University of Salahaddin-Hawler
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Mathematics III

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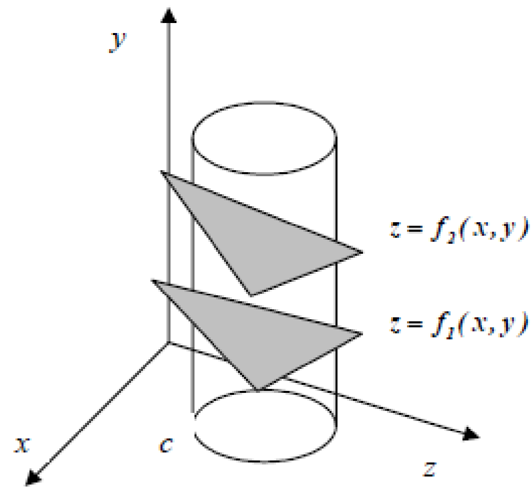
Lecture 6&7: Triple integrals (Volume) & length of curve

7-3- Triple integrals (Volume):

Consider a region N in xyz-space bounded below by a surface $z = f_1(x,y)$, above by the surface $z = f_2(x,y)$ and laterally by a cylinder c with elements parallel to the z-axis. Let A denote the region of the xy-plane enclosed by cylinder c (that is, A is the region covered by the orthogonal projection of the solid into xy-plane). Then the volume V of the region V can be found by evaluating the triply iterated integral:-

$$V = \iint_A \int_{f_1(x,y)}^{f_2(x,y)} dz\, dy\, dx$$



Let z-limits of integration indicate that for every (x,y) in the region A,Z may extend from the lower surface $z = f_1(x,y)$ to the surface $z = f_2(x,y)$. The y- and x-limits of integration have not been given explicitly in equation above, but are indicated as extending over the region A.

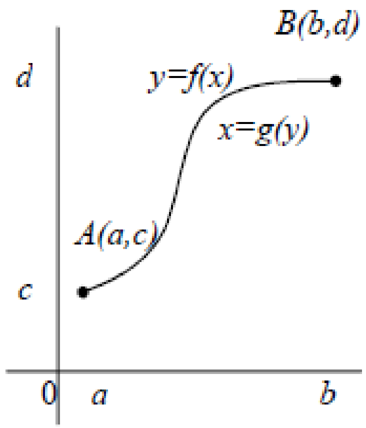
We can find the equation of the boundary of the region A by eliminating z between the two equations $z = f_1(x,y)$ and $z = f_2(x,y)$, thus obtaining an equation $f_1(x,y) = f_2(x,y)$ which contains no z, and interpret it as an equation in the xy-plane.

EX1: The volume in the first octant bounded by the cylinder $x = 4 - y^2$, and the planes $z = y$, $x = 0$, $z = 0$.

EX2: The volume enclosed by the cylinders $z = 5 - x^2$, $z = 4x^2$ and the planes $y = 0$, $x + y = 1$.

The length of a plane curve:-

The length of the curve $y = f(x)$ from point $A(a,c)$ to $B(b,d)$ is:-



$$L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}\, dx$$

If x can be expressed as a function of y then the length is:-

$$L = \int_c^d \sqrt{1 + (\frac{dx}{dy})^2}\, dy$$

EX1: - **Find the length of the curve:**

1) $y = \frac{1}{3}(x^2 + 2)^{\frac{3}{2}}$ *from* $x = 0$ *to* $x = 3$

2) $9x^2 = 4y^3$ *from* $(0,0)$ *to* $(2\sqrt{3},3)$