Salahaddin Univ.-Erbil	**College of Science**	**Group Theory**
Dept. of Math.	**(2023-2024)**	**Third Class**
First Trail	**Final Exam (First Semester)**	**Time: 2 hours**

Q1/ (i) Define "Isomorphic between two groups". Show that $(\mathbf{Z}, +) \cong (\mathbf{Z}_e, +)$, where $\mathbf{Z}$ is the set of integers and $\mathbf{Z}_e$ is the set of even integers. (7+10) marks

(ii) Define "Quotient group". Show that quotient group satisfies all properties of groups.

Q2/ (i) Let (G, *) be a group. Show that G is commutative **iff** $(a*b)^{-1} = a^{-1}*b^{-1}$, for each $a, b \in G$. (9+10) marks

(ii) What is mean by Dihedral group ? Explain.

Q3/ (9+7) marks

(i) Suppose $a \in \mathbf{R} \setminus \{0, 1\}$ and let $G = \{a^k : k \in \mathbf{Z}\}$, where **R** is the set of real numbers and **Z** is the set of integer numbers. Is (G, .) a group or not ? Explain.

(ii) Define "Homomorphism group". Let $f: (G, *) \rightarrow (G', .)$ be a group homomorphism. If H is a subgroup of G, then show that $f(H)$ is a subgroup of G′.

Q4/ Write "**True**" or "**False**" of the following: (8) marks

(1) If (H, *) is a subgroup of the group (G, *) and $\phi \neq K \subseteq G$, then $H*K \subseteq H$ implies $K \subseteq H$.

(2) Let (H, *) be s subgroup of index 2 in the group (G, *), then (H, *) is a normal subgroup.

(3) The original motivation for group theory was the quest for solutions of polynomial equations of degree higher than 3.

(4) Let n be a fixed positive integer and a, b be arbitrary integers. If $a \equiv b \bmod (n)$, then $a^k \equiv b^k \bmod (n)$.

(5) Let (G, *) be a group. Then, the order of an element $x \in G$ is the positive integer n such that $x^n = e$.

(6) Let (G, *) be a group. Every subgroup of the Z(G) is normal.

(7) Let (G, *) be a group, then G is commutative **iff** [a, b] = e.

(8) Let n be a fixed positive integer and a, b be arbitrary integers, then $a \equiv b \bmod (n)$ **iff** a and b have different remainder when divided by n.

Good Luck

Prof. Dr. Abdullah M. Abdul-Jabbar