University of Salahaddin-Hawler
College of Engineering
Chemical and Ptrochemical Department

Mathematics IV

Lecturer: **Ahmed A. Zardoey**
M.Sc. **Mechanical Engineering**
Contact:
Email: Ahmed.ahmed3@su.edu.krd

Lecture 1: Formation of differential equation

Ordinary Differential Equations

Introduction:

the study of ordinary differential equations (ODEs) by deriving them from physical or other problems (***modeling***), solving them by standard mathematical methods, and interpreting solutions and their graphs in terms of a given problem. The simplest ODEs to be discussed are ODEs ***of the first order*** because they involve only the first derivative of the unknown function and no higher derivatives. These unknown functions will usually be denoted by or when the independent variable denotes time t. The chapter ends with a study of the existence and uniqueness of solutions of ODEs

Understanding the basics of ODEs requires solving problems by hand (paper and pencil, or typing on your computer, but first without the aid of a CAS). In doing so, you will gain an important conceptual understanding and feel for the basic terms, such as ODEs, direction field, and initial value problem. If you wish, you can use your Computer Algebra System (CAS) for checking solutions.

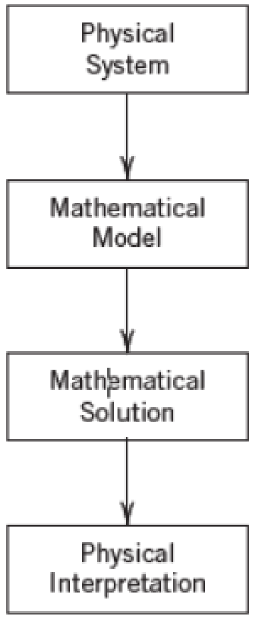
Basic Concepts. Modeling

If we want to solve an engineering problem (usually of a physical nature), we first have to formulate the problem as a mathematical expression in terms of variables, functions, and equations. Such an expression is known as a mathematical **model** of the given problem.

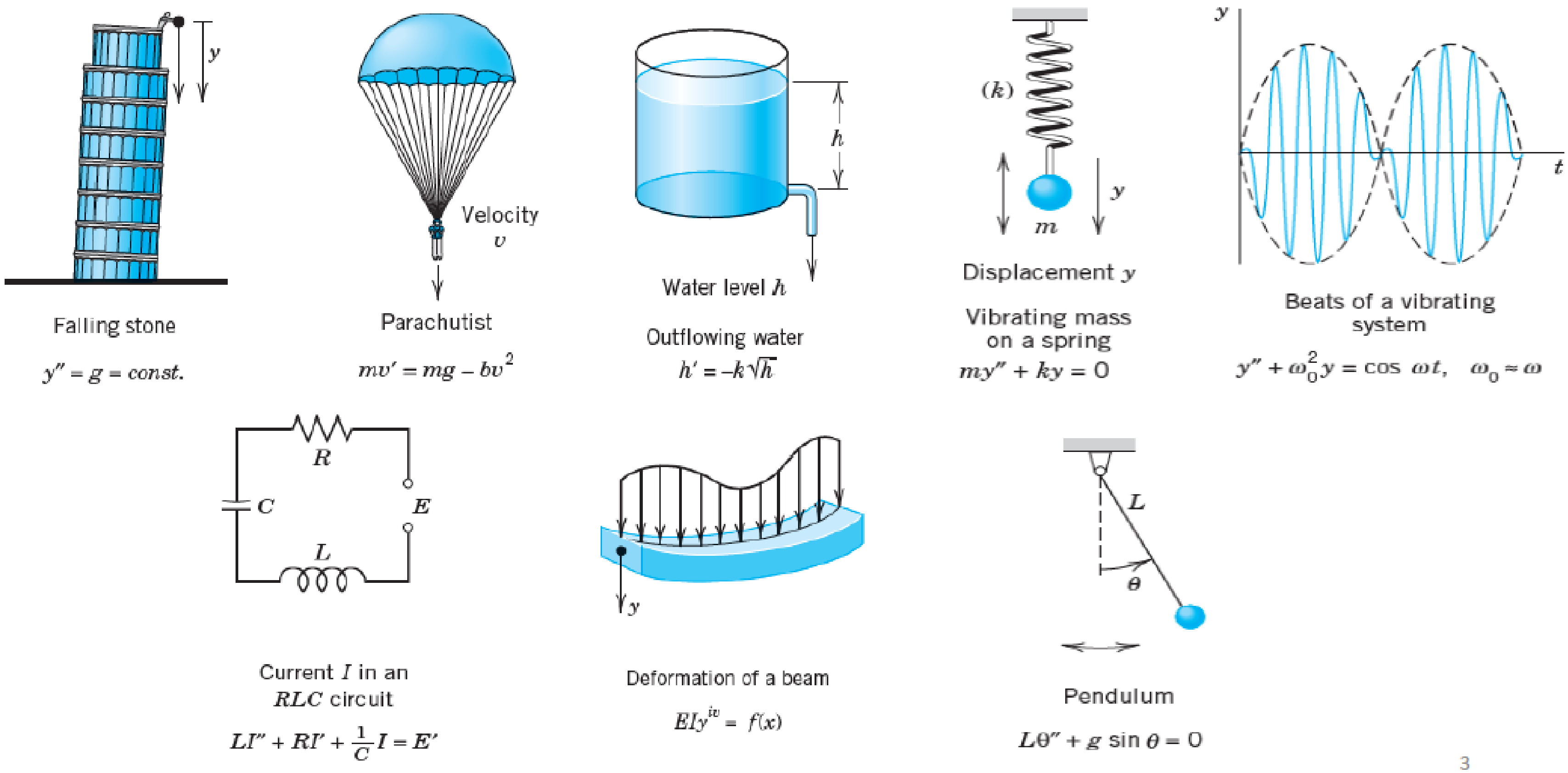
The process of setting up a model, solving it mathematically, and interpreting the result in physical or other terms is called *mathematical modeling* or, briefly, **modeling**.

Modeling needs experience, which we shall gain by discussing various examples and problems. (Your computer may often help you in *solving* but rarely in *setting up* models.)

Now many physical concepts, such as velocity and acceleration, are derivatives. Hence a model is very often an equation containing derivatives of an unknown function. Such a model is called a **differential equation**. Of course, we then want to find a solution (a function that satisfies the equation), explore its properties, graph it, find values of it, and interpret it in physical terms so that we can understand the behavior of the physical system in our given problem.



Basic Concepts. Modeling



Some applications of differential equations

Ordinary differential equation (ODE)

An **ordinary differential equation (ODE)** is an equation that contains one or several derivatives of an unknown function, which we usually call $y(x)$(or sometimes $y(t)$ if the independent variable is time t). The equation may also contain y itself, known functions of x (or t), and constants. For example,

(1) $$y' = \cos x$$

(2) $$y'' + 9y = e^{-2x}$$

(3) $$y'y''' - \frac{3}{2}y'^2 = 0$$

are ordinary differential equations (ODEs). Here, as in calculus, y' denotes dy/dx and $y''=d^2y/dx^2$,etc. The term ordinary distinguishes them from partial differential equations (PDEs), which involve partial derivatives of an unknown function of two or more variables. For instance, a PDE with unknown function u of two variables x and y is $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$

PDEs have important engineering applications, but they are more complicated than ODEs.

Ordinary differential equation (ODE)

An ODE is said to be of order n if the nth derivative of the unknown function y is the highest derivative of y in the equation. The concept of order gives a useful classification into ODEs of first order, second order, and so on.

$x\frac{dy}{dx} - y^2 = 0$ is an equation of the 1st order

$xy\frac{d^2y}{dx^2} - y^2 \sin x = 0$ is an equation of the 2nd order

$\frac{d^3y}{dx^3} - y\frac{dy}{dx} + e^{4x} = 0$ is an equation of the 3rd order

EX1) Form a differential equation from the function $\boldsymbol{x^2 - 2ax + y^2 = 0}$

EX2) Find a differential equation whose is general solution is $\boldsymbol{y = a\, e^{4x} + b\, e^{-x}}$

EX3) Find a differential equation whose is general solution is $\boldsymbol{y = e^{x}\,(a \cos x + b\, sinx)}$

$\Rightarrow \qquad x^2 - 2ax + y^2 = 0 \qquad \ldots(i)$

On differentiating both sides w.r.t. x, we get

$$2x - 2a + 2y\frac{dy}{dx} = 0$$

$\Rightarrow \qquad a = x + y\frac{dy}{dx}$ **(1)**

On putting above value of a in Eq. (i), we get

$$x^2 + y^2 - 2\left(x + y\frac{dy}{dx}\right)x = 0$$

$\Rightarrow \qquad 2xy\frac{dy}{dx} + x^2 - y^2 = 0$

$\Rightarrow \qquad 2xyy' + x^2 - y^2 = 0$

or $\qquad 2xy\frac{dy}{dx} + x^2 - y^2 = 0$

which is the required differential equation. **(1)**

Given $y = ae^{4x} + be^{-x}$. (1)

Here a and b are arbitrary constants

From (1), $\frac{dy}{dx} = 4ae^{4x} - be^{-x}$ (2)

and $\frac{d^2y}{dx^2} = 16ae^{4x} + be^{-x}$ (3)

(1)+(2) $\Rightarrow$ $y + \frac{dy}{dx} = 5ae^{4x}$ (4)

(2)+(3) $\Rightarrow$ $\frac{dy}{dx} + \frac{d^2y}{dx^2} = 20ae^{4x}$

$$= 4\left(5ae^{4x}\right)$$

$$= 4\left(y + \frac{dy}{dx}\right)$$

$$\frac{dy}{dx} + \frac{d^2y}{dx^2} = 4y + 4\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 0,$$

which is the required differential equation.

$$y = e^x\left(a\cos x + b\sin x\right) \qquad (1)$$

Differentiating (1) w.r.t x, we get

$$\frac{dy}{dx} = e^x\left(a\cos x + b\sin x\right) + e^x\left(-a\sin x + b\cos x\right)$$

$$= y + e^x(-a\sin x + b\cos x) \quad \text{(from (1))}$$

$$\Rightarrow \quad \frac{dy}{dx} - y = e^x(-a\sin x + b\cos x) \qquad (2)$$

Again differentiating, we get

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = e^x(-a\sin x + b\cos x) + e^x(-a\cos x - b\sin x)$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = e^x(-a\sin x + b\cos x) - e^x(a\cos x + b\sin x)$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = \left(\frac{dy}{dx} - y\right) - y \qquad \text{(from (1) and (2))}$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0,$$ which is the requited differential equation.