Salahaddin Univ.-Erbil | **College of Science** | **First Semester**
Dept. of Math. | **(2023-2024)** | **MSc Exam.**
First Trail | **Non-commutative Algebra (Prof. Dr. Abdullah M.)** | **Time: 2 hours**

Q1/ (i) State and prove "Correspondence Theorem". **(12+8) marks**

(ii) Let I and J be two ideals of the ring R. Define "The right quotient of I by J, $I:_rJ$". Show that $I:_rJ$ is an ideal of R.

Q2/ **(8+12) marks**

(i) Show that the set of elements of the form ar+na is a right ideal of the ring R.

(ii) Let $I_1, I_2, \ldots, I_n$ be n ideals of a ring R. Show that $I_1+I_2+ \ldots +I_n = \langle I_1 \cup I_2 \cup \ldots \cup I_n \rangle$.

Q3/ (i) What is mean by **(12+8) marks**

(1) Internal direct sum (2) Let $\{I_i\}$ be an arbitrary indexed collection of ideals of a ring R, what is $\sum I_i$? (3) The evaluation homomorphism at a point. (4) n-fold sum of 1.

(ii) Let R be a commutative ring with unity. If every ideal of R is prime, then show that R is a field.

Q4/ (i) Suppose that R is an integral domain. Determine all the idempotent elements of R. **(6+6+8) marks**

(ii) Consider the map f: **C**→**C** such that f (a+ib) = a-ib. Is f a ring homomorphism or not ? Explain, where **C** = complex numbers.

(iii) Write "**True**" or "**False**" of the following:

(1) In a ring (R, +, .) with identity. No divisor of zero can posses a multiplicative inverse.

(2) Let (R, +, .) be a ring which has the property that $a^2 = a$, for every $a \in R$. Then, (R,+, .) is commutative ring.

(3) In an integral domain the zero element is the only idempotent.

(4) The ring of real numbers (**R**, +, .) is a simple ring.

(5) Given that (I, +, .) is an ideal of the ring (R,+, .). If (R,+, .) is a principal ideal ring, then so is the quotient ring (R / I,+, .).

(6) If R is any ring with identity 1, then R has characteristic n > 0 **iff** n is the positive integer for which n1 = 0.

(7) $I:_r(JK) = (I:_rK):_rJ$, where I, J, K are ideals in R.

(8) If I is an ideal of the ring R and J is an ideal of I, then J is an ideal of R.