University of Salahaddin-Hawler
College of Engineering
Chemical and Ptrochemical Department

Mathematics I

Lecturer: **Ahmed A. Zardoey**
M.Sc. **Mechanical Engineering**
Contact:
Email: Ahmed.ahmed3@su.edu.krd

Lecture 1.2.3: Revision

Reffrences

1) Thomas & Finney " Calculus and Analytic Geometry " (1988) , 7th edition , Addison Wesley.

2) Ford , S.R. and Ford , J.R. " Calculus " , (1963) McGraw-Hill.

3) J.K.Back house and S.P.T. Houldsworth " Pure Mathematics a First Course " (1979) , S1 Edition , Longman Group .

Chapter one

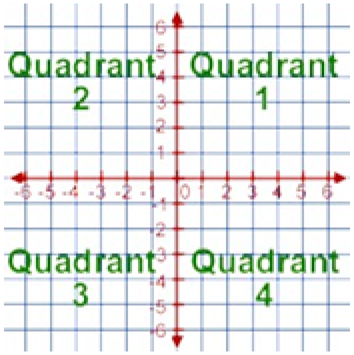
Revision

1-1 Coordinates and Graphs in the Plain:

Cartesian coordinate:- Two number lines, one of them horizontal [A horizontal line in the plane, extending indefinitely to the left and to the right is chosen as the *x-axis*] and the other vertical [A vertical line in the plane, extending indefinitely up and down, this line becomes the *y-axis*]. Each line is assumed to represent the real number.

The point where the lines cross is the *origin*. *This is termed by "O".*

You can locate any point on the coordinate plane by an ordered pair of numbers (x,y), called the coordinates.



Points in Quadrant 1 have positive x and positive y coordinates. Points in Quadrant 2 have negative x but positive y coordinates. Points in Quadrant 3 have negative x and negative y coordinates. Points in Quadrant 4 have positive x but negative y coordinates.

1-2 The Slope of a line:

Increment

When particle moves from one point (x_1, y_1) to (x_2, y_2), the increments are:

$\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$

Example

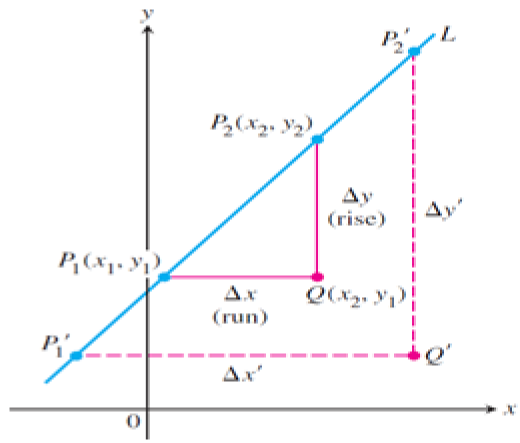
- Find the net changes in coordinates, when particle move from (0,2) to (3,- 1).

- If a particle starts at S (-2, 3) and its coordinates receive increments $\triangle x$ = 5 and $\triangle y$ = -6, what will its new position be? Ans. D (3,-3)

Slopes of nonvertical line

The slope of the straight line is the ratio of rise to run, so when point $p_1 (x_1, y_1)$ and $p_2 (x_2, y_2)$ are points on a non-vertical line L, the slope of L is:

$$slope = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



***Notes*:**

- Any non-vertical line in the plane has the same value for every choice of the two points $P_1(x_1, y_1)$ and P_2 (x_2, y_2) on the line (figure 2). This is because the ratios of corresponding sides for similar triangle are equal.
- A line that goes uphill as x increases has a positive slope. A line that goes downhill as x increases has a negative slope.

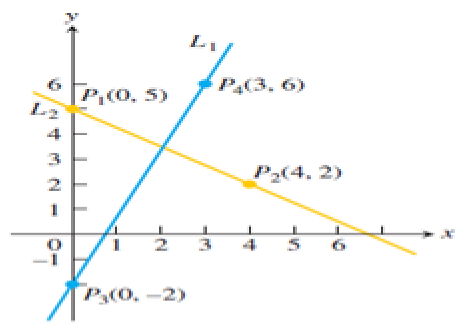
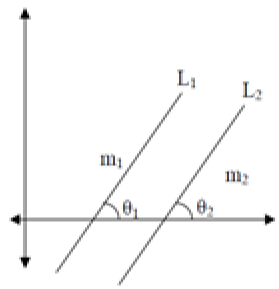
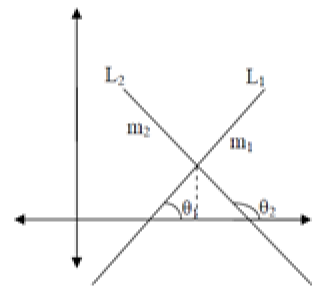


FIGURE 1.9 The slope of L_1 is
$$m = \frac{\Delta y}{\Delta x} = \frac{6 - (-2)}{3 - 0} = \frac{8}{3}.$$

- If $y_1=y_2$ or $\Delta y = 0$ (A horizontal line) then the line has zero slope.
- The slope of a vertical line is undefined (no slope) because $\Delta x=0$
  {we cannot evaluate the slope ratio (m)} ***the denominator is zero***
- Parallel lines have the same slope.
- For two perpendicular lines, like L_1 and L_2 of their slope m_1 and m_2 are related by the equation: $m_1 . m_2 = -1$

Parallel and Perpendicular Lines





If $L_1//L_2$ then $\theta_1=\theta_2$ and $m_1=m_2$

If $L_1 \perp L_2$ then $m_1 = -1/m_2$

Example

Line one goes through the origin and is parallel to the line through (-2, 3) and (4, -5). Find the slope of the line one.

Sol.

Example

Line one contains the point (1,6) and is perpendicular to the line through (-4, 1) and (3, -2). Find the slope of the line one.

Sol.

Angle of inclination

The angle of inclination of a line that crosses the x-axis is the smallest counterclockwise angle from the x-axis to the line (figure 4).

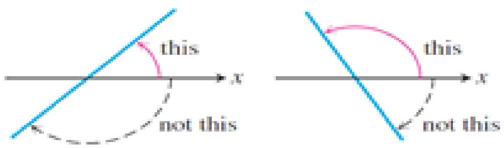
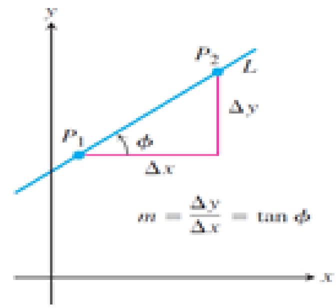


FIGURE 1.10 Angles of inclination are measured counterclockwise from the x-axis.

The inclination of horizontal line= 0°

The inclination of vertical line= 90°

If ϕ (phi) is the inclination of a line, then $0 \leq \phi \leq 180$, the relationship betwe the slope m of a non-vertical line and the lines angle's of inclination Φ is shown in figure 5.



__Example:__

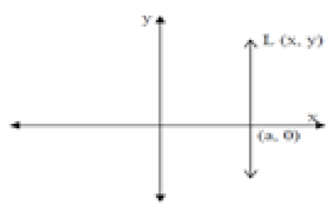
Find the angle of the inclination line determined by two points *A (2, -1)* and *B (-2, 3)* and find the slope of the line perpendicular to AB.

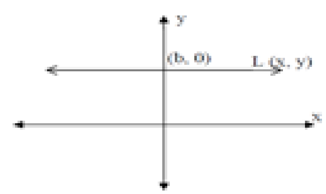
__Sol.__

1.3 Equations of a straight line:

__Vertical lines:__ The standard form of equation of vertical lines is: x = a

__Horizontal lines:__ The standard form of equation of horizontal lines is: y = b





Point–slope equation: The general form of point–slope equation of the point (x_1, y_1) with slope m is:

$(y-y_1)=m(x-x_1)$ ⟹ $y = m(x - x_1) + y_1$

Example

Write an equation for the line thought (-2, -1) to (3, 4).

Solution

Slope – intercept equation

The general form of slope-intercepts equation of line L with slope m and y-intercept b is: $y = mx + b$ *where m is the slope of the line and b is the constant*

Example

Find the slope and y-intercept of the line $10x + 5y = 20$

Sol.

General linear equation: The general linear equation is:

$Ax + By + C = 0$ where A, B and C are constants.

Example

Find the formula relating Fahrenheit and Celsius temperature, then find the Celsius equivalent of 90℉ and the Fahrenheit equivalent of -5 ℃.

F = m ℃ + b (linear relationship)

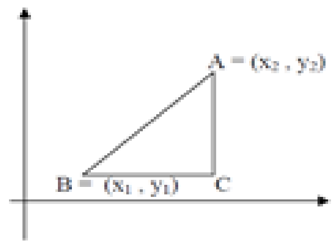
The freezing point of water is TF =32 or TC= 0 and boiling point is TF=212 or TC=100.

Sol.

EX2) The pressure P experienced by a diver under water is related to the diver's depth d by an equation of the form P = k d + 1 where k a constant. When d = 0 meters, the pressure is 1 atmosphere. The pressure at 100 meters is about 10.94 atmosphere. Find the pressure at 50 meters.

The distance from a point to a point:

The distance between points in the plane is calculated with a formula that comes from the Pythagorean theorem (Figure 6).



The distance between two (x_1, y_1) and (x, y) points is found by using

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example:

Find the distance between *A*(-7, 11) and *B*(1, 3)

Ans.

Remark

- If the line parallel to the x-axis

Distance = $\overline{x_2 - x_1}$

- If the line parallel to the y-axis

Distance = $y_2 - y_1$

The distance from a point to a line:

To find the distance from the point $P(x_1, y_1)$ to the line L , we follow :

- Find an equation for the line L' through P perpendicular to L :

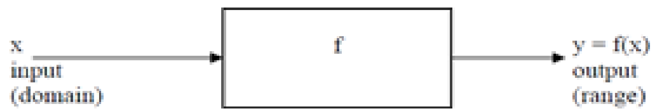
$y - y_1 = m'(x - x_1)$ where $m' = -1 / m$

- Find the point $Q(x_2, y_2)$ by solving the equation for L and L' simultaneously.
- Calculate the distance between P and Q.

1.4 Functions and Graphs

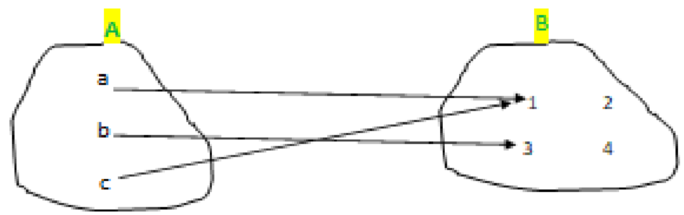
Functions

- A function is like a machine that assigns a unique output to every allowable input.
- A function describes a specific relationship between two variables; where an independent (input) variable has exactly one dependent (output) variable. Every element in the domain maps to only one element in the range.

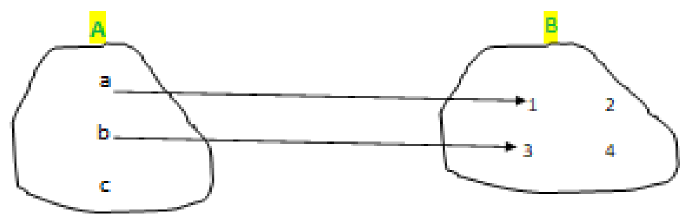


<u>Think about the function at these examples:</u>

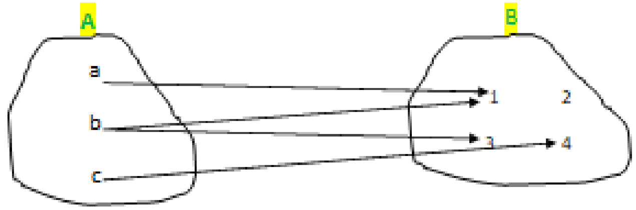
1. Let A={a,b,c}, B={1,2,3,4} let *f*={(a,1),(b,3),(c,1)} is <u>a function</u>.



2. Let g={(a,1),(b,3)} is <u>not a function</u>.



3. Let h={(a,1),(b,3),(c,4),(b,1)} is <u>not a function</u>.



H.W:

Is the equation of the circle $x^2+y^2=1$ is a function?

Sol.

Interval

1. The open interval consisting of all real numbers between two fixed numbers a and b :

$$(a,b)\equiv a<x<b$$

open interval (a, b)

2. Half open interval is the set of all real numbers that contain one endpoint but not both :

$$[a,b)\equiv a\le x<b$$

$$(a,b]\equiv a<x\le b$$

half-closed interval [a, b) *half-closed interval (a, b]*

3. The closed interval is the set of all real numbers that contain both end points :

$$[a,b] \equiv a \le x \le b$$

closed interval $[a, b]$

Remark:

The end points of interval are called the <u>boundary points</u>, the remaining points make up the interval called <u>interior points</u>.

1.5 Domain and Range

Given a function $y = f(x)$, the Domain of the function is the set of inputs and the Range is the set of resulting outputs.

<u>Domain of a function (D_f):</u>

Def.

Is the set of real numbers over which x may vary and makes the value of y true (input to the function).

Def. One

The <u>polynomials</u> of degree "n" can be written $y=p_n(x)=a_0+a_1x+a_2x^2+\dots\dots a_nx^n$ where the numbers $a_0, a_1, a_2 \dots a_n$ are called the coefficients of the polynomials and n is the positive integer numbers (n=1,2,3,.....).

<u>Note</u>:

Domains of polynomials are real numbers

$D_{P_n}(x)=R$

It can also be written $(-\infty,\infty)$

Example: find the domain of function $y=x^2+2x+1$

Sol.

Even and Odd Functions

Aim

To show how to determine if a function is even or odd.

Learning Outcomes

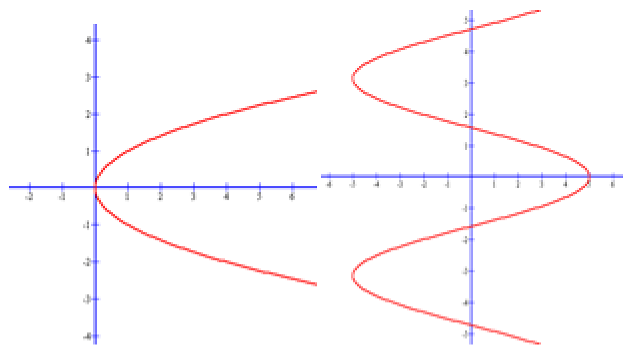
At the end of this section you will be able to:

- Tell the difference between even and odd functions,
- Test a given function to see if it is even or odd.

There are three types we address here: *x-axis* symmetry, *y-axis* symmetry, and *origin* symmetry.

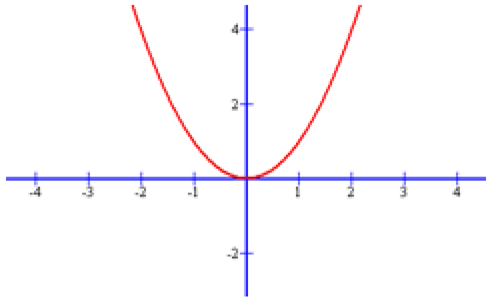
<u>X-*axis* Symmetry</u>

We say that a graph has "x-axis symmetry", or is "symmetric about the x-axis", when its graph would look the same if it were reflected about the x-axis. So a graph is symmetric about the x-axis if whenever the point *(x,y)* is on the graph, so is the point *(x,-y)*. For example, these two graphs have x-axis symmetry.



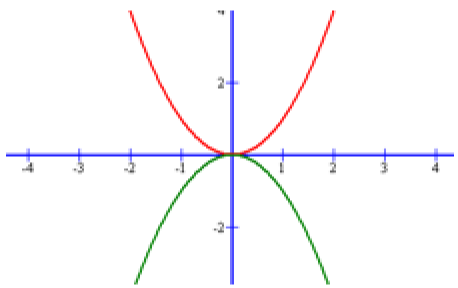
<u>*Y-axis* Symmetry</u>

We say that a graph has "y-axis symmetry", or is "symmetric about the y-axis", when its graph would look the same if it were reflected about the y-axis. So, a graph is symmetric about the y-axis if whenever the point **(x,y)** is on the graph, so is the point **(-x,y)**. For example, our standard parabola $y=x^2$ has y-axis symmetry.

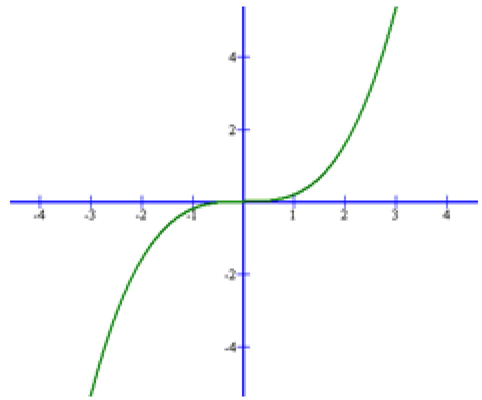


Origin Symmetry

A graph is said to have "origin symmetry", or is "symmetric around the origin", when its graph would look the same if it were rotated 180 degrees around the origin. So, a graph is symmetric about the origin if whenever the point (x,y) is on the graph, so is the point (-x,-y). Our standard parabola does not have this symmetry, since this is what we would get if we rotated it in such a way:



Here are a couple of examples of graphs that are symmetric about the origin:



If $f(-x) = f(x)$ f(x) is even function, even functions are symmetric about y- axis.

If $f(-x) = -f(x)$ f(x) is odd function, odd functions are symmetric about Origin.

If $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, the function is neither even nor odd.

<u>Example</u>

1) Is $f(x) = \frac{x^2+4}{x^3-x}$ be odd or even function?
2) Is $y=x^4-3x^2+7$ be odd or even function?
3) Is $f(x)= x^2-5x$ be odd or even function?

Sol.

1.6 Function in Pieces

While some functions are defined by single formula, others are defined by applying different formulas to different parts of their domain.

A piecewise defined function *is one which is defined using* ***two or more*** *formulas.*

Example:

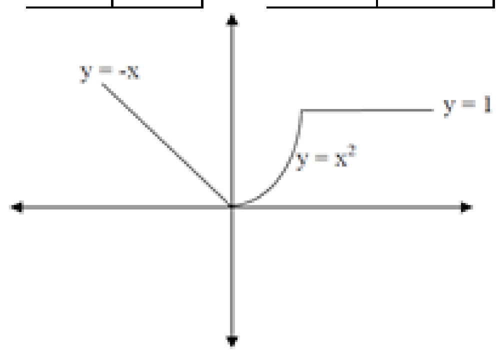
Graph

$$y=f(x)=\begin{cases} -x & x<0 \\ x^2 & 0 \le x \le 1 \\ 1 & x>1 \end{cases}$$

Sol.

$y=x^2$	
0	0
1/2	1/4
1	1

$y=-x$	
-1	1
-2	2
-3	3



1.7 Sum, Difference, Product and Quotients of Function

Example

Example

If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$ find domain of (f+g), (f-g), (g-f), (f.g), (f/g)

Solution

The domain of $f(x) = \sqrt{x}$ is $x >= o$ Or $[0, \infty)$

The domain of $g(x) = \sqrt{1-x}$ is $x <= 1$ Or $(-\infty, 1]$

$(f+g)(x) = \sqrt{x} + \sqrt{1-x}$ **Domain : $0 <= x <= 1$** Or $[0, 1]$

$(f-g)(x) = \sqrt{x} - \sqrt{1-x}$ Domain : $0 <= x <= 1$ Or $[0, 1]$

$(g-f)(x) = \sqrt{1-x} - \sqrt{x}$ Domain : $0 <= x <= 1$ Or $[0, 1]$

$(f.g)(x) = \sqrt{x} . \sqrt{1-x}$ Domain : $0 <= x <= 1$ Or $[0, 1]$

$\frac{f}{g}(x) = \sqrt{\frac{x}{1-x}}$ Domain: $0 <= x < 1$ Or $[0, 1)$

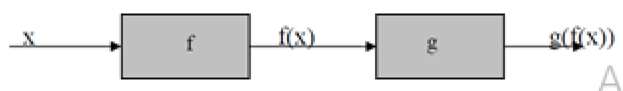
$\frac{g}{f}(x) = \sqrt{\frac{x-1}{x}}$ Domain: $0 < x <= 1$ Or $(0, 1]$

1.8 Composition of Functions

Suppose that the outputs of a function *f* can be used as inputs of a function *g*. We can then hook *f* and *g* together to form a new function whose inputs are the inputs of *f* and whose outputs are the numbers:

$$(g o f)(x) \cdot g(f(x))$$

If *f* is a function of *x* and its an input of another function *g* then we can link *f* and *g* together to form a new function denoted as *(f o g)(x) or (g o f)(x).*



Example

If $f(x) = 1/x$ and $g(x)=1/\sqrt{x}$, find fog and gof? fog (4)

Sol.

1.9 The Absolute Function

When you take the absolute value of a number, you always end up with a positive number (or zero). Whether the input was positive or negative (or zero), the output is always positive (or zero). For instance, $|3| = 3$, and $|-3| = 3$ also.