

### Q1/

(A) Define "Neighbourhood of a point". Show that a subset  $G$  of a space  $X$  is open **iff** it is a neighbourhood of each of its points.

(B) Define "continuous function in topology". Let  $X$  and  $Y$  be two topological spaces. Show that a function  $f: X \rightarrow Y$  is continuous **iff** the inverse image of every member of a sub-base of  $Y$  is open in  $X$ .

### Q2/

(A) Define "Disconnected space". Let  $(Y, \tau_Y)$  be a subspace of a space  $(X, \tau)$  and  $A \subset Y$ . Show that  $A$  is  $\tau$ -disconnected **iff**  $A$  is  $\tau_Y$ -disconnected.

(B) Define "open function". Let  $(X, \tau)$  and  $(Y, \sigma)$  be two topological spaces. Show that a mapping  $f: X \rightarrow Y$  is open **iff**  $f(\text{Int}(A)) \subset \text{Int}(f(A)), \forall A \subset X$ .

### Q3/

(A) Define "Locally connected space". Show that locally connectedness is a topological property.

(B) Write "**True**" or "**False**" of the following:

Let  $(X, \tau)$  be a topological space and  $A, B \subset X$ .

(1) Let  $(X, d)$  be a metric space. If  $Y$  is an infinite subset of  $X$ , then the subspace  $Y$  has the discrete topology.

(2) If  $Y$  is a subspace of  $X$ . If  $A \subset Y$ , then  $\text{Cl}(A)$  in  $Y = Y \cap (\text{Cl}(A)$  in  $X$ ).

(3) In general, the number of elements of topology is equal to the number of elements of its subspace.

(4)  $A^\circ = A - b(A)$ .

(5)  $D(\mathbf{Q}) = \mathbf{Q}$ , where  $\mathbf{Q}$  is the set of all rational numbers and  $D$  is the derived set.

(6) Every component of a locally connected space is a closed set.

(7) A space  $X$  is said to have Bolzano-Weierstrass property if every finite subset of  $X$  has a limit point in  $X$ .

(8) The union of a family of connected subsets with nonempty intersection is connected.

### Q4/

(A) Define "Base for a topology". Let  $(X, \tau)$  be a topological space. Show that a subcollection  $\mathbf{B}$  of  $\tau$  is a base for  $\tau$  **iff** every  $\tau$ -open set can be expressed as a union of some members of  $\mathbf{B}$ .

(B) Let  $\tau$  be a topology on a set  $X$  consisting of four sets, i. e.,

$\tau = \{X, \phi, A, B\}$ , where  $A$  and  $B$  are non-empty distinct proper subsets of  $X$ . What conditions must  $A$  and  $B$  satisfy to become  $\tau$  a topology on  $X$ .