Salahaddin Univ.-Erbil	**College of Science**	**Group Theory**
Dept. of Math.	**(2023-2024)**	**Third Class**
Second Trail	**Final Exam (First Semester)**	**Time: 2 hours**

Q1/ (i) Define "The natural map". Show that the natural map is homomorphism group.

(ii) Let (G, *) be a group. Define "Centre of a group, Z(G)". Show that (Z(G), *) is a normal subgroup of G. (6+9) marks

Q2/ (i) Let (G, *) be a group. Given $a^2 = e$, $\forall\ a \in G$, then show that G is abelian, where e is the identity element of G. (9+9) marks

(ii) What is mean by Klein group ? Explain.

Q3/ (i) Show that every quotient group of a cyclic group is cyclic. (10+9) marks

(ii) Define "Partition of a set". If (H, *) is a subgroup of the group (G, *), then show that the left (right) cosets of H in G form a partition of the set G.

Q4/ Write "**True**" or "**False**" of the following: (8) marks

(1) Given that (H_1, *) and (H_2, *) are both normal subgroups of the group (G, *), then the subgroup ($H_1 \cap H_2$,*) is also normal.

(2) If the equation $x^2 \equiv a \bmod (n)$ has a solution x_1, then $x_2 = n - x_1$ is also a solution.

(3) The concept of a group a rose from the study of polynomial equations, starting with Cayley in the 1830.

(4) To find the negative elements of cyclic group, we use the following formula:
$a^{-i} = a^{i-n}$, where n is the order of the group.

(5) Let (G, *) be a group and a, b $\in$G. The commutator of a and b in G as follows:
$[a, b] = a^{-1}*b^{-1}*a*b$.

(6) A group G is said to be simple if it has non-trivial normal subgroup.

(7) ($\mathbf{Z}_n$, $+_n$) is an abelian group, where $\mathbf{Z}_n$ is the set of integers modulo n.

(8) Let (G, *) be a group, then
End (G) = {f: G→G, f is isomorphism}.

Good Luck

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