Department of Mathematics

College of Science

Salahaddin University -Erbil

Subject: Non-Commutative Algebra

Course Book – (MSc)

Lecturer's name Abdullah M. PhD

Academic Year: 2023/2024

Units: 3 (First Semester)

Course Book

1. Course name	**Non-commutative Algebra**
2. Lecturer in charge	**Professor Dr. Abdullah M. Abdul-Jabbar**
3. Department/ College	**Mathematics / Science**
4. Contact	**e-mail:abdullah.abduljabbar@su.edu.krd** **e-mail:m1abdullah.math71@gmail.com**
5. Time (in hours) per week	**Theory: 3** **Practical: 0** **Tutorial:**
6. Office hours	**Availability of the lecturer to the student during the week**
7. Course code	
8. Teacher's academic profile	**i. BSc. (1992-1993)** **ii. M. Sc. (Topology) (2000)** **iii. Assistant Lecture (2000)** **iv. Lecture (2004)** **v. PhD. (Algebra) (2007)** **vi. Assistant Professor (2009)** **vii. Head of Mathematics in (Basic Education College) (2009-2010)** **viii. (46) Published paper about Algebra and General Topology.** **ix. Reviews at Mathematical Reviews in American Mathematical Society (USA) and Members of Editorial Board in (16) Journals outside Iraq.** **x. Professor (2019)**
9. Keywords	**Non-commutative Algebra, rings, ideals**

10. Course overview:

In algebra, ring theory is the study of rings[1]—algebraic structures in which addition and multiplication are defined and have similar properties to those operations defined for the integers. Ring theory studies the structure of rings, their representations, or, in different language, modules, special classes of rings (group rings, division rings, universal enveloping algebras), as well as an array of properties that proved to be of interest both within the theory itself and for its applications, such as homological properties and polynomial identities.

In mathematics, and more specifically in ring theory, an ideal of a ring is a special subset of its elements. Ideals generalize certain subsets of the integers, such as the even numbers or the multiples of 3. Addition and subtraction of even numbers preserves evenness, and multiplying an even number by any integer (even or odd) results in an even number; these closure and absorption properties are the defining properties of an ideal. An ideal can be used to construct a quotient ring in a way similar to how, in group theory, a normal subgroup can be used to construct a quotient group.

Among the integers, the ideals correspond one-for-one with the non-negative integers: in this ring, every ideal is a principal ideal consisting of the multiples of a single non-negative number. However, in other rings, the ideals may not correspond directly to the ring elements, and certain properties of integers, when generalized to rings, attach more naturally to the ideals than to the elements of the ring. For instance, the prime ideals of a ring are analogous to prime numbers, and the Chinese remainder theorem can be generalized to ideals. There is a version of unique prime factorization for the ideals of a Dedekind domain (a type of ring important in number theory).

The related, but distinct, concept of an ideal in order theory is derived from the notion of ideal in ring theory. A fractional ideal is a generalization of an ideal, and the usual ideals are sometimes called integral ideals for clarity.

11. Course objective:

The objectives of this course are to:
1. Introduce students to the concepts of definition of rings, commutative rings, rings with identity with some examples.
2. Introduce students to the left (right) zero divisors, integral domain, subrings with some strong examples.
3. Introduce students to center of a ring with some properties, examples.
4. Introduce students to the nth power a^n and negative powers of a with some properties.
5. Introduce students to characteristic of a ring, some properties, examples, related with integral domain and finite integral domain.
6. Introduce students to the set Z1 of integral multiples of the identity with some properties.
7. Introduce students to the ideals and their operations.
8. Introduce students to the map (X, R), where X is non-empty and (R, +, .) is a ring with some properties ans examples.
9. Introduce students to the matrix ring with some properties, ideal generates by a subset, finitely generated ideal, principal ideal, n-fold sum of a with some properties and examples, principal ideal ring.
10.Introduce students to the sum of ideals for finite and infinite with some properties, Internal direct sum with some properties, product of n ideals with some properties.
11.Introduce students to the right (left) quotient of two ideals with several properties and examples.
12. Introduce students to the regular rings with some properties and examples, Boolean ring, homomorphic image, endomorphism.
13. Introduce students to the evaluation homomorphism at a point with some properties.

14. Several others concepts about rings and ideals

12. Student's obligation

In this course, every lecture we review all topics with students which we gave in the previous lecture and when we teach, we try to contribute all students.

13. Forms of teaching

We use data-show with white board and give a copy of my lecture about Non-commutative Algebra for all students step by step.

14. Assessment scheme

Your course grade will be determined as follows:

Midtearm examination: 20%

Seminar: 5%

Article review: 15%

Quiz: 5%

Homework: 5%

Final Exam: 50%

15. Student learning outcome:

In first year in our Department of Mathematics, students studies Foundation of Mathematics, which included set theory and functions. In this course we discuss Non-commutative Algebra which is applications of set theory and functions step by step to understand them.

16. Course Reading List and References:

[1] David M. Burton, A first course in Rings and Ideals, Addison-Wesley Publishing Company, 1970.

[2] McCoy N. H. and Berger T. R., Algebra: Groups, Rings and Other Topics, Allyn and Bacon, Inc. Boston London Sydney Toronto, 1977.

[3] Fraleigh J. B., A first Course in Abstract Algebra, ADDISON-WESLEY PUBLISHING COMPANY, 1982.

[4] Herstei I. N., Topics in Algebra, JOHN WILEY & SONS New York Chichester Brisbane Toronto Singapore, 1975.

[5] Allenby R B J T, Rings, Fields and Groups, Edward Arnold, 1983.
[6] Dummit D. S. and Foote R. M., Abstrat Algebra, John Wiley & Sons, Inc., 2003.
[7] Singh S. and Zameeruddin Q., Modern Algebra, VIKAS PUBLISHING HOUSE PVT LTD, 1972.
[8] Durbin J. R., Modern Algebra, JOHN WILEY & SONS
New York Chichester Brisbane Toronto Singapore, 1985.

[9] Gallian J. A., Contemporary Abstract Algebra, HOUGHTON MIFFLIN COMPANY, 1998.

17. The Topics:	Lecturer's name
Non-Commutative Algebra **Chapter 1: Introduction to Rings** • Subring and Ideals • Quotient and Homomorphism Rings • Prime Ideals with Examples • Primary Ideals with Examples • Maximal Ideals with Examples **Ch 2: Introductory Concepts** 2.1 Definition of rings. 2.2 Finite ring. 2.3 Commutative ring, A ring with unity, invertible element (unit element), two-sided inverse, multiplicative inverse. 2.4 Definition of symbol R* and the system (R*, .) forms a group. 2.5 Examples of rings. 2.5.1 The set of integers, rational and real numbers are all examples of rings. 1.5.2 (P(X), $\cap$, $\cup$) forms a commutative ring with identity. 2.5.3 (M, +,.) (resp., Mnn, +, .))is a ring with identity with no commutative .	Non-Commutative Algebra (3 hours per week)

2.6 Definition of left (right) zero divisor with example. 2.7 Q(H. W.). Give an example of zero divisor for non-commutative ring. 2.8 A result of non-zero divisor interms of cancellation law. 2.9 Integral domain and subrings with some new examples. 2.10 Center of a ring with some results. 2.11 The nth power of an element of a ring with some results. 2.12 Characteristic of a ring with examples. 2.13 A characterization of characteristic of a ring. 2.14 Order of a group 2.15 Definition of Char R with some results. 2.16 A result which connection between finite integral domain with Char R. 2.17 Definition of Z1 with some excercises. **Ch 3: Ideals with their operations.** 3.1 Two-sided ideal 3.2 (a), the set consisting of integral multiples of a. 3.3 Subrings and ideals for non-commutative rings. 3.4 Let $X \neq \phi$ and (R, +, .) be a ring. The definition of notation (X, R). 3.5 The definition of the sum and product of f and g. 3.6 We show that ((X, R), +,.) is a ring. 3.7 For a fixed element x, we denote and define Ix, the set of all mappings which take on the value zero 0 at x. 3.8 More generally, define an ideal of (X, R). 3.9 Two results about the ideal which defined in 3.8. 3.10 The definition of simple ring. 3.11 Q: Show that the matrix ring Mn(R) is a simple ring, where Mn(R)	

is n×n matrices over the real numbers.

3.12 Consider an arbitrary ring R and a non-empty subset S of R. Define the symbol (S), the collection of ideals which contains S.

3.13 The definition of finitely generated ideal.

3.14 The definition of principal ideal.

3.14.1 The principal right ideal, denoted by (a)r.

3.15.2 Likewise for the principal left ideal, denoted by (a)l.

3.16.3 The two-sided ideal (a) generated by a.

3.17.4 Q/ Show that the set of elements of the form ar+na is a right ideal of R.

3.18.5 We take some concepts about principal ideal.

3.19 The definition of principal ideal ring.

3.20 The ring of integer Z is a principal ideal ring.

3.31 Definition of the sum of finite number of ideals I1, I2, …, In.

3.32 A result about finite number of ideals.

3.33 The definition of the sum of two ideals.

3.34 More generally, let {Ii}be an arbitrary index collection of ideals of R. The sum of this collection Σ Ii.

Ch 4: More on Ideals with their operations.

4.1 The definition of internal direct sum of n ideals.

4.2 A characterization of internal direct sum of n ideals.

4.3 The definition of the product of two ideals.

4.4 A result for the product of two ideals.

4.5 A remark about the product of two ideals.

4.6 A special case where all the ideals are equal to the ideal I.

4.7 If I is a right ideal and S a non-empty subset of the ring R, define SI and it is a right ideal of R.

4.8 Let I and J be two ideals of a ring R. The right (left) quotient of I and J , denoted by the symbol I :r J (I :l J). 4.9 We show that I :r J is an ideal of R. 4.10 Likewise we can prove I :l J is an ideal of R. 4.11 Some results on 3.43. 4.12 Q/ If I is an ideal of the ring R and J is an ideal of I, then J need not be an ideal of the ring R, in general. 4.13 A condition which will ensure that J is an ideal of R is to take R to be a regular ring. 4.14 Definition of regular ring. 4.15 Some examples about regular rings interms of fields and Boolean rings. 4.16 A result on regular rings. 4.17 Definition of homomorphic mapping and the homomorphic image of a ring. 4.18 Definition of endomorphism, isomorphism and automorphism of a ring. 4.19 Definition of hom(R, R′) and hom(R, R), where R and R′ are two rings. 4.20 Definition of End(R), where R is a ring. 4.21 Some examples on homomorphim ring. 4.22 In the ring map(X, R), define $\tau\alpha$ and we show that $\tau\alpha$ is a homomorphism from map (X, R) into R. 4.23 Some properties of homomorphim ring ingeneral and interms of subrings and ideals. 4.24 Definition of kernel of a homomorphim ring. 4.25 Some important remarks on homomorphim ring. 4.26 A result on homomorphim ring, which connection between Ker(f) with an ideal.	

4.27 Correspondence Theorem. 4.28 Imbedded ring with examples. 4.29 Dorroh Extension Theorem 4.30 A result on homomorphim ring, which connection between Cent (R) with an ideal. 4.31 Let R1, R2, …, Rn be a finite number of rings. There is another type of direct sum is called external direct sum. 4.32 A remark on external direct sum. 4.33 Internal direct sum with some remarks. 4.34 Definition of direct summand with some remarks. **Ch 5: The Classical Isomorphism Theorems** 5.1 Equivalence class of an element of a ring with some properties. 5.2 natural map with some properties. 5.3 Equivalent between Zn with Z / (n) with examples and application on it. 5.4 Factorization of Homomorphism 5.5 Fundamental Isomorphism Theorem with examples. 5.6 the identity map I sub z. 5.7 Several properties on homomorphism, quotient ring, Kernel of a function. 5.8 Some properties related to 5.5 5.9 Second Isomorphism Ring Theorem with some examples 5.10 nil ideal, nilpotent ideal, examples, relation between them, some properties. **Ch 6: Integral Domains and Fields** 6.1 Definition of fields with examples, division rings and the relation between them.	

6.2 Standard example of division ring. 6.3 Some new examples on fields 6.4 Some properties related between field with an integral domain. 6.5 Relatively prime elements with some properties. 6.6 The relation between the field with n is a prime number in the ring Zn. 6.7 An appication on 6.6. 6.7 Euler Phi-function, example, some properties. 6.8 Euler-Fermat Theorem. 6.9 Subfields, examples, some properties. 6.10 Some equivalent statements od subfields. 6.11 A classical ring of quotients of a ring with some properties and construct it, some properties. **Ch 7: Some more other concepts**	
18. Practical Topics (If there is any)	

19. Examinations:

Q;/ What is the relation between nil ideal and nilpotent ideal ?.
Q:/ What is mean by principal ideal ring. Explain by an example.
Q:/ Define the right (left) quotients of two ideal. Then write 4 properties of it.

2. True or false type of exams:

We use this type in exam.

3. Multiple choices:

We cannot use this type of exams.

20. Extra notes:

Also we have some applications of general topology and combine it with biology and computer science such as digital topology, digital line and digital n space.

21. Peer review