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Mathematics II

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Lecture 7 : Derivatives

Derivatives

<u>Rules of derivatives :</u> Let *c* and *n* are constants, *u* , *v* and *w* are differentiable functions of x :

1. $\frac{d}{dx}c = 0$

2. $\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx} \Rightarrow \frac{d}{dx}\left(\frac{1}{u}\right) = -\frac{1}{u^2}\frac{du}{dx}$

3. $\frac{d}{dx}cu = c\frac{du}{dx}$

4. $\frac{d}{dx}(u \mp v) = \frac{du}{dx} \mp \frac{dv}{dx}$; $\frac{d}{dx}(u \mp v \mp w) = \frac{du}{dx} \mp \frac{dv}{dx} \mp \frac{dw}{dx}$

5. $\frac{d}{dx}(u.v) = u.\frac{dv}{dx} + v\frac{du}{dx}$

and $\frac{d}{dx}(u.v.w) = u.v\frac{dw}{dx} + u.w\frac{dv}{dx} + v.w\frac{du}{dx}$

6. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ *where* $v \neq 0$

Exponential functions : **If u is any differentiable function of x , then :**

$$7)\quad \frac{d}{dx}a^u = a^u . \ln a . \frac{du}{dx} \qquad and \qquad \frac{d}{dx}e^u = e^u . \frac{du}{dx}$$

Logarithm functions : **If u is any differentiable function of x , then :**

$$8)\quad \frac{d}{dx}\log_a u = \frac{1}{u . \ln a} . \frac{du}{dx} \qquad and \qquad \frac{d}{dx}\ln u = \frac{1}{u} . \frac{du}{dx}$$

_EX-2_- Find $\frac{dy}{dx}$ for the following functions :

$a)\ y = (x^2+1)^5$ $\qquad$ $b)\ y = [(5-x)(4-2x)]^2$

$c)\ y = (2x^3-3x^2+6x)^{-5}$ $\qquad$ $d)\ y = \frac{12}{x} - \frac{4}{x^3} + \frac{3}{x^4}$

$e)\ y = \frac{(x^2+x)(x^2-x+1)}{x^3}$ $\qquad$ $f)\ y = \frac{x^2-1}{x^2+x-2}$

$a)\ y = \log_{10} e^x$ $\qquad$ $b)\ y = \log_5 (x+1)^2$

$c)\ y = \log_2 (3x^2+1)^3$ $\qquad$ $d)\ y = [\ln (x^2+2)^2]^3$

$a)\ y = 2^{3x}$ $\qquad$ $b)\ y = 2^x . 3^x$

$c)\ y = (2^x)^2$ $\qquad$ $d)\ y = x . 2^{x^2}$

<u>Trigonometric functions</u> : **If u is any differentiable function of x , then :**

9) $\frac{d}{dx} \sin u = \cos u . \frac{du}{dx}$

10) $\frac{d}{dx} \cos u = -\sin u . \frac{du}{dx}$

11) $\frac{d}{dx} \tan u = \sec^2 u . \frac{du}{dx}$

12) $\frac{d}{dx} \cot u = -\csc^2 u . \frac{du}{dx}$

13) $\frac{d}{dx} \sec u = \sec u . \tan u . \frac{du}{dx}$

14) $\frac{d}{dx} \csc u = -\csc u . \cot u . \frac{du}{dx}$

<u>EX-9</u>- **Find** $\frac{dy}{dx}$ **for the following functions :**

a) $y = \tan(3x^2)$ b) $y = (\csc x + \cot x)^2$

c) $y = 2\sin\frac{x}{2} - x\cos\frac{x}{2}$ d) $y = \tan^2(\cos x)$

e) $x + \tan(xy) = 0$ f) $y = \sec^4 x - \tan^4 x$

EX-10- Prove that :

a) $\frac{d}{dx}\tan u = \sec^2 u.\frac{du}{dx}$ b) $\frac{d}{dx}\sec u = \sec u.\tan u.\frac{du}{dx}$

Proof :

a) $$L.H.S. = \frac{d}{dx}\tan u = \frac{d}{dx}\frac{\sin u}{\cos u} = \frac{\cos u.\cos u.\frac{du}{dx} - \sin u.(-\sin u)\frac{du}{dx}}{\cos^2 u}$$

$$= \frac{\cos^2 u + \sin^2 u}{\cos^2 u}.\frac{du}{dx} = \frac{1}{\cos^2 u}.\frac{du}{dx} = \sec^2 u.\frac{du}{dx} = R.H.S.$$

b) $$L.H.S. = \frac{d}{dx}\sec u = \frac{d}{dx}\frac{1}{\cos u} = -\frac{1}{\cos^2 u}(-\sin u)\frac{du}{dx}$$

$$= \frac{1}{\cos u}.\frac{\sin u}{\cos u}.\frac{du}{dx} = \sec u.\tan u.\frac{du}{dx} = R.H.S.$$

Hyperbolic functions **: If u is any differentiable function of x , then :**

21) $\frac{d}{dx}\sinh u = \cosh u.\frac{du}{dx}$

22) $\frac{d}{dx}\cosh u = \sinh u.\frac{du}{dx}$

23) $\frac{d}{dx}\tanh u = \operatorname{sech}^2 u.\frac{du}{dx}$

24) $\frac{d}{dx}\coth u = -\operatorname{csch}^2 u.\frac{du}{dx}$

25) $\frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u.\tanh u.\frac{du}{dx}$

26) $\frac{d}{dx}\operatorname{csch} u = -\operatorname{csch} u.\coth u.\frac{du}{dx}$

EX-13 **- Find $\frac{dy}{dx}$ for the following functions :**

a) $y = \coth(\tan x)$ b) $y = \sin^{-1}(\tanh x)$

c) $y = \ln\left|\tanh\frac{x}{2}\right|$ d) $y = x.\sinh 2x - \frac{1}{2}.\cosh 2x$

e) $y = \operatorname{sech}^3 x$ f) $y = \operatorname{csch}^2 x$