Salahaddin Univ.-Erbil	**College of Science**	**Abstract Algebra**
Dept. of Math.	**(2023-2024)**	**Third Class**
First Trail	**First Semester**	**Time: 75 minutes**

Remark: Each question carry (4) marks

Q1/ Define "Normal subgroup". Let (H, *) be a subgroup of a group (G, *). Show that (H, *) is normal **iff** $a*H*a^{-1} \subseteq H, \forall a \in G$.

Q2/ (i) State and prove "Lagrange Theorem". **(3+1) marks**

(ii) Let (G, .) be a finite group and let (H, .) and (K, .) be subgroups of G such that $K \subseteq H \subseteq G$. Show that [G:K] = [G:H].[H:K].

Q3/ Write "**True**" or "**False**" of the following:

Let (G, *) be a group and (H, *) be a subgroup of G.

(1) $K_1 = \{e, a, b\}$ is a subgroup of the Klein group.

(2) Every group of prime order is cyclic.

(3) $H \nabla G$ if [G:H] = 2, where ∇ is normal.

(4) If gcd (r, n) = 1, then r is generator of **Z**, where n is a fixed positive integer and **Z** is the set of integers.

(5) Every subgroup of a cyclic group is cyclic.

(6) Dihedral group is abelian for n > 2.

(7) If order of a cyclic group is prime, then order of any element of this group is equal to the order of the group.

(8) Let (G, *) be a group. Given $a^2 = e, \forall a \in G$, then G is cyclic.

Q4/ If the equation $x^2 \equiv a \mod n$ has a solution x_1, show that $x_2 = n - x_1$ is also a solution.

Q5/ If (H, *) is a subgroup of the group (G, *) and (K, *) is a subgroup of (H, *). Show that (K, *) is also a subgroup of (G, *).

Good Luck

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