

Q1/

(A) Define "Totally disconnected space". Show that every discrete space X is a totally disconnected space.

(B) Define "Homeomorphism in topology". State and prove an equivalent statement of homeomorphism.

Q2/

(A) State and prove "Left-ray topology".

(B) Define "Subspace". Show that the subspace satisfies all conditions of topology.

Q3/

(A) Define "Sequential continuity in topology". What is the relation between continuity and sequential continuity in topology? Prove it.

(B) Define "Base for a topology". Let $X = \{a, b, c\}$ and $\mathbf{B} = \{\{a\}, \{c\}, \{a, b\}, \{b, c\}\}$. Show that \mathbf{B} is not a basis for any topology on X .

Q4/

(A) Define "Separated set". What is the relation between separated sets and disjoint? Under what condition the converse of above relation is true? Prove it.

(B) Write "True" or "False" of the following:

Let (X, τ) be a topological space and $A, B \subset X$.

- (1) Every metric space is topological space.
- (2) Every closed subset of a locally compact space is locally compact.
- (3) A subset A of a space (X, τ) is called dense in itself if $A \subset \text{Cl}(A)$.
- (4) Every discrete space is a locally connected space.
- (5) In topological space, every convergent sequence has unique limit.
- (6) X is connected iff X can be expressed as union of two disjoint non-empty closed sets.
- (7) $\text{Int}(A \cup B) = \text{Int}(A) \cup \text{Int}(B)$.

(C) Define "open function". Let \mathbf{B} be a base for a space X . If $f: X \rightarrow Y$ has the property that $f(B)$ is open, for every $B \in \mathbf{B}$, then show that f is an open function.