Salahaddin Univ.-Erbil	**College of Science**	**Ring Theory**
Dept. of Math.	**(2023-2024)**	**Third Class**
First Trail	**Second Semester**	**Time: 75 minutes**

Q1/ (3) marks

Let (G, .) be a group, $a \in G$. Define "σ_a". Show that σ_a is an isomorphism.

Q2/ (4) marks

State and prove "First Isomorphism Group Theorem".

Q3/ (5) marks

Define "Integral Domain". What is the relation between Integral domain with field? Prove it. For the converse part, give an example. Under what condition the converse part is true? Prove it.

Q4/ (4) marks

Write "**True**" or "**False**" of the following:

(1) The definition of "principal ideal" is true only for commutative rings.

(2) The center of a ring R, C(R) is a subring of R.

(3) Three is the only idempotent element in $\mathbf{Z}_6$.

(4) A commutative division ring is a ring with identity such that each nonzero element has an inverse.

(5) The characteristic of an integral domain (R, +, .) is either zero or prime number.

(6) In a field (F, +, .), the equation $a^2 = a$ implies either $a = 0$ or $a = 1$.

(7) A ring (R, +, .) is commutative **iff** $(a+b)^2 = a^2 + 2(a.b) + b^2$, for every pair of elements $a, b \in R$.

(8) Let $(\mathbf{Z}, +)$ and $(\mathbf{Z}_n, +_n)$ be two groups, then $\mathbf{Z}_n / (n) \cong \mathbf{Z}$.

Q5/ (4) marks

Let I be an ideal of a commutative ring R. Define the radical of I, . Show that is an ideal of R.

Good Luck

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