

**Q1/** Define "Limit point". Show that a subset  $A$  of a space  $(X, \tau)$  is closed if and only if  $A$  contains all its limit points.

**Q2/**

(i) Consider  $(X, \mathbf{D})$ , where  $\mathbf{D}$  is the discrete topology. What is the base for  $\mathbf{D}$ ? Explain it.

(ii) Prove that  $\mathbf{D}(\mathbf{Q}) = \mathbf{R}$ , where  $\mathbf{Q}$  is the set of all rational numbers and  $\mathbf{R}$  is the set of all real numbers.

**Q3/**

(i) Write "True" or "False" of the following:

Let  $(X, \tau)$  be a topological space and  $A, B \subset X$ .

(1) Every metric space is topological space.

(2)  $b(A) = (A^c \cup \text{ext}(A))^c$ .

(3) A subset  $A$  of a space  $(X, \tau)$  is called dense in itself if  $A \subset \text{Cl}(A)$ .

(4) If  $x$  is not an adherent point of  $A$  then it is not a limit point of  $A$ .

(5)  $\text{Cl}(A \cup B) = \text{Cl}(A) \cup \text{Cl}(B)$ .

(6)  $b(A) \subset b(A^\circ)$ .

(7) If  $A^\circ \cap A^c = \emptyset$ , then  $A^c \cap A^\circ \setminus \{x\} = \emptyset$ .

(8)  $A^\circ = \bigcup \{G: G \text{ is open set and } A \subset G\}$ .

(9)  $A \cap \text{ext}(A) = \emptyset$ .

(10) A point  $x \in X$  is an exterior point of  $A$  if and only if  $x$  is not an adherent point of  $A$ .

(ii) Define "Base for a topology". Let  $\mathbf{B}$  be a base for a topology  $\tau$  on  $X$  and let  $\mathbf{B}^*$  be a class of open sets containing  $\mathbf{B}$ , i.e.,  $\mathbf{B} \subset \mathbf{B}^* \subset \tau$ . Show that  $\mathbf{B}^*$  is also a base for  $\tau$ .

(iii) Let  $\tau$  be a topology on a set  $X$  consisting of four sets, i.e.,  $\tau = \{X, \emptyset, A, B\}$ , where  $A$  and  $B$  are non-empty distinct proper subsets of  $X$ . What conditions must  $A$  and  $B$  satisfy to become  $\tau$  a topology on  $X$ ? Check it.