Question Bank General Topology 2022-2023

Q1/ Suppose a singleton set {p} is an open subset of a topological space X. Show that for any topological space Y and any function f :X→Y, f is cont. at p ∈ X.

Q2/ State a theorem**,** which is equivalent to cont. fun. interms of interior of a set.

Q3/ Show that, in topology, every continuity is sequential continuity.

Q4/ Show that the identity function i: (X, τ)→(X, σ) is continuos iff τ is finer than σ, i.e., σ ⊂ τ.

Q5/ Let $\{\tau_i\}$ be a collection of topologies on a set X. If a fun. f :X→Y is continuous with respect to each τ_i, then show that f is continuous with respect to the intersection topology $\tau = \bigcap_i \tau_i$.

Q6/ Under what conditions will a function f :X→Y not be continuous at a point p ∈X ?

Q7/ Let X and Y be topological spaces. Then show that a fun. f :X→Y is continuous iff it is continuous at every point p ∈X.

Q8/ Let the fun. f :X→Y and g: Y→Z be continuous. Then the composition fun. g o f : X→Z is also continuous.

Q9/ State and prove an equivalent statement of open function.

Q10/ State and proof an equivalent statements of homeomorphism.

Q11/ Let **B** be a base for a topological space X. Show that if f : X→Y has the property that f [B] is open for every B ∈ **B**, then f is an open fun.

Q12/ Show that the closed interval A = [a, b] is homeomorphic to the closed unit interval I = [0, 1].

Q13/ Give an example of a real function f :**R**→**R** such that f is continuous and closed, but not open.

Q14/ What is the relation between separated sets and disjoint ?

Q15/ State and prove a theorem which is equivalent to two separated sets interms of subspace.

Q16/ Show that if (X, τ) is disconnected and τ' is finear than τ (i.e., $\tau \subset \tau'$). Then, (X, τ') is disconnected.

Q17/ Show that if (X, τ) is connected and τ' is coarser than τ (i.e., $\tau' \subset \tau$). Then, (X, τ') is connected.

Q18/ Show that connectedness being a topological property.

Q19/ State and prove four equivalent statements of connectedness.

Q20/ State and prove a theorem which is equivalent statement of disconnectedness in terms of the discrete space $\{0, 1\}$.

Q21/ Show that every discrete space X is a totally disconnected space.

Q22/ Let $X = \{a, b, c, d, e\}$ and $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$. Show that (X, τ) is disconnected space.

Q23/ Show that (**R**, **U**) is connected space, where **R** is the set of real numbers and **U** is the usual topology.

Q25/ Show that every finite set is compact.

Q26/ Show that compactness being a topological property.

Q27/ Let τ be the cofinite topology on any set X. Show that (X, τ) is a compact space.

Q28/ Show that any infinite subsets A of a discrete topological space X is not compact.

Q29/ Consider the following class of open intervals:

A = {(0, 1), (0, 1/2), (0, 1/3), (0, 1/4), …}. Show that **A** has the FIP.

Q30/ State and prove an equivalent statement of T_0-space.

Q31/ Every T_1-space is a T_0-space, but the converse is not true in general. Give an example.

Q32/ State and prove an equivalent statement of T_1-space.

Q33/ State a property of a T_1-space.

Q34/ T_2-space being hereditary property.

Q35/ Show that a finite subset of a T_1-space X has no accumulation points.

Q36/ Show that every finite T_1-space X is a discrete space.

Q37/ Let τ be the topology on the real line R generated by the open-closed interval (a, b]. Show that (R, τ) is T_2-space.