CHAPTER 3
Manipulator kinematics

4rth Year C/C

12 / 12/ 2018

CHAPTER OUTLINE

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3.7 FRAMES WITH STANDARD NAMES
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INTRODUCTION

- **Kinematics** is the science of motion that treats the subject without regard to the **forces** that cause it.
- Within the science of **kinematics**, one studies the position, the **velocity, the acceleration, and all higher order derivatives of the position variables.**

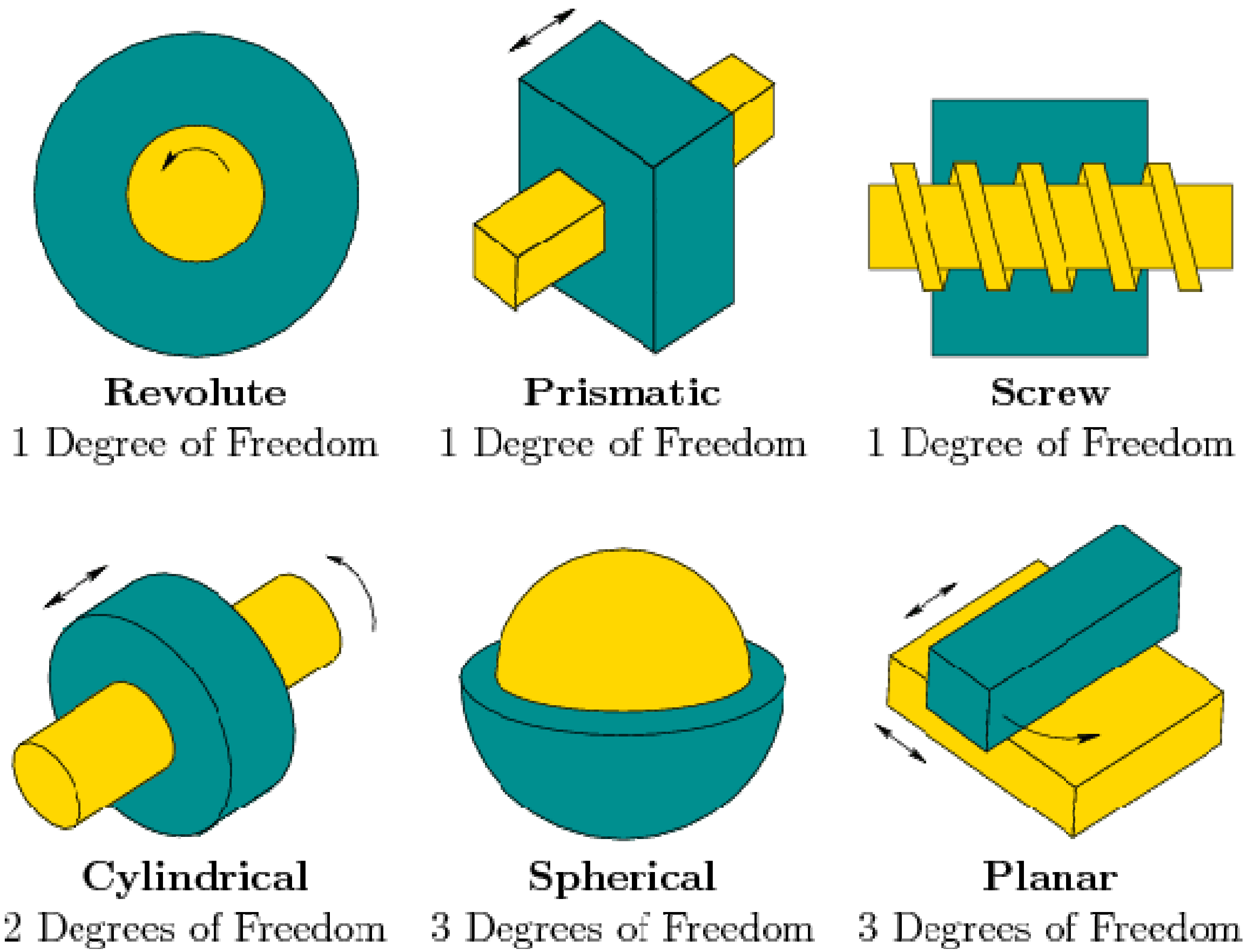
INTRODUCTION

Our focus is on a method to compute the position and orientation of the manipulator's end-effector relative to the base of the manipulator as a function of the joint variables

LINK DESCRIPTION

- A manipulator may be thought of as a set of **bodies** connected in a **chain** by **joints**.
- These **bodies** are called **links**.
- **Joints** form a **connection** between a neighboring **pair** of **links**.
- The term **lower pair** is used to describe the connection between a **pair of bodies** when the relative motion is characterized by two surfaces **sliding over one another**.

LINK DESCRIPTION



LINK DESCRIPTION

- Manipulators' generally being constructed from **joints** that exhibit just **one degree** of freedom.
- Most manipulators have **revolute joints** or have sliding joints called **prismatic joints**.
- In the mechanism is built with a joint having **n** degrees of freedom, it can be modeled as **n joints** of one degree of freedom connected with **n — 1 links** of zero length.

LINK DESCRIPTION

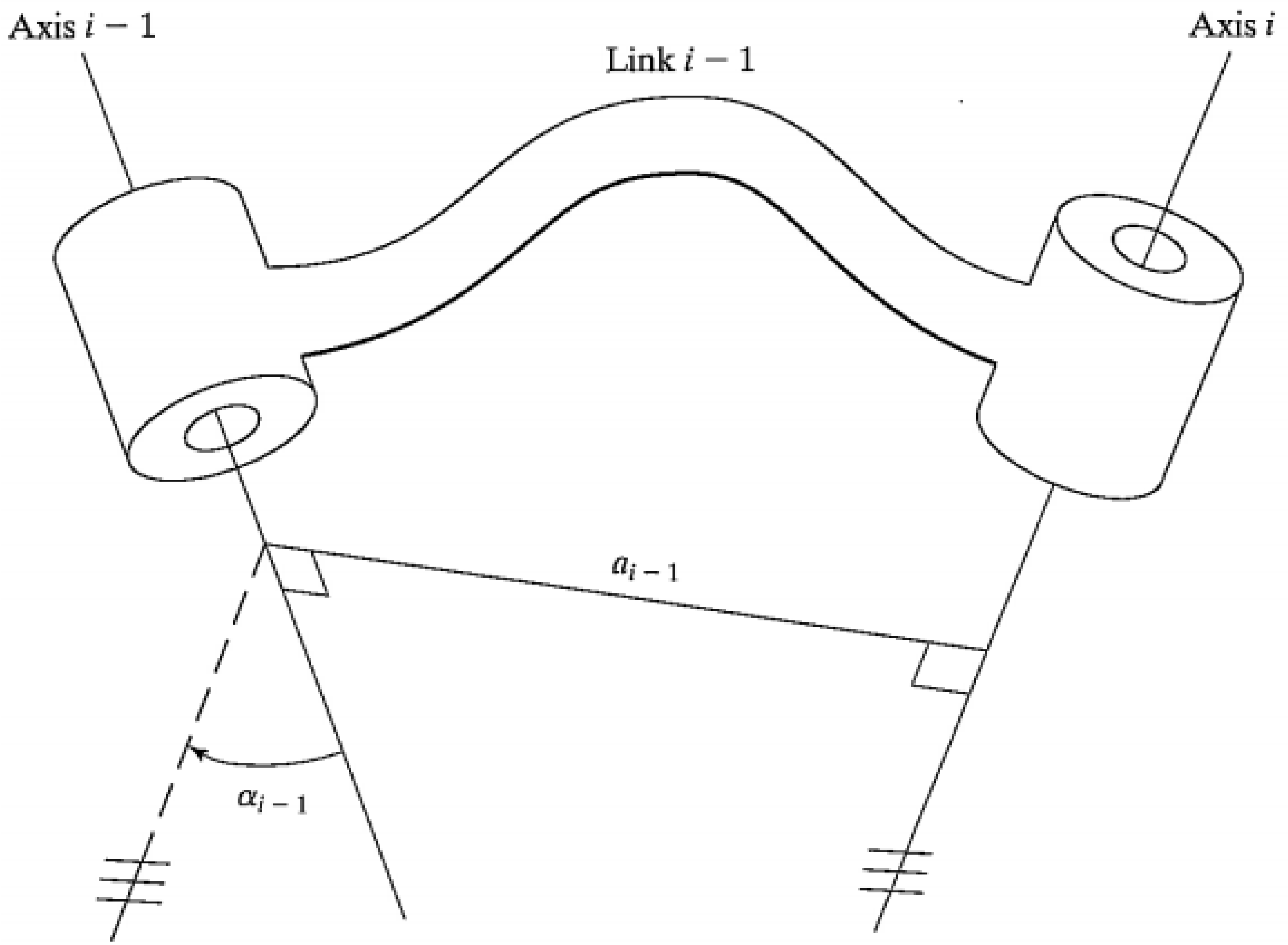
- The **links** are numbered starting from the immobile base of the arm, which might be called **link 0** .
- The first moving body is **link 1**, and so on, out to the **free end** of the arm, which is **link n**.
- In order to position an **end-effector** generally in **3-space**, a minimum of **six joints** is required.

because the description of an object in space requires six parameters—three for position and three for orientation.

LINK DESCRIPTION

- Typical manipulators have **five or six joints**.
- for the purposes of obtaining the **kinematic equations** of the mechanism, a **link** is considered only as a **rigid body** that defines the relationship **between two neighboring joint** axes of a manipulator.
- **Joint axes** are defined by lines in space.

LINK DESCRIPTION



LINK DESCRIPTION

The kinematic function of a link is to maintain a fixed relationship between the two joint axes it supports. This relationship can be described with two parameters: **the link length, a**, and the **link twist,** α.

Joint axis ***i*** is defined by a line in space, or a vector direction, about which link ***i*** rotates relative to link ***i*** **-1**.

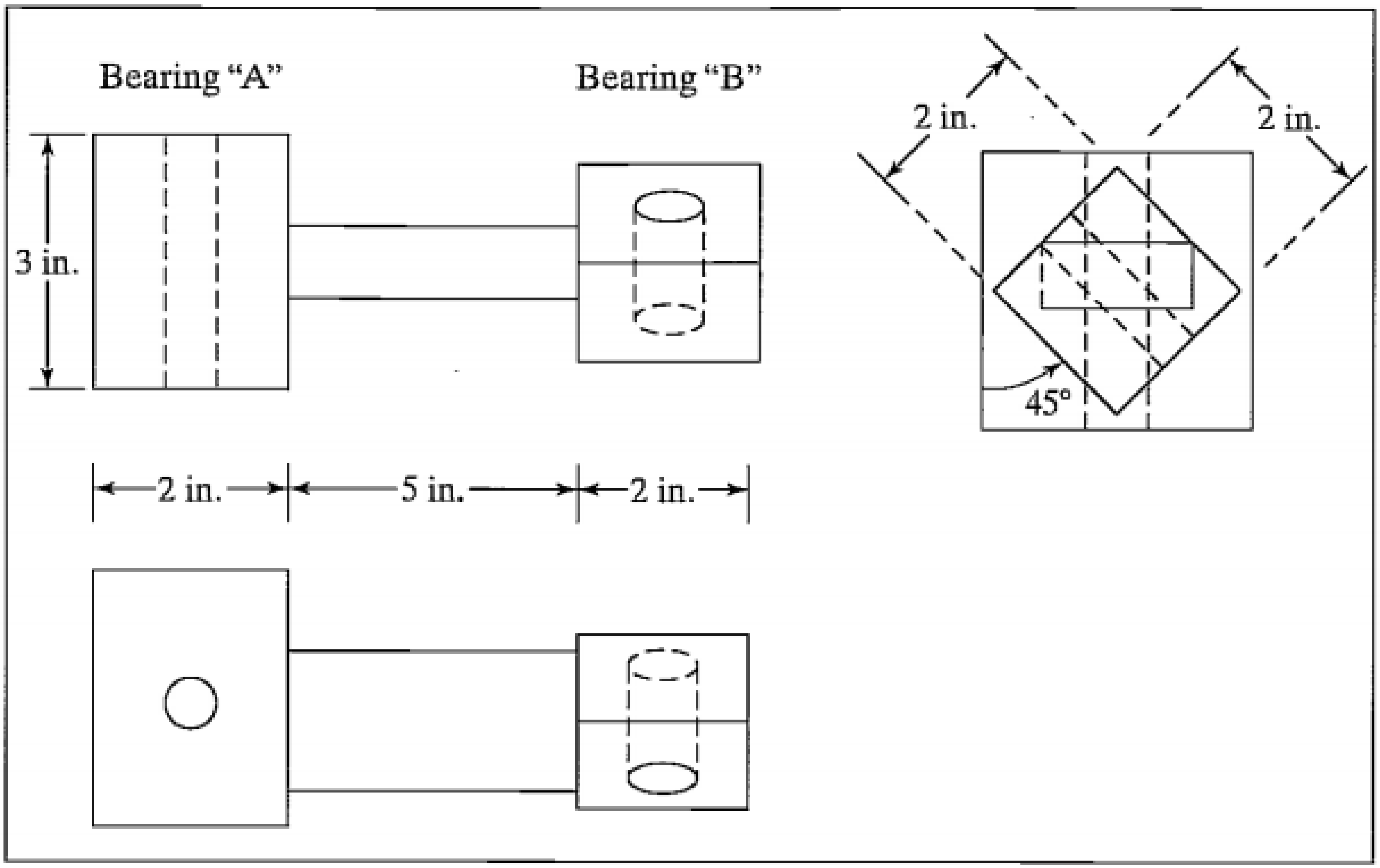
LINK DESCRIPTION

- For any two axes in 3-space, there exists a well-defined measure of distance between them.
- **This distance** is measured along a line that is mutually **perpendicular to both axes**.

LINK DESCRIPTION

- Link ***i — 1*** and the mutually perpendicular line along which the **link length (a_i -1)**, is measured.
- Another way to visualize the link parameter is to imagine an expanding cylinder whose axis is the joint ***i — 1*** axis—when it just touches joint axis ***i***, the radius of the cylinder is equal to **(a_i -1).**
- The **second parameter** needed to define the relative location of the two axes is called the **link twist.**

LINK DESCRIPTION

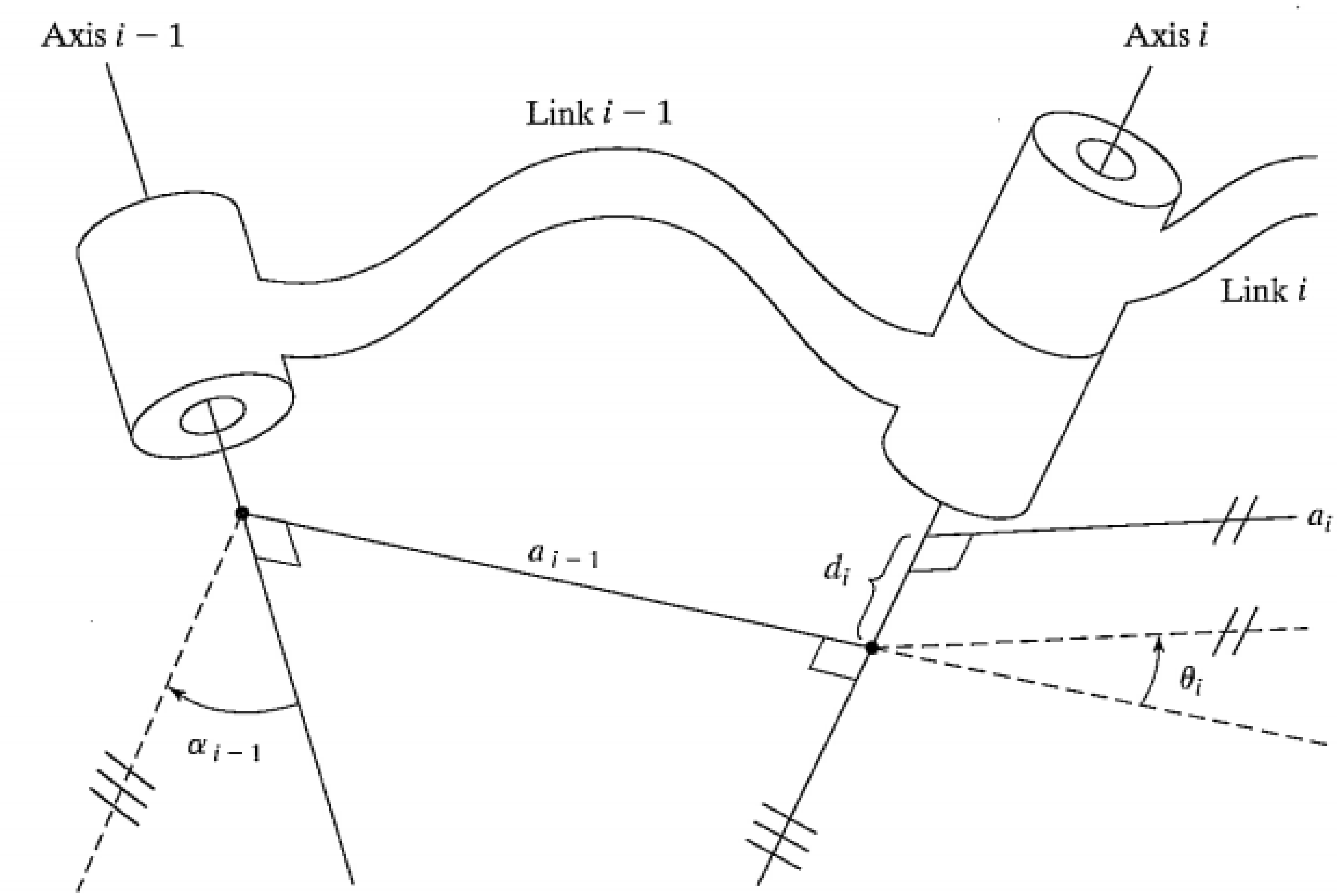


A simple link that supports two revolute axes

LINK-CONNECTION DESCRIPTION

- For the investigation of kinematics, we need only worry about two quantities, which will completely specify the way in which links are connected together.
- Neighboring links have a common joint axis between them.
- One parameter of interconnection has to do with the distance along this common axis from one link to the next. This parameter is called the link offset.

LINK-CONNECTION DESCRIPTION



LINK-CONNECTION DESCRIPTION

- The offset at joint axis ***i*** is called d_i .
- The second parameter describes the amount of **rotation** about this common axis between **one link and its neighbor**.
- This is called the joint angle, θ_i .
- The Figure in slide 16 shows the interconnection of link (***i-1)*** and link ***i.***

LINK-CONNECTION DESCRIPTION

- Recall that a_{i}_1 is the mutual perpendicular between the two axes of link i — 1.
- Likewise, a_i is the mutual perpendicular defined for link i.
- The first parameter of interconnection is the link offset, $\mathbf{d_i}$ which is the signed distance measured along the axis of joint **i** from the point where **(a_i -1)**, intersects the axis to the point where $\mathbf{a_i}$ intersects the axis.

LINK-CONNECTION DESCRIPTION

- The link offset is **variable** if joint **i** is **prismatic**.
- The second parameter of interconnection is the angle made between an extension of **(a_i - 1)**, and **a_i** measured about the axis of joint **i**.
- This parameter is named θ_i and is variable for a **revolute joint**.

First and last links in the chain

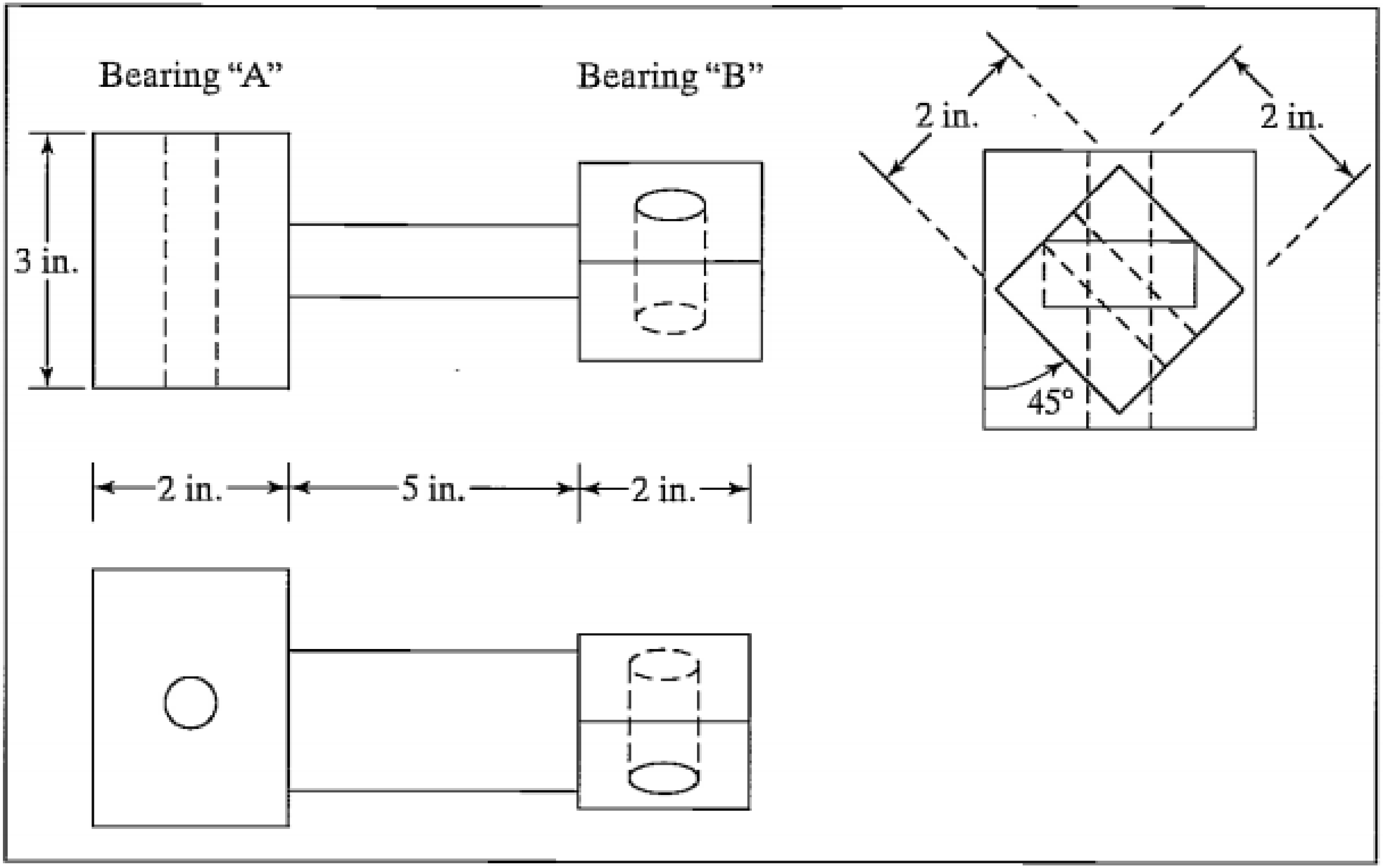
- Hence, any robot can be described kinematically by giving the values of **four quantities** for each link.
- Two describe the link itself, and two describe the link‘s connection to a neighboring link.
- In the usual case of a **revolute joint**, is called the **joint variable**, and the other three quantities would be fixed **link parameters**.

- For **prismatic joints**, d_i is the **joint variable**, and the other three quantities are fixed **link parameters**.
- The definition of mechanisms by means of these quantities is a convention usually called the **Denavit—Hartenberg** notation .
- At this point, we could inspect any mechanism and determine the **Denavit—Hartenberg** parameters that describe it

- For a six-jointed robot, 18 numbers would be required to describe the fixed portion of its kinematics completely.

Example

Figure below shows the mechanical drawings of a robot link. If this link is used in a robot, with bearing "A" used for the lower-numbered joint, give the length and twist of this link. Assume that holes are centered in each bearing.


Bearing "A"
Bearing "B"
3 in.
2 in.
5 in.
2 in.
2 in.
2 in.
45°

Solution

By inspection, the common perpendicular lies right down the middle of the metal bar connecting the bearings, so the link length is 7 inches. The end view actually shows a projection of the bearings onto the plane whose normal is the mutual perpendicular. Link twist is measured in the right-hand sense about the common perpendicular from axis i — 1 to axis i, so, in this example, it is clearly +45 degrees.

Example

Two links, as described in Fig. (Slide No. 25) , are connected as links 1 and 2 of a robot. Joint 2 is composed of a "B" bearing of link 1 and an "A" bearing of link 2, arranged so that the flat surfaces of the "A" and "B" bearings lie flush against each other. What is d2?

Solution

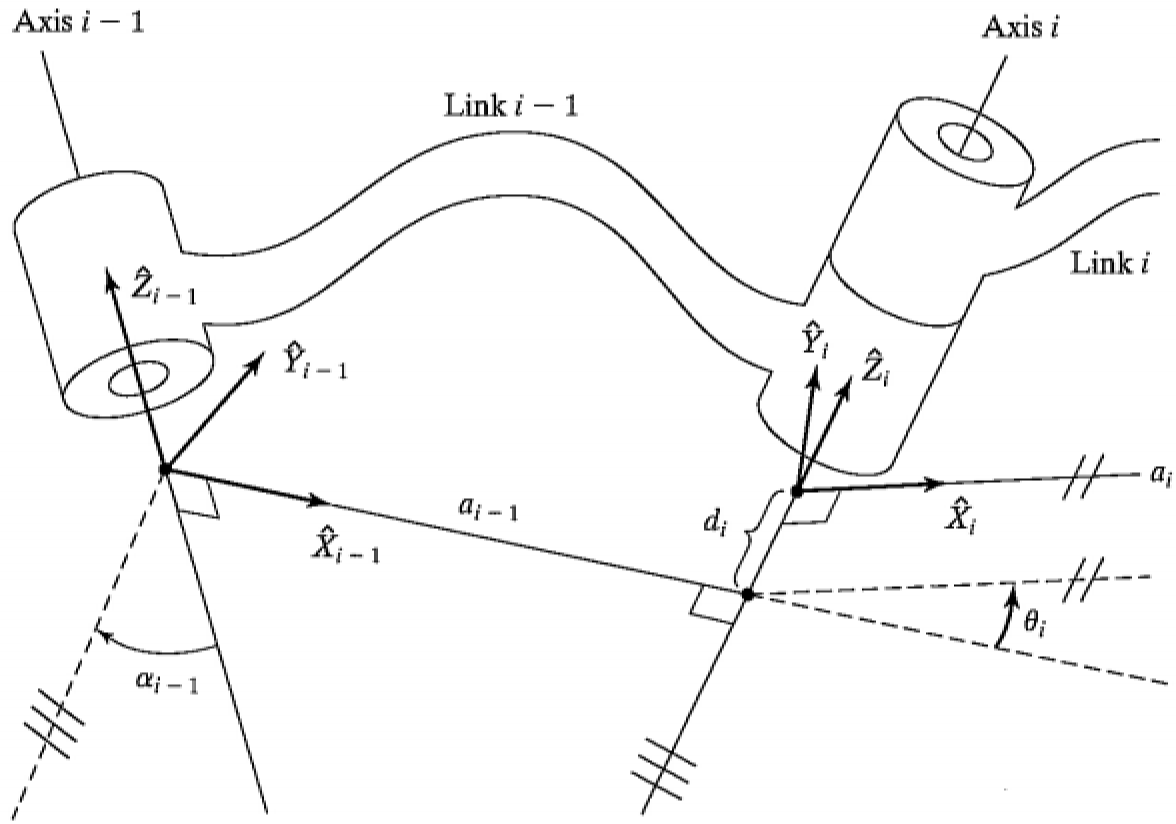
The link offset d2 is the offset at joint 2, which is the distance, measured along the joint 2 axis, between the mutual perpendicular of link 1 and that of link 2. From the drawings in Fig, this is 2.5 inches.

CONVENTION FOR AFFIXING FRAMES TO LINKS

- In order to describe the location of each link relative to its neighbors, we define a frame attached to each link.
- The link frames are named by number according to the link to which they are attached.
- frame {i } is attached rigidly to link i.

CONVENTION FOR AFFIXING FRAMES TO LINKS

The convention we will use to locate frames on the links is as follows:



CONVENTION FOR AFFIXING FRAMES TO LINKS

The convention we will use to locate frames on the links is as follows: The Z-axis of frame {i}, called Z_i , is coincident with the joint axis i.

The origin of frame {i} is located where the a_i perpendicular intersects the joint i axis.

X_i points along a_i in the direction from joint I to joint i + 1.

CONVENTION FOR AFFIXING FRAMES TO LINKS

- We attach a frame to the base of the robot, or link 0, called frame {0}.
- This frame does not move; for the problem of arm kinematics, it can be considered the reference frame.
- We may describe the position of all other link frames in terms of this frame.

CONVENTION FOR AFFIXING FRAMES TO LINKS

- Frame {0} is arbitrary, so it always simplifies matters to choose Z_0 along axis 1 and to locate frame {0} so that it coincides with frame {1} when joint variable 1 is zero.
- Using this convention, we will always have a_0 = 0.0, α_0 = 0.0. Additionally, this ensures that d_1 = 0.0 if joint 1 is revolute, or θ_1 = 0.0 if joint 1 is prismatic.

CONVENTION FOR AFFIXING FRAMES TO LINKS

- For joint n revolute, the direction of X_N is chosen so that it aligns with X_{N_1} when θ_n= 0.0, and the origin of frame {N} is chosen so that d_n= 0.0.
- For joint n prismatic, the direction of X_N is chosen so that θ_n = 0.0, and the origin of frame {N} is chosen at the intersection of X_{N-1} and joint axis n when d_n = 0.0.

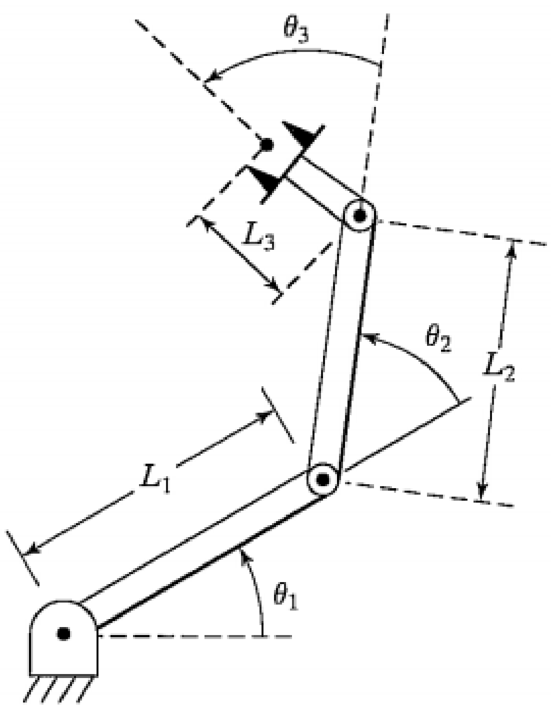
Summary of link-frame attachment procedure

The following is a summary of the procedure to follow when faced with a new mechanism, in order to properly attach the link frames:

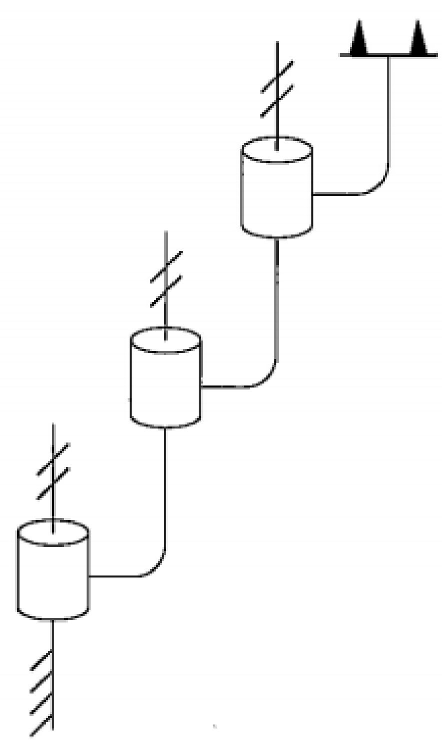
1. Identify the joint axes and imagine (or draw) infinite lines along them. For steps 2 through 5 below, consider two of these neighboring lines (at axes i and $i+1$).
2. Identify the common perpendicular between them, or point of intersection. At the point of intersection, or at the point where the common perpendicular meets the ith axis, assign the link-frame origin.
3. Assign the $\hat{Z}_i$ axis pointing along the ith joint axis.
4. Assign the $\hat{X}_i$ axis pointing along the common perpendicular, or, if the axes intersect, assign $\hat{X}_i$ to be normal to the plane containing the two axes.
5. Assign the $\hat{Y}_i$ axis to complete a right-hand coordinate system.
6. Assign $\{0\}$ to match $\{1\}$ when the first joint variable is zero. For $\{N\}$, choose an origin location and $\hat{X}_N$ direction freely, but generally so as to cause as many linkage parameters as possible to become zero.

EXAMPLE

Figure below shows a three-link planar arm. Because all three joints are revolute, this manipulator is sometimes called an RRR (or 3R) mechanism.



(a)



(b)

EXAMPLE

Note the double hash marks indicated on each of the three axes, which indicate that these axes are parallel.

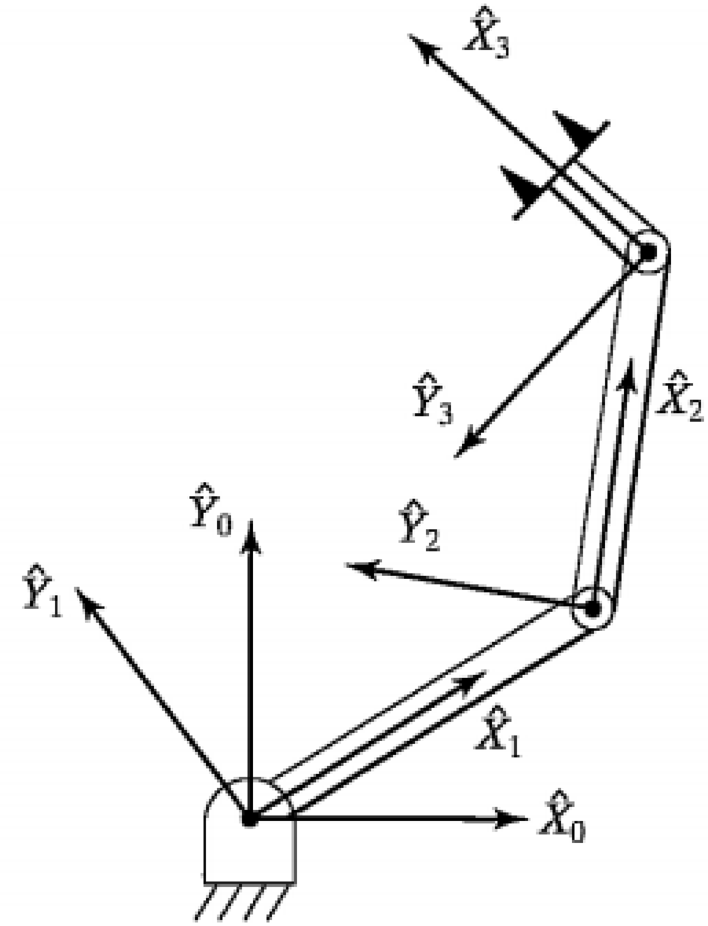
Assign link frames to the mechanism and give the Denavit—Hartenberg parameters.

EXAMPLE

- We start by defining the reference frame, frame {0}.
- It is fixed to the base and aligns with frame {i} when the first joint variable (θ_1) is zero.
- Therefore, we position frame {0} as shown in Fig below with Z_0 aligned with the joint-1 axis.
- For this arm, all joint axes are oriented perpendicular to the plane of the arm.

EXAMPLE

- Because the arm lies in a plane with all Z axes parallel, there are no link offsets—all d_i are zero.
- All joints are rotational, so when they are at zero degrees, all X axes must align.



Link parameters of the three-link planar manipulator.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3

Example

- Note that, because the joint axes are all parallel and all the Z axes are taken as pointing out of the paper.
- All α_i are zero.
- Note also that our kinematic analysis always ends at a frame whose origin lies on the last joint axis; therefore, L_3 does not appear in the link parameters. Such final offsets to the end-effector are dealt with separately later.

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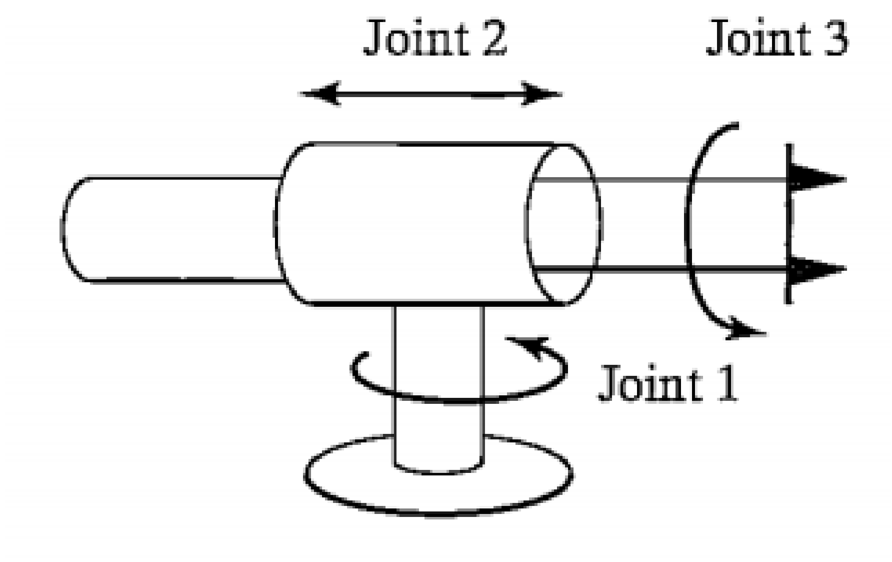
Example

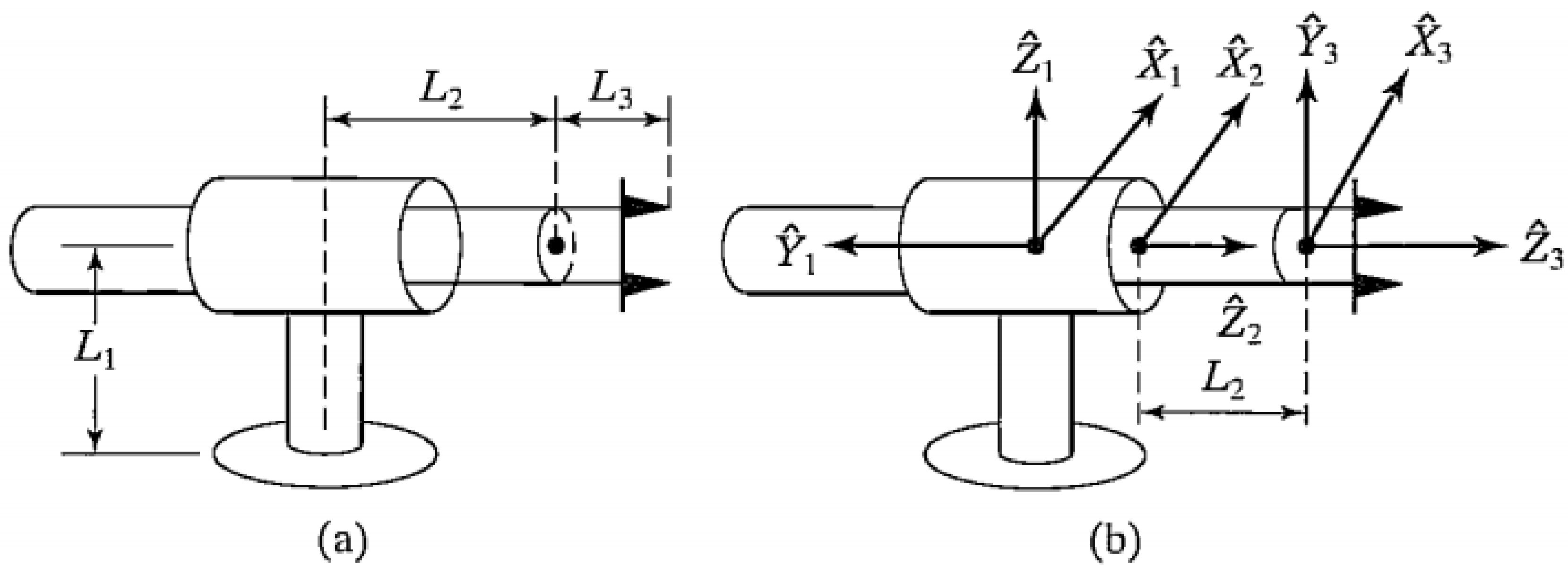
A robot having three degrees of freedom and one prismatic joint. Find

1 - D-H parameters.

2- Compute the individual transformations for each link



Example



L_2
L_3
L_1
(a)
$\hat{Z}_1$
$\hat{X}_1$
$\hat{X}_2$
$\hat{Y}_3$
$\hat{X}_3$
$\hat{Y}_1$
$\hat{Z}_3$
$\hat{Z}_2$
L_2
(b)

Example

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3

Example

General form of

$$ {}^{i-1}_{\ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

$$ {}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^1_2T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

Concatenating link transformations

Once the link frames have been defined and the corresponding link parameters found, developing the kinematic equations is straightforward. From the values of the link parameters, the individual link-transformation matrices can be computed. Then, the link transformations can be multiplied together to find the single transformation that relates frame [N} to frame {0}:

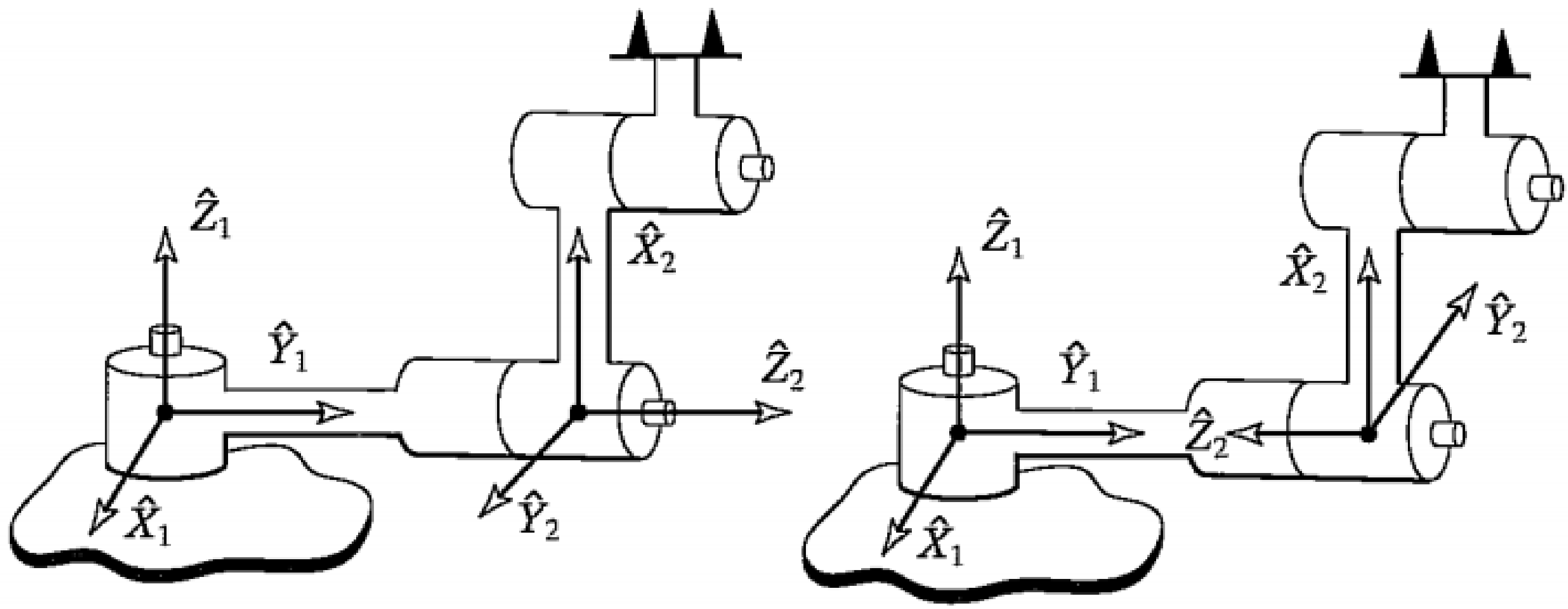
Concatenating link transformations

$$ {}^{0}_{N}T = {}^{0}_{1}T \, {}^{1}_{2}T \, {}^{2}_{3}T \ldots {}^{N-1}_{N}T. $$

This transformation, will be a function of all n joint variables .

If the robot's joint-position sensors are queried, the Cartesian position and orientation of the last link can be computed by above transformation equation.

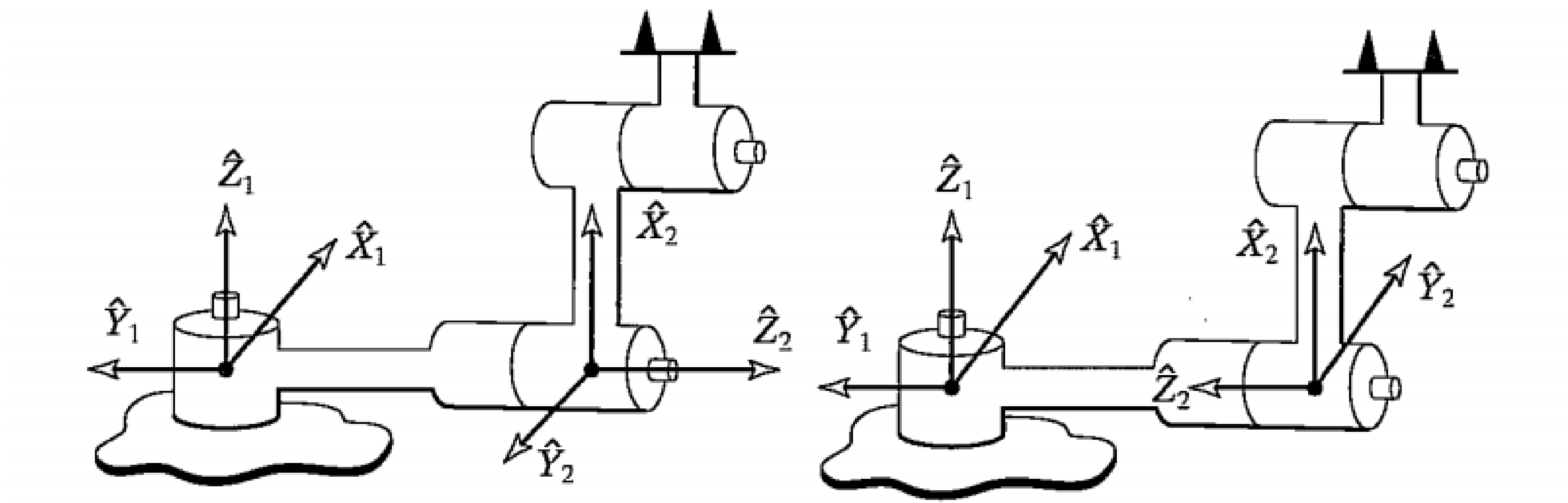
MANIPULATOR KINEMATICS



$a_1 = 0$	$a_2 = L_2$		$a_1 = 0$	$a_2 = L_2$	
$\alpha_1 = -90°$	$\alpha_2 = 0$	$\theta_2 = -90°$	$\alpha_1 = 90°$	$\alpha_2 = 0$	$\theta_2 = 90°$
$d_1 = 0$	$d_2 = L_1$		$d_1 = 0$	$d_2 = -L_1$	

Two possible frame assignments.

MANIPULATOR KINEMATICS



$a_1 = 0$ $\quad a_2 = L_2$

$\alpha_1 = 90°$ $\quad \alpha_2 = 0$ $\quad \theta_2 = 90°$

$d_1 = 0$ $\quad d_2 = L_1$

$a_1 = 0$ $\quad a_2 = L_2$

$\alpha_1 = -90°$ $\quad \alpha_2 = 0$ $\quad \theta_2 = -90°$

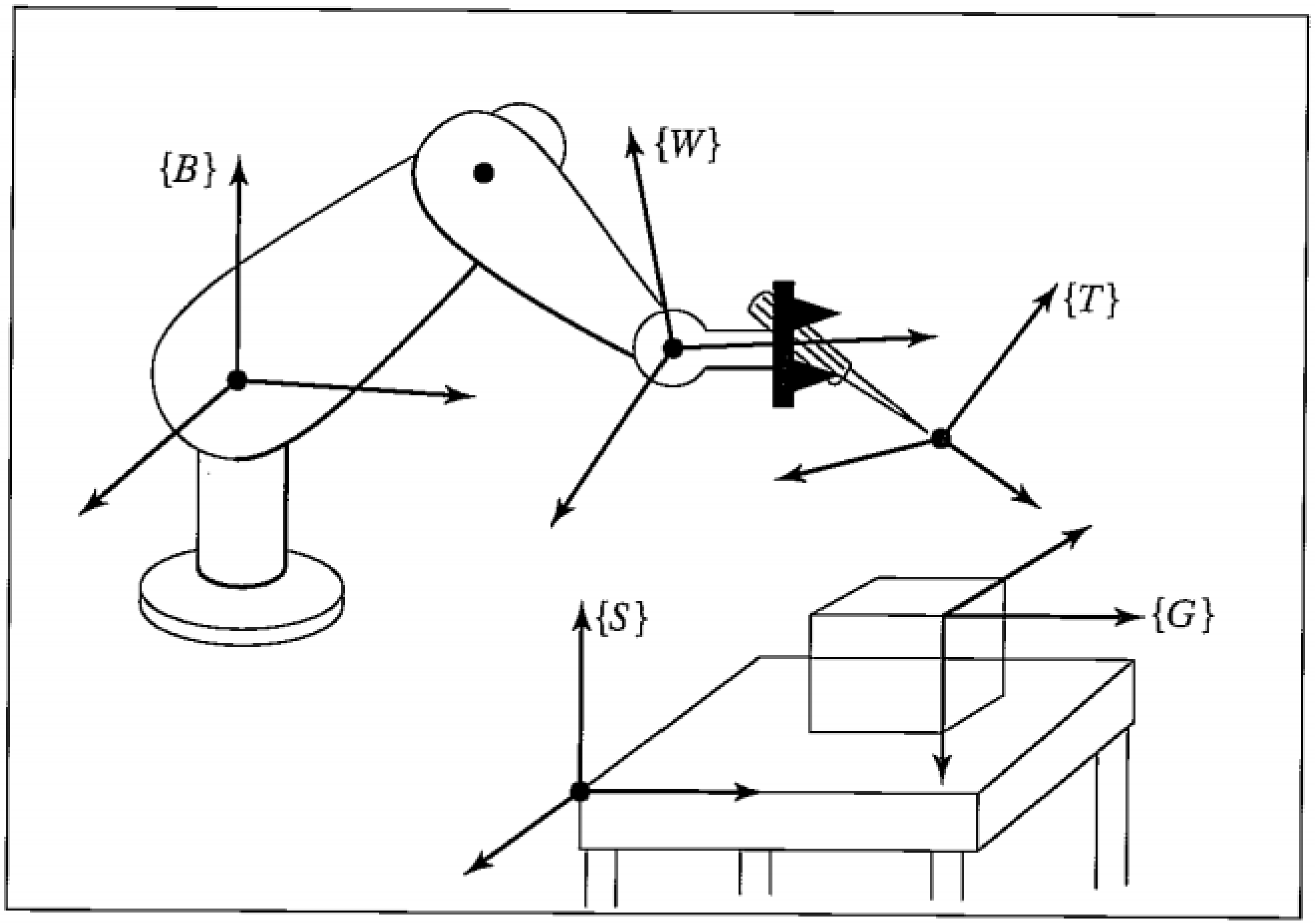
$d_1 = 0$ $\quad d_2 = -L_1$

Two possible frame assignments.

FRAMES WITH STANDARD NAMES

- As a matter of convention, it will be helpful if we assign specific names and locations to certain "standard" frames associated with a robot and its workspace.
- shows a typical situation in which a robot has grasped some sort of tool and is to position the tool tip to a user-defined location.

FRAMES WITH STANDARD NAMES



The base frame, {B}

{B} is located at the base of the manipulator. It is merely another name for frame {0}.

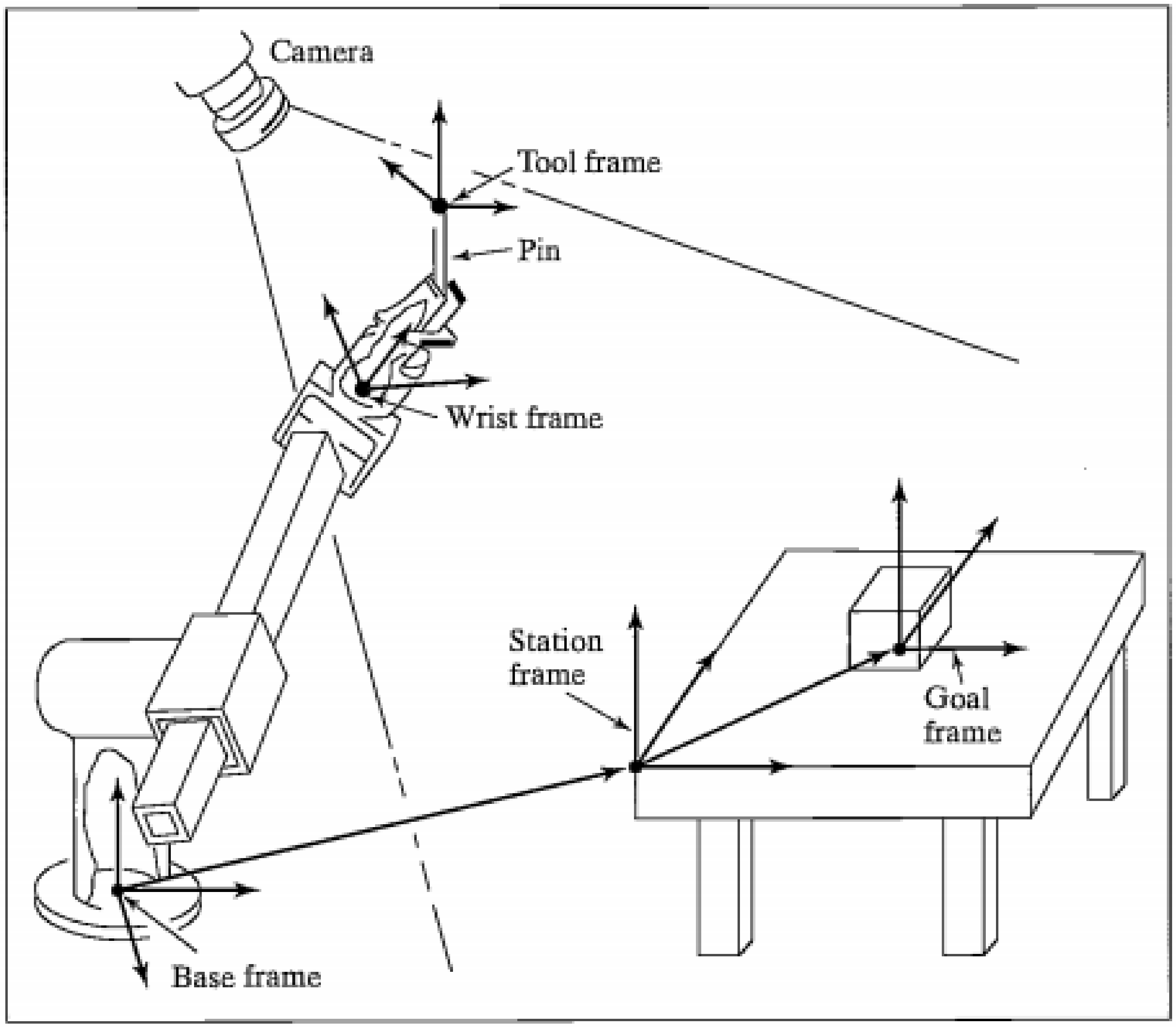
It is affixed to a nonmoving part of the robot, sometimes called link 0.

The station frame, {S}

- {S} is located in a task-relevant location.
- sometimes called the task frame, the world frame, or the universe frame.
- The station frame is always specified with respect to the base frame, that is,

$$ {}^{B}_{S}T. $$

The station frame, {S}



The wrist frame, {W}

- It is another name for frame {N}, the link frame attached to the last link of the robot.
- very often, {W} has its origin fixed at a point called the wrist of the manipulator, and {W} moves with the last link of the manipulator.
- It is defined relative to the base frame—that is, {W} $= {}^{B}_{W}T = {}^{0}_{N}T.$

The tool frame, {T}

- {T} is affixed to the end of any tool the robot happens to be holding.
- When the hand is empty, {T} is usually located with its origin between the fingertips of the robot. The tool frame is always specified with respect to the wrist frame.

The goal frame, {G}

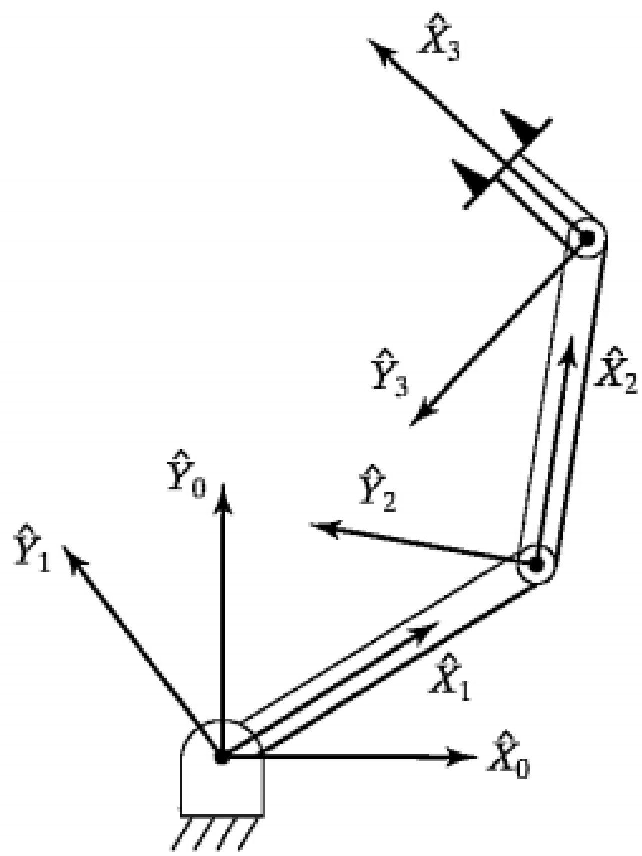
- {G} is a description of the location to which the robot is to move the tool.
- Specifically this means that, at the end of the motion, the tool frame should be brought to coincidence with the goal frame.
- {G} is always specified relative to the station frame.

WHERE IS THE TOOL?

$$ {}^{S}_{T}T = {}^{B}_{S}T^{-1} \; {}^{B}_{W}T \; {}^{W}_{T}T. $$

EXAMPLE

- Compute the kinematics of the planar arm



Solution

α_{i-1}	a_{i-1}	d_i
0	0	0
0	L_1	0
0	L_2	0

$$
{}^0_1T = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^1_2T = \begin{bmatrix} C_2 & -S_2 & 0 & L_1 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^2_3T = \begin{bmatrix} C_3 & -S_3 & 0 & L_2 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_3T = {}^0_1T\,{}^1_2T\,{}^2_3T = \begin{bmatrix} C_{123} & -S_{123} & 0 & L_1C_1 + L_2C_{12} \\ S_{123} & C_{123} & 0 & L_1S_1 + L_2S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$C_{123} = \cos(\theta_1 + \theta_2 + \theta_3)$

$S_{123} = \sin(\theta_1 + \theta_2 + \theta_3)$, etc.