as PE, store $t_E(i) = t$

<90 deg., N.D > 0, mark the point as PL, store $t_L(i) = t$

Find the maximum value of t_E, and minimum value of t_L for a line.

If $t_E < t_L$ choose pair of parameters as valid intersections on the line. Else NULL.



P1
P1
P1
PE
L3
PL
PL
PL
P0
PL
L1
PE
P0
L2
PE
PE
P0

Calculations for parametric line Clipping

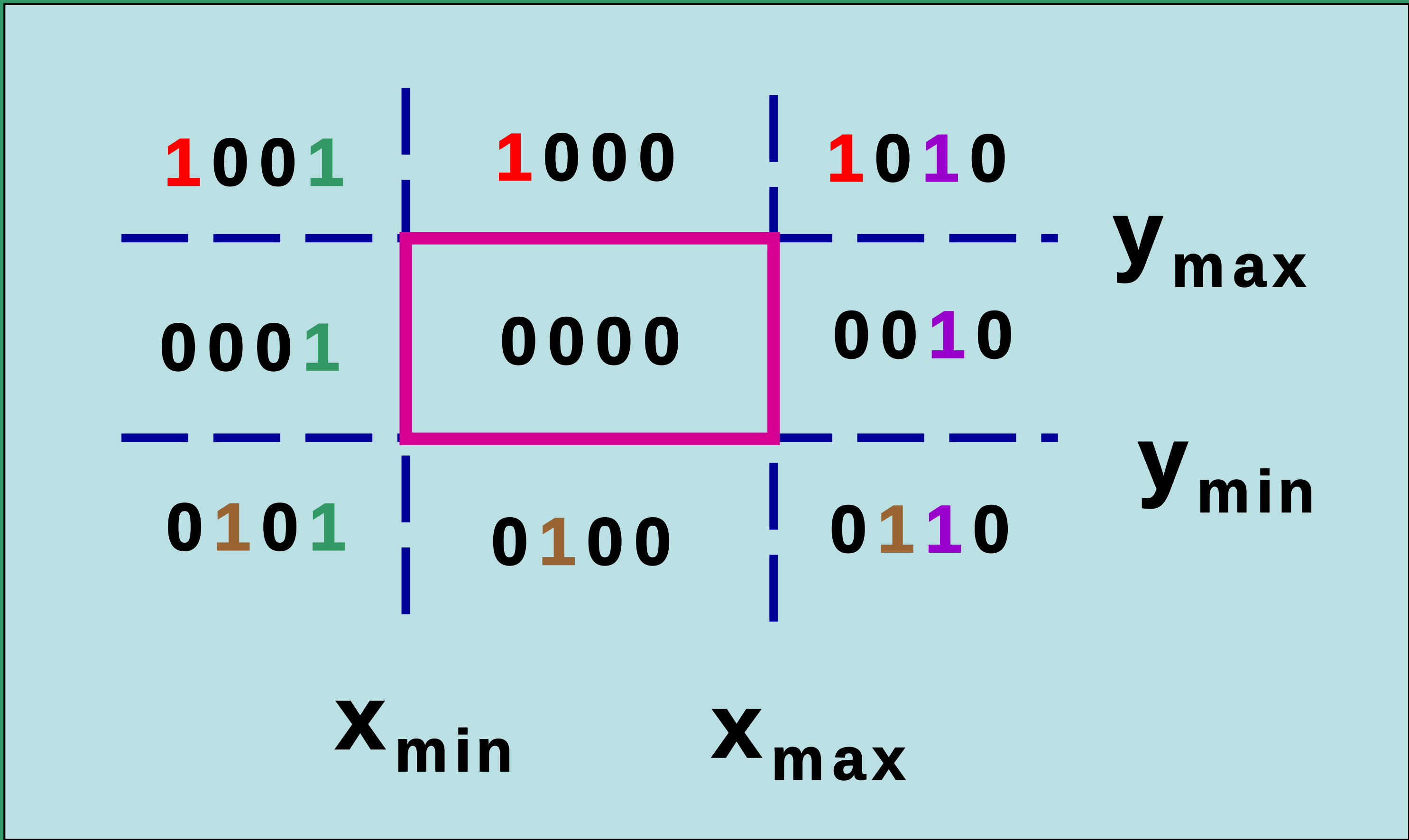
Clip Edge	Normal N	P_E §	P_0 - P_E	$t = \frac{N.[P_0 - P_E]}{-N.D}$
Left: $X = X_{min}$	(-1, 0)	(X_{min}, Y)	$(X_0 - X_{min}, Y_0 - Y)$	$\frac{-(X_0 - X_{min})}{(X_1 - X_0)}$
Right: $X = X_{max}$	(1, 0)	(X_{max}, Y)	$(X_0 - X_{max}, Y_0 - Y)$	$\frac{(X_0 - X_{max})}{-(X_1 - X_0)}$
Bottom: $Y = Y_{min}$	(0, -1)	(X, Y_{min})	$(X_0 - X, Y_0 - Y_{min})$	$\frac{-(Y_0 - Y_{min})}{(Y_1 - Y_0)}$
Top: $Y = Y_{max}$	(0, 1)	(X, Y_{max})	$(X_0 - X, Y_0 - Y_{max})$	$\frac{(Y_0 - Y_{max})}{-(Y_1 - Y_0)}$

§ - Exact coordinates for P_E is irrelevant.

Cohen-Sutherland

Line Clipping

Region Outcodes:



Bit Number	1	0
FIRST (MSB)	Above Top edge $Y > Y_{max}$	Below Top edge $Y < Y_{max}$
SECOND	Below Bottom edge $Y < Y_{min}$	Above Bottom edge $Y > Y_{min}$
THIRD	Right of Right edge $X > X_{max}$	Left of Right edge $X < X_{max}$
FOURTH (LSB)	Left of Left edge $X < X_{min}$	Right of Left edge $X > X_{min}$

First Step: Determine the bit values of the two end-points of the line to be clipped.

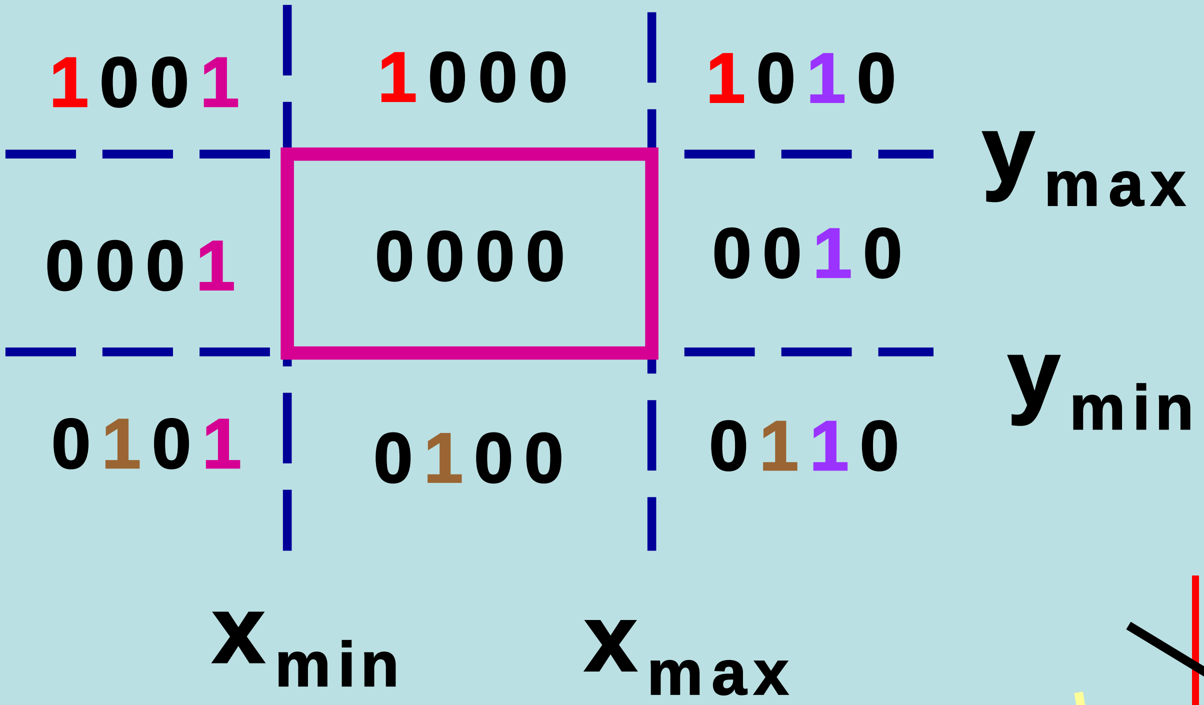
To determine the bit value of any point, use:

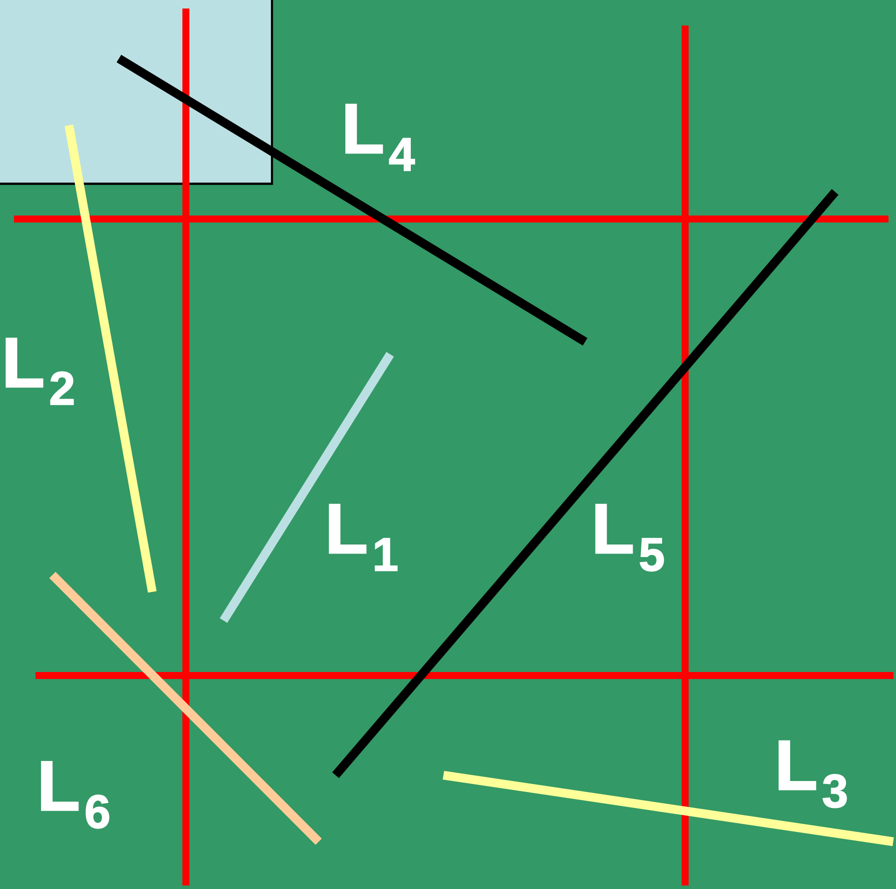
$$b_1 = sgn(Y_{max} - Y); \quad b_2 = sgn(Y - Y_{min});$$
$$b_3 = sgn(X_{max} - X); \quad b_4 = sgn(X - X_{min});$$

Use these end-point codes to locate the line. Various possibilities:

- **If both endpoint codes are [0000], the line lies completely inside the box, no need to clip. This is the simplest case (e.g. L_1).**

- **Any line has 1 in the same bit positions of both the endpoints, it is guaranteed to lie outside the box completely (e.g. L_2 and L_3).**





- Neither completely reject nor inside the box:

Lines: L_4 and L_5, - needs more processing.

- What about Line L_6 ?

Processing of lines, neither Completely IN or OUT; e.g. Lines: L_4, L_5 and L_6.

Basic idea:

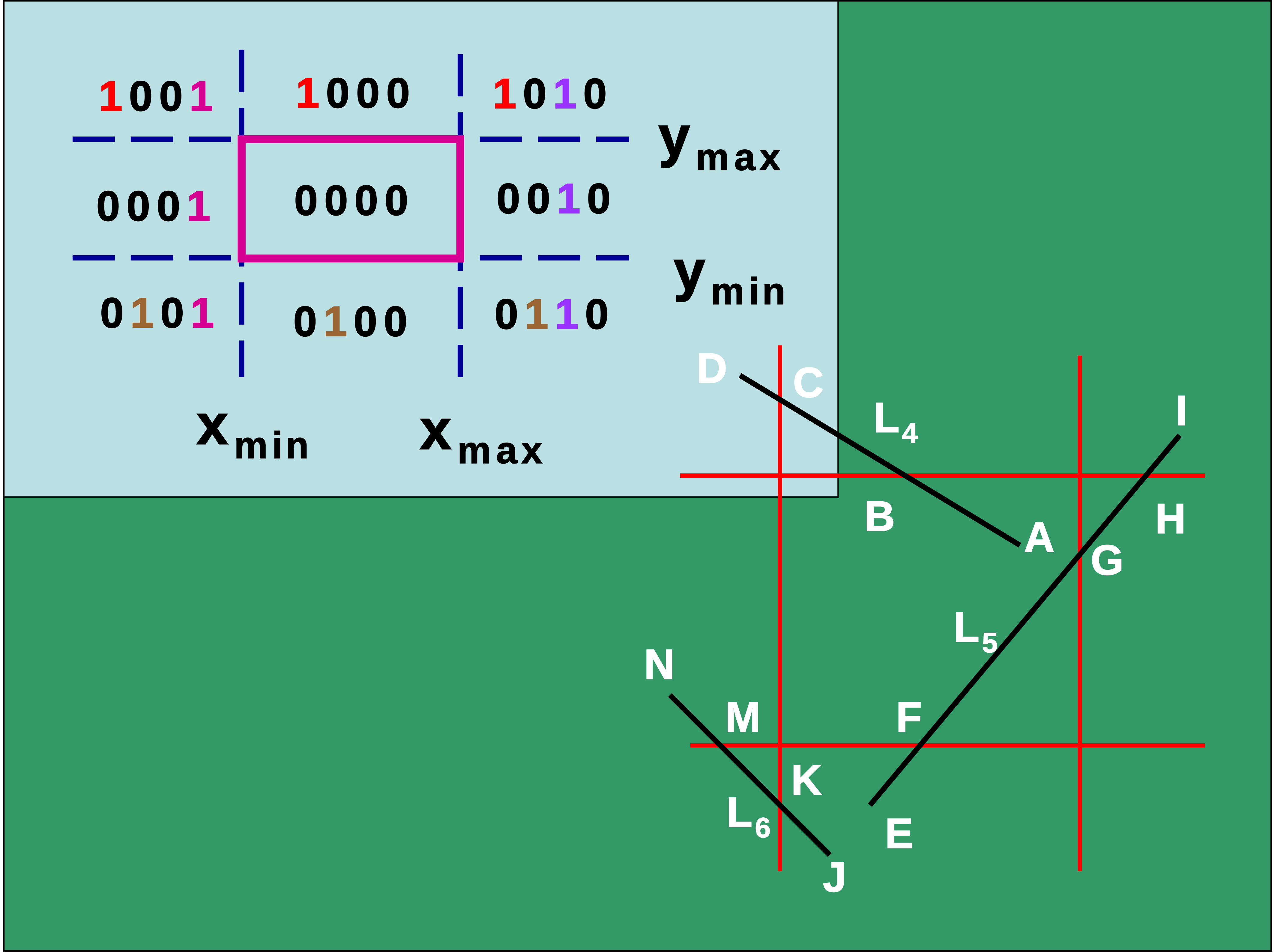
Clip parts of the line in any order (consider from top or bottom).

Algorithm Steps:

- Compute outcodes of both endpoints to check for trivial acceptance or rejection (AND logic).

- If not so, obtain an endpoint that lies outside the box (at least one will ?).

- Using the outcode, obtain the edge that is crossed first.


1001
1000
1010
0001
0000
0010
0101
0100
0110
y_{max}
y_{min}
x_{min}
x_{max}
D
C
L_4
I
B
A
H
G
L_5
N
M
F
K
L_6
E
J

Coordinates for intersection, for clipping w.r.t edge:

Inputs: **Endpoint coordinates:**
(X_0, Y_0) and (X_1, Y_1)

OUTPUT:

Edge for clipping (obtained using outcode of current endpoint).

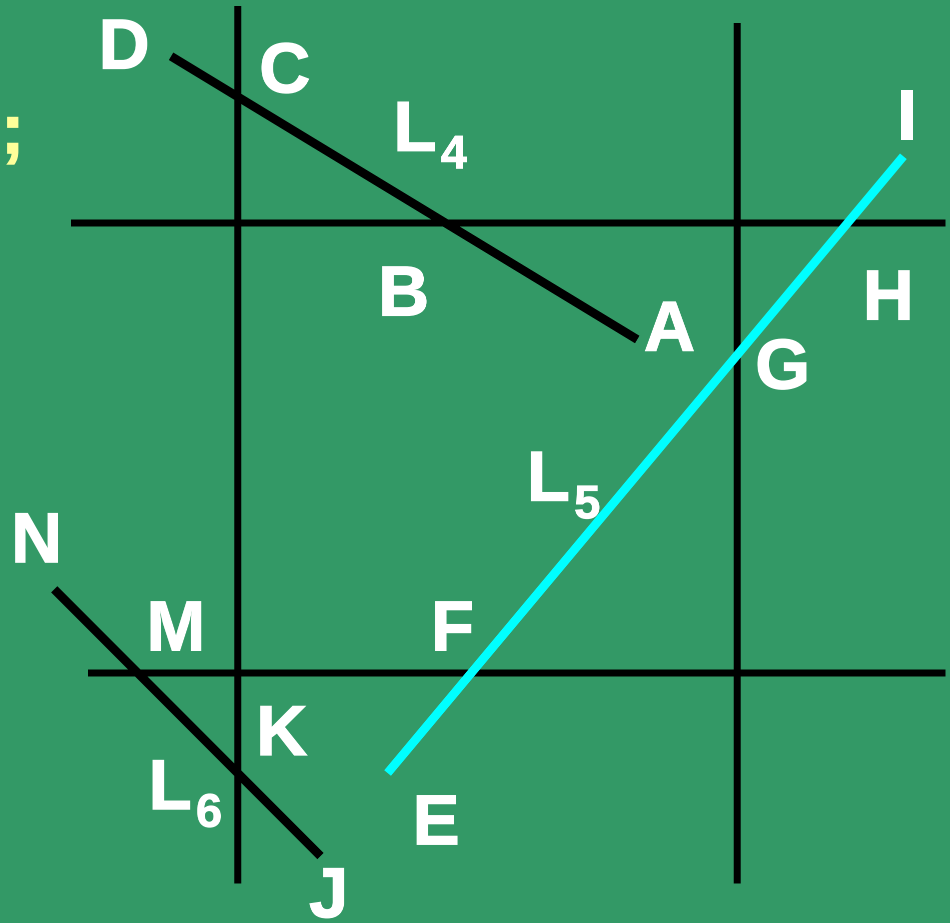
Obtain corresponding intersection points

- CLIP (replace the endpoint by the intersection point) w.r.t. the edge.
- Compute the outcode for the updated endpoint and repeat the iteration, till it is 0000.
- Repeat the above steps, if the other endpoint is also outside the area.

e.g. Take Line L_5 (endpoints - E and I):
E has outcode 0100 (to be clipped w.r.t. bottom edge);

So EI is clipped to FI;
Outcode of F is 0000;
But outcode of I is 1010;
Clip (w.r.t. top edge) to get FH.
Outcode of H is 0010;
Clip (w.r.t. right edge) to get FG;

Since outcode of G is 0000, display the final result as FG.



Formulas for clipping w.r.t. edge, in cases of:

Top Edge :

$$X = X_0 + (X_1 - X_0) * \frac{(Y_{max} - Y_0)}{(Y_1 - Y_0)}$$

Bottom Edge:

$$X = X_0 + (X_1 - X_0) * \frac{(Y_{min} - Y_0)}{(Y_1 - Y_0)}$$

Right Edge:

$$Y = Y_0 + (Y_1 - Y_0) * \frac{(X_{max} - X_0)}{(X_1 - X_0)}$$

Left edge:

$$Y = Y_0 + (Y_1 - Y_0) * \frac{(X_{min} - X_0)}{(X_1 - X_0)}$$

Let's compare with Cyrus-Beck formulation →

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Top: $Y = Y_{max}$	(0, 1)	(X, Y_{max})	$(X_0 - X, Y_0 - Y_{max})$	$\frac{(Y_0 - Y_{max})}{-(Y_1 - Y_0)}$

§ - Exact coordinates for P_E is irrelevant.