Department of Mathematics

College of Science

Salahaddin University -Erbil

Subject: General Topology

Course Book – (Year 4)

Lecturer's name Abdullah M. PhD

Academic Year: 2022/2023

Units: 2 (Second Semester)

Course Book

1. Course name	**General Topology**
2. Lecturer in charge	**Professor Dr. Abdullah M. Abdul-Jabbar**
3. Department/ College	**Mathematics / Science**
4. Contact	**e-mail:abdullah.abduljabbar@su.edu.krd** **e-mail:m1abdullah.math71@gmail.com**
5. Time (in hours) per week	**Theory: 2** **Practical: 0** **Tutorial:**
6. Office hours	**Availability of the lecturer to the student during the week**
7. Course code	**54XX**
8. Teacher's academic profile	**i. B.Sc. (1992-1993)** **ii. M. Sc. (Topology) (2000)** **iii. Assistant Lecture (2000)** **iv. Lecture (2004)** **v. Ph. D. (Algebra) (2007)** **vi. Assistant Professor (2009)** **vii. Head of Mathematics in (Basic Education College) (2009-2010)** **viii. (38) Published paper about Algebra and General Topology.** **ix. Members of Editorial Board in (13) journals outside Iraq.** **x. Professor (2019)**
9. Keywords	**General topology**

10. Course overview:

It is often said that idea of evolution to biology is same as the ideas of topology to mathematics.

Topology refers to the relationship between spatial features or objects. In terms of functionality, topology is important in (at least) three important ways:

First, topology is necessary for certain spatial functions such as network routing through linear networks. Here the idea is that if line features do not share common nodes, that routes can not be established through the network.

Second, topology can be used to create datasets with better quality control and greater data integrity. Topology rules can be created so that edits made to a dataset can be 'validated' and show errors in that dataset. An example would be the creation of a new manhole/sewer access feature outside a polygon dataset of road features.

Third, by creating topological relationships between feature classes, features can be shared across feature classes. In other words, if you open one dataset and edit/move a line feature that is shared between two feature classes, then both feature classes will be updated to reflect the edits. This is massively helpful for keeping datasets syncrhonized. An example would be a river feature that defines a administrative boundary (where the river moves over time), or the boundary of a muncipal area and zoning polygons.

Topological spaces show up naturally in almost every branch of mathematics. This has made topology one of the great unifying ideas of mathematics.

The branch of geometry concerned with the study of continuity and limits at the natural level of generality determined by the nature of these concepts. The initial concepts of general topology are the concepts of a topological space and a continuous mapping, introduced by F. Hausdorff in 1914.

A particular case of a continuous mapping is a homeomorphism — a continuous one-to-one mapping between topological spaces that has a continuous inverse mapping. Spaces that can be mapped onto each other by a homeomorphism (that is, homeomorphic spaces) are regarded as the same in general topology. One of the basic problems in general topology is to find and investigate natural topological invariants — properties of spaces preserved under homeomorphisms (cf. Topological invariant). Of course, any property of a space that can be formulated entirely in terms of its topology is automatically a topological invariant. Proof of the topological invariance of a property of a space is only required when it is formulated with the aid of additional structures defined on the set of points of the space, in some way related to its topology. The topological invariance of the homology groups may serve as an example.

A topological invariant is not necessarily expressible by a number; for example, connectedness, (Hausdorff) compactness and metrizability are topological invariants. Among the numerical invariants (taking numerical values on every topological space), the most important are the dimensional invariants: the small inductive dimension ind, the large inductive dimension Ind and the Lebesgue dimension dim (dimension in the sense of coverings).

Topological invariants of another kind, with cardinal numbers (cf. Cardinal number) as values, play an important role. These cardinal characteristics include the weight of a topological space.

Related to a system of topological invariants there are classes of topological spaces, each class being determined by restriction of one or another topological invariant. The most important classes are metrizable spaces, compact spaces, Tikhonov spaces, paracompact spaces, and feathered spaces (cf. Metrizable space; Compact space; Tikhonov space; Paracompact space; Feathered space).

Fundamental "intrinsic" problems in general topology include: 1) the isolation of important new classes of topological spaces; 2) the comparison of different classes of topological spaces; and 3) the study of spaces within such a class and of categorical properties of this class as a whole. Problem 2) is undoubtedly central in this group, directed to ensuring the internal unity of general topology.

The isolation of important new classes of topological spaces (that is, new topological invariants) is often related to the consideration of additional structures on the space (numerical, algebraic, order), naturally compatible with its topology. Thus, one distinguishes metrizable spaces, ordered spaces, spaces of topological groups, symmetric spaces, etc. The method of coverings plays an important role in solving 1), 2) and 3). In the language of coverings and relations between coverings, the most important of which are the relations of refinement and star refinement, the fundamental classes of compact and paracompact spaces can be singled out, and topological properties like compactness can be formulated. The method of coverings plays an important part in dimension theory.

For the solution of the central problem 2), the method of mutual classification of spaces and mappings is particularly important. It is concerned with establishing connections between various classes of topological spaces by means of continuous mappings subject to certain simple restrictions. Spaces of quite general nature can in this way be described as the images of simpler spaces under "good" mappings. For example, spaces satisfying the first axiom of countability can be characterized as images of metric spaces under continuous open mappings. Connections of this kind establish an effective system of reference in the consideration of classes of topological spaces.

General topology is important in methodical respects in mathematical education. The fundamental concepts of continuity, convergence and continuous transformation can only be explained and become transparent within the framework of the concepts and constructions of topology. It is hard to name any area of mathematics in which the concepts and language of general topology are not used at all. In particular, its unifying role in mathematics becomes apparent in this. The position of general topology in mathematics is also determined by the fact that a whole series of principles and theorems of general mathematical importance find their natural (i.e. corresponding to the nature of these principles or theorems) formulation only in the framework of general topology. As examples one can mention the concept of compactness — an abstraction from the Heine–Borel lemma on extracting a finite subcovering of an interval, the theorem on the compactness of a product of compact spaces (which generalizes the assertion that a finite-dimensional cube is compact), and the theorem that a continuous real-valued function on a compact space is bounded and attains its least upper and greatest lower bounds. This series of examples could be extended: the concept of a set of the second category, the concept of completeness, the concept of an extension (the very character of these concepts and the results related to them, important for mathematics as a whole, makes their investigation most natural and transparent within the framework of general topology).

11. Course objective:

The objectives of this course are to:
1. Introduce students to the concepts of open and closed sets.
2. Introduce students to the base and sub-base of a topology.
3. Introduce students how to generate new topologies from a given set with bases.
4. provide the awareness of tools for students to carrying out advanced research work in Pure mathematics.
5. Introduce students to the concepts of limit points, subspace.
6. Introduce students to the concepts of continuous functions which is an important tools in mathematics.
7. Introduce students to the concepts of open, closed maps and homeomorphisms.
8. Introduce students to the concepts of connctedness, compactness, separation axioms.
9. Introduce students to the concepts of first and second countable spaces, separable spaces, product spaces and metrizable spaces.

12. Student's obligation

In this course, every lecture we review all topics with students which we gave in the previous lecture and when we teach, we try to contribute all students.

13. Forms of teaching

We use data-show with white board and give a copy of my lecture about general topology for all students step by step.

14. Assessment scheme

Your course grade will be determined as follows:

First examination: 30%

Tutorial: 10%

Pursuit Grade: First Examination + Tutorial = 30% + 10% = 40%

Note: Other activity add to the First Examnation over 5%

Final Examination: 60%

Total: (Pursuit Grade + Final Examination) = (40%+60%) = 100%

15. Student learning outcome:

In first year in our Department of Mathematics, students studies Foundation of Mathematics, which included set theory and functions. In this year we discuss general topology which is applications of set theory and functions step by step to understand them.

16. Course Reading List and References:

[1] Lynn Arthur Steen and J. Arthur Seebach, Counterexamples in Topology, Springer-Verlag New York-Heidelberg-Berlin, 1978.

[2] William J. Pervin, Foundations of General Topology, New York Academic Press London, 1972.

[3] Yu. Borisovich, N. Bliznyakov, YA. Izrailevich, T. Fomenko, Introduction to Topology, MIR Pulishers. Moscow, 1985.

[4] Nicolas Bourbaki, Elements of Mathematics General Topology (part 2), Addison-Wesley Publishing Company, 1966.

[5] George F. Simmons, Topology and Modern Analysis, McGROW-HILL Book Company, INC, 1963.

[6] J. Dugundji, Topology, Allyn and Bacon, Boston, 1966.

[7] Stewort R. Munkres, General Topology, Pretice Hall, Inc., 2000.

[8] V. A. Rokhlin, D. B. Fuks, Beginners Course in Topology Geometric, Chapters. Springer-Verlag, 1984.

[9] Nicolas Bourbaki, General Topology, Chapters 1-4, Elements of Mathematics. Springer-Verlag, Berlin, 1998.

[10] Internet. Lecture Notes. http://www.math.cornell.edu

17. The Topics:	**Lecturer's name**
General Topology **Chapter 4 (Section 2): Continuous functions** • Continuity in topological space • Continuous functions interms of open set • Continuous functions interms of neibourhoods • Continuous functions interms of closed set • Continuous functions interms of a sub-base • Continuous functions interms of a base • Continuous functions interms of closure of a set • Continuous functions interms of closure of inverse function • Some examples on continuous functions • Continuous functions interms of inverse function of interior of a set • If two functions are continuous. Is composition of them continuous ? • Is restriction function continuous? • Convergent sequence in topology • Sequential continuity in topology • The relation between continuity and sequential continuity Note: some of the above theorems we take without proof **Chapter 5: Open mappings and Homeomorphisms** • Open mappings with example • Closed mappings with example • Bicontinuous with example • Homeomorphisms with examples • Homeomorphic spaces • Topological property with examples • Equivalent statement of homeomorphisms	General Topology (2 hours per week)

- Equivalent statement of open mappings
- Equivalent statement of closed mappings
- Equivalent statement of homeomorphisms interms of closed set

Note: some of the above theorems we take without proof

Chapter 6: Connected Space

- Separated set with example
- Connected between subspace with separated sets
- Connected between separated subsets with disjoint sets
- Definition of disconnectedness interms of separated sets with example
- Connected of two points
- Connection between disconnected interms of topology with disconnected interms of subspace
- Characterization of disconnectedness interms of open and closed subsets
- Disconnectedness interms of finear topology
- Connectedness interms of coarser topology
- Connectedness is a topological property
- Some equivalent statements of connectedness
- Property of connectedness interms of family of connected subsets of the whole space
- Continuity and connectedness
- Totally disconnected space
- Connection between totally disconnected with discrete space

Note: some of the above theorems we take without proof

Chapter 7: Compact Space

- Open cover (covering)
- Compact space
- The connection between finite space with compact
- The connection between usual topology with compact
- The connection between discrete space with compact
- The relation between compact relative to whole space with compact relative to a subspace
- The connection between compact of the whole space with compact relative to a subspace under closed subset
- Compact space interms of basic open cover
- Finite intersection property (FIP) with example
- Compact space interms of FIP
- Continuity and compactness

- Compact space is a topological property

Note: some of the above theorems we take without proof

Chapter 8: Separation Axioms

- T_0-space (Kolomogorov axiom) with example
- An equivalent statement of T_0-space
- Hereditary property
- T_0-space is hereditary property
- T_0-space is topological property
- T_1-space (Frechet space) with example
- The relation between T_1-space with T_0-space
- An equivalent statement of T_1-space
- T_1-space is topological property
- T_2-space (Hausdorff space)
- The connection between discrete space with T_2-space
- The connection between T_2-space with T_1-space
- T_2-space is hereditary property

Note: some of the above theorems we take without proof

18. Practical Topics (If there is any)

In this section the lecturer shall write titles of all practical topics he/she is going to give during the term. This also includes a brief description of the objectives of each topic, date and time of the lecture

We have not applications of it but it is theoretical.

19. Examinations:

Q;/ Define T1-space. State and prove an equivalent statement of T1-space.
Q:/ Prove or disprove.
Q:/ Show that the T2 space is a topological property.
Q:/ What is the relation between T4-space with T3-space ?

2. True or false type of exams:

May be we use this type of exam.

3. Multiple choices:

We cannot use this type of exams because it is not benefit for general topology.

20. Extra notes:

Also we have some applications of general topology and combine it with biology and computer science such as digital topology, digital line and digital n space.

21. Peer review