زانكۆی سەلاحەدین - هەولێر
Salahaddin University-Erbil

Probability theory and application

Research Project

Submitted to the department of (mathematics)in partial fulfillment of the requirements for the degree of BSc in (mathematic)

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April-2023

Certification of the supervisors:

I certify that this work was prepared under my supervision at the department of mathematics /college of education/Salahaddin university –Erbil in partial fulfilment of the requirements for the degree of bachelor of philosophy of science in mathematics.

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Acknowledgement

I would like to thank Allah for giving me the power to complete this work.

I would like to present and thanks to supervisor Dr.Awaz Kakamam for her kind and valuable suggestion that head assisted me to a complete this work.

I would also to extend my gratitude to head of the Mathematics Department Dr.Rashad Rashid Haji.

Thank my family for all the love and support they provided, and then all my friend loves, and thank all Teacher Mathematic Department.

Abstract

In this project, we study probability theory. We review basic concepts on Bayes theorem and probability theorem. Bayes' Theorem is a way of calculating conditional probabilities.

In this project, I applied Bayes probability theory on dataset which collected by myself. I got some experiment from this data set.

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Introduction:

The Bayes Theorem is a mathematical formula used to determine the condition probability of events. The theorem is named after English statistical (Tomas Bayes) who discovered in 1763. A theorem describing how the condition probability of each of a set of possible causes for a given observed outcome can be computed from knowledge of the probability of each cause. And the condition probability of the outcome of each cause.

As an example, Bayes theorem can be used to determine the accuracy of medical test results, tossing a coin, rolling a die, drawing a card of as well. It can be used to rate the risk of lending money to potential borrowers. Bayes theorem can help us to determine the probability of a given B denoted by P(A\B). Bayes rule is used in various occasions including a medical testing for a rare disease.

Beside statistical the Bayes theorem is also used in various disciplines with medicine and pharmacology. The researchers were described how Bayes' theorem can be applied to improve the interpretation of exercise tests used in the detection of coronary artery disease [3].

This project includes two chapter. In chapter one we review on probability theory and give some major theorem on probability theorem. we illustrate that what is probability tree? We also examination conditional probability and Independence between two events. Conditional probabilities are those probabilities whose value depends on the value of another probability.

In chapter two, we review on Bayes theorem and explain what is it? and how Bayes theorem calculate a conditional probability? I collected a data in (Iza laboratory) in Erbil. This data contains 20 examples which are asked to people about gender, blood pressure, sugary, cholesterol, and have Vitamin- D or not? We take several experiment on the dataset. The result show that probability of one person who have vitamin-D is 0.913 if you know the person who chosen is men. We found that the probability of one person who have vitamin-D is 0.931 if you know the person who selected has cholesterol. Moreover, the probability of person who have vitamin-D is 0.923 if you know the person who selected has Sugary.

Chapter one

Probability Theory:

In this chapter we review basic information on probability theory.

Random Experiment: any experiment when the results are unknown is called random experiment and it is denoted by R.E.

Sample Space: Is the set of all possible outcomes of a random experiment and denoted by S.

Example: R.E. In the toss of a coin, let the outcome tails be denoted by T and let the outcome heads be denoted by H. $S = \{T, H\}$.

Example: R.E. In the toss of a die twice, the sample space consists of the 36 ordered pairs:

$S = \{(1,1), \dots, (1,6), (2,1), \dots, (2,6), \dots, (6,6)\}$.

Events: Any subset of sample space is called event and denoted by E. There are two types of event: simple event and compound event.

Example: Tossing a fir dice once, the faces 1, 2, 3, 4, 5, 6 one equal likely event.

P(1)=P(2)=P(3)=P(4)=P(5)=p(6)=1/6.

Probability:

Suppose that an event E can happen in h ways out of a total of n possible equally likely ways. Then the probability of occurrence of the event (called its *success*) is denoted by

$$p = P_r\{E\} = \frac{h}{n}$$

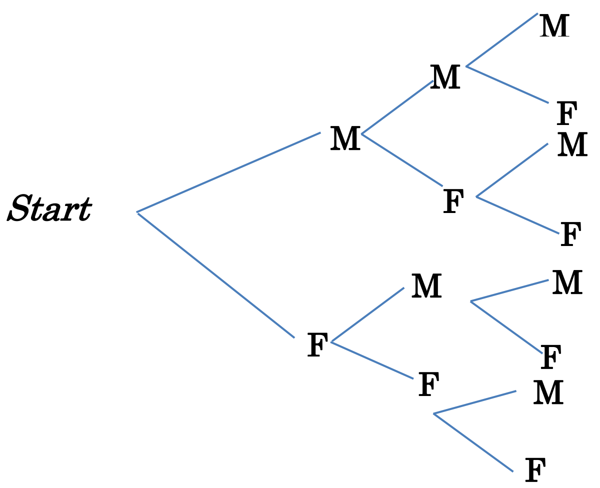
The probability of nonoccurrence of the event (called its *failure*) is denoted by

$$q = P_r\{notE\} = 1 - \frac{h}{n} = 1 - p$$

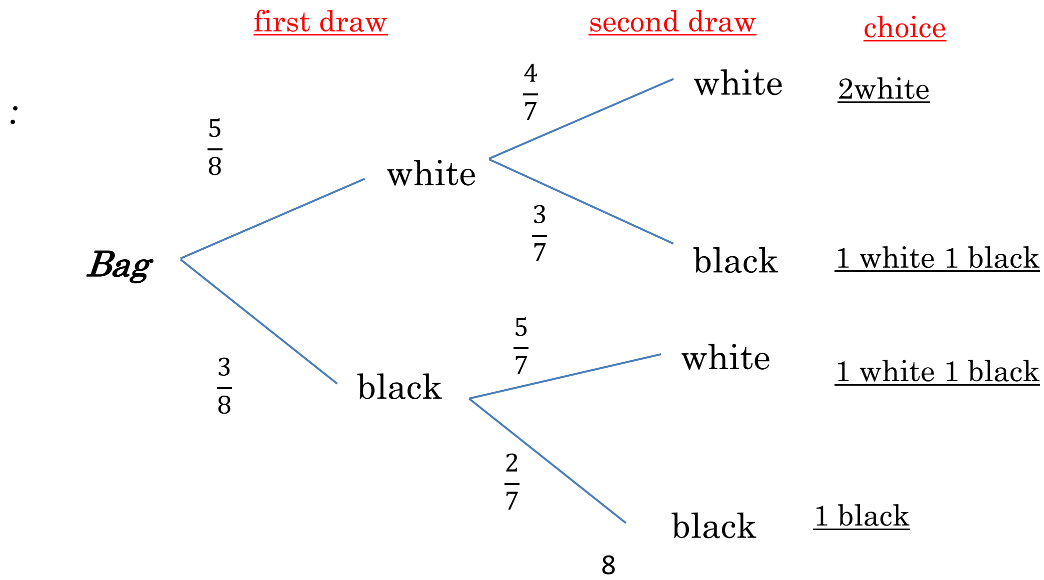
Thus $p + q = 1, or \Pr\{E\} + P_r\{notE\} = 1$.

Definition: (probability tree): The tree diagram helps to organize and visualize the different possible outcomes. branches and ends of the tree are two main position. Probability of each branch is written on the branch.

Example: There are three children in a family.
How many outcomes can represent the order in which the children many have been in relation to their gender
How many outcomes will be in the sample space indicating the gender of the children?
Assume that the probability of Male (M) and the probability of female (F) are each 1/2.



Example: There are eight balls in a bag.
How many outcomes can represent the order in which the balls many have been in relation to their bag.
How many outcomes will be in the sample space indicating the balls of the bag?
Assume that the probability of white balls (5/8) and the probability of black balls are (3/8)



Conditional probability and Independence:

let A and B be two events such that $P(A) \neq 0$. Then the probability of event B given the knowledge of event A is denoted by

$$P(B|A) = \frac{P(\mathrm{A} \cap \mathrm{B})}{P(A)}.$$

From the definition of the conditional probability set function, we observe that

$$P(\mathrm{A} \ \cap \ \mathrm{B}) = P(A)P(B|A)$$

Definition: Two events A and B are ***independent*** if the knowledge of the occurrence of one event does not affect of the other event occurring. That is,

Let A and B be two events. We say that A and B are independent if

$$P(A \ \cap \ B) \ = \ P(A)P(B).$$

It means, when P(A) > 0, P(B|A) = P(B).

Dependent events: two events $A \ and \ B$ are dependent events if the knowledge of occurrence of one event does effect of the other event occurring.

Mutually events: Tow events A and B are mutually events if they cannot occur at the same time

Example: A bowl contains ten balls. Five of the balls are red, two of the ball are white, and the remaining three are blue. If we select two balls, what is the probability that the second ball is red given the knowledge that the first ball is red too?

Solution: $P(B|A) = \frac{P(\mathrm{A} \cap \mathrm{B})}{P(A)}$, Let $A = P(1Red) = \frac{5}{10}$, $P(\text{A and B}) = \frac{5}{10} . \frac{4}{9}$.

$$P(B|A) = \frac{P(\mathrm{A} \cap \mathrm{B})}{P(A)} = \frac{\frac{5}{10} . \frac{4}{9}}{\frac{5}{10}} = \frac{4}{9}.$$

Example: A standard deck of playing cards consists of 52 cards

- Four suits: Hearts, Diamonds (red) , and Spades , Clubs (black) .
- Each suit has 13 cards, whose denomination is 2, 3, · · ·, 10, Jack, Queen, King, Ace
- The Jack, Queen, and King are called face cards.

Suppose we draw a card from a shuffled set of 52 playing cards.

1- What is the probability of drawing a Queen, given that the card drawn is of suit Hearts?
2- What is the probability of drawing a Queen, given that the card drawn is a Face card?

Answer: 1- $P(Q|H) = \frac{P(Q\cap H)}{P(H)} = \frac{P(Q)\cap P(H)}{P(H)} = \frac{\frac{4}{52}.\frac{13}{52}}{\frac{13}{52}} = \frac{4}{52} = \frac{1}{13}$

2- $P(Q|F) = \frac{P(Q\cap F)}{P(F)} = \frac{P(Q)}{P(F)} = \frac{\frac{4}{52}}{\frac{12}{52}} = \frac{1}{3}$. $(Here\ Q \subset F, so\ that\ Q \cap F = Q)$

Example: If P (A) =0.15, P (B) =0.38, and P (A∩B) =0.04, is the independent?

P (A∩B) =P (A) P (B), then

0.04=0.15*0.38

0.04 ≠0.06

Therefore, A and B are not independent.

Total probability Theorem: -

Let $E_1, E_2, \ldots\ldots E_n$ be n mutually events in sample space S such that $S = \bigcup_{i=1}^{n} E_i$, and let B be any event in S such that $P(B) \neq 0$. Then

$P(B) = \sum_{i=1}^{n} P(B|AE_i).\mathrm{P}(E_i)$

Proof: We have $S = \bigcup_{i=1}^{n} E_i$,

S∩B=$\bigcup_{i=1}^{n} E_i$, ∩B

B = ($\bigcup_{i=1}^{n} E_i$)∩B

B= $\bigcup_{i=1}^{n}(E_i$∩B) =$\bigcup_{i=1}^{n}$(B ∩ Ei) by commutative law

P (B) =P ($\bigcup_{i=1}^{n}$(B ∩ Ei))

P (B) = ∑ P(B∩Ei) by condition 2 probability function

Since $P\ (B|Ei) = P(B \cap Ei)\ /P(Ei)$, then

$P\ (B \cap Ei) = P(B|Ei) * P$(Ei), then we get

$P(B) = \sum_{i=1}^{n} P(B|AE_i) * \mathrm{P}(E_i)$

Example: In a mathematical department, consider three groups of computer A, B, and C in third class, Group A contains 19 students which 11 girls and 8 boys. Group B contains 15 students which 6 girls and 9 boys. Group C contains 17 students 6 girls and 11 boys. The probability of three Groups are 0.3, what is probability of selecting a girl student?

Solution:

Let R1→be the event of Group A.

R2→be the event of Group B.

R3→be the event of Group C.

G→be the event of girl student.

P (G)=∑p (Ri)*P (G|Ri)

=P (R1)*P (G|R1) + P (R2)*P (G|R2) + P (R3)*P (G|R3)

= (0.3*11/9) + (0.3*6/15) + (0.3*6/17) =0.40

Bayes Theorem:

Let $A_1, A_2, , \cdots A_n$ be a collection of mutually events in sample space S and let B be any event in S such that $P(B) > 0$. Then

$$P(A_k|B) = \frac{P(B|A_k).\mathrm{P}(A_k)}{\sum_{i=1}^{n} P(B|A_i).\mathrm{P}(A_i)}.$$

Proof: Based on the definition of conditional probability, we have

$P(A_k|B) = \frac{P(A_k \cap \mathrm{B})}{P(B)}$, then $P(A_k \cap \mathrm{B}) = P(A_k|B)P(B) - - - - - (1)$

$P(B|A_k) = \frac{P(\mathrm{B} \cap A_k)}{P(A_k)}$, then $P(\mathrm{B} \cap A_k) = P(B|A_k)P(A_k) - - - - - -(2)$

Since $P(A_k \cap \mathrm{B}) = P(\mathrm{B} \cap A_k)$, then from equation (1) and (2), we get

$$P(A_k|B)P(B) = P(B|A_k)P(A_k)$$

Then $P(A_k \cap \mathrm{B}) = P(B|A_k)P(A_k) - - - - - -(3)$

Then from equation (3) and (1), we get

$P(A_k|B) = \frac{P(B|A_k)P(A_k)}{P(B)}$, By Total probability theorem, we get

$$P(A_k|B) = \frac{P(B|A_k).\mathrm{P}(A_k)}{\sum_{i=1}^{n} P(B|A_i).\mathrm{P}(A_i)}.$$

Example: let is we have three boxes, Box -1- contain 9 balls numbered from 1 to 9, Box -2- contain 5 balls numbered from 3 to 7, and Box 3 contain 9 balls numbered from 2 to 10. The player toss a die if the occur number is 3 the player select the ball from box -1-. If the occur number 2 or 5 the player select a ball from box -2-, otherwise the player select the ball from box -3-

1) What is the probability that select ball is even numbered ball?
2) If the select ball is odd numbered ball, what is the probability that it select from box-3-?

Solution:

1) $P(\text{even numbered ball}) = P(even\ |box\ 1)P(box1) + P(even\ |box\ 2)P(box2) + P(even\ |box\ 3)P(box3)$

$$= \frac{4}{9} \times \frac{1}{6} + \frac{2}{5} \times \frac{2}{6} + \frac{5}{9} \times \frac{3}{6} = ?$$

2) $P(box3\ |odd\ numbered\ ball) = \dfrac{\frac{4}{9}\times\frac{3}{6}}{\frac{5}{9}\times\frac{1}{6}+\frac{3}{5}\times\frac{2}{6}+\frac{4}{9}\times\frac{3}{6}}$

Example: Let bowl A1 contains 5 red, 2 white, 4 black balls and bowl A2 contains 4 red, 5 white,1 black balls, if we selected one bowl only and select one white ball on it, what is the probability that the white ball is selected from bowl A2?

Solution: By Bayes' Theorem, we have

$$P(A_k|B) = \frac{P(B|A_k).\mathrm{P}(A_k)}{\sum_{i=1}^{n} P(B|A_i).\mathrm{P}(A_i)}, \quad \text{then}$$

$$P(bowl\ A2|White) = \frac{P(White|bowl\ A2).\mathrm{P}(bowl\ A2)}{P(White|bowl\ A1)\mathrm{P}(bowl\ A1) + P(White|bowl\ A2)\mathrm{P}(bowl\ A2)}.$$

$$P(bowl\ A2|White) = \frac{\frac{5}{10}\times\frac{1}{2}}{\frac{2}{11}\times\frac{1}{2}+\frac{5}{10}\times\frac{1}{2}} = \frac{\frac{5}{20}}{\frac{2}{22}+\frac{5}{20}} = 0.73.$$

Example: In orange country, %51 of the adults are males, and %49 females.

a) Find the prior probability that the selected person is a male.

b) %9.5 of males smoke cigars, where %1.7 of females smoke cigars, Find the probability that the selected subject is a male.

Solution: M→ male Ḿ→ female

C→ cigar smoker Ć→ not cigar smoker

a) we know only that %51 of adult's orange country are male, so the probability that the selected is a male is P(M)=0.51.

b) P (M) =0.51, P (Ḿ) =0.49

P (C|M) =0.095, P (C|Ḿ) =0.017

Lets Now apply Bayes theorem

P(M|C)=P(M)*P(C|M)/P(M)*P(C|M)+P(Ḿ)*P(C|Ḿ)

=0.51*0.095/0.51*0.095+0.49*0.017

=0.853 probability that the cigar smoking.

Chapter two

Application on Bayes Theorem

In this chapter we applied Bayes probability theory on dataset which collected by myself. We study some experiment on this data set.

Description of dataset:

I collected a data in (Iza laboratory). This data contains 20 examples which are asked to people about gender, blood pressure, sugary, vitamin-D, and have cholesterol or not?

Table2.1 shows the dataset. This data contains 20 observation (cases). From these 20 observations: -

Gender: gender is divided into eleven women and nine men.

Blood pressure: eight of person have Blood pressure five men with three women (we assume that Normal range for Blood pressure is: 120/80 mm/Hg

Blood Sugar (Sugary): ten of the person have Blood Sugar (Sugary) four men and six women (we assume that Normal range for sugary for fasting is 80-100 mg\dl, After eating is 170-200 mg\dl, and 2-3 hours after eating is 120-140 mg\dl).

Cholesterol: eleven of person have not normal cholesterol (normal or not normal).

The information about having VITAMIN-D normal or not from this table we can see that fifteen cases have deficiency (not normal) VITAMIN-D and the reminder cases are five.

Each the variables with vitamin-D we can draw a 2×2 tables for example gender with vitamin-D drawing 2×2 tables as follows:

Gender\VITAMIN-D	Female	Male
Normal	3	2
Deficiency	8	7

Table 2.1: data table of 20 person who asked Blood pressure, sugary, Cholesterol and show that which of the person have a Vitamin-D normal or not.

person	Blood Pressure By mm\Hg	Blood sugar (Sugary) by mg\dl	Gender	Cholesterol	Vitamin-D
1	11/8	90	women	150	35
2	13/7	117	men	190	8
3	10/7	110	women	100	20
4	16/10	130	women	230	5
5	14/9	200	men	200	45
6	18/10	180	women	300	55
7	11/6	80	women	120	70
8	12/8	100	women	100	10
9	15/8	300	men	210	48
10	11/6	280	women	100	30
11	18/11	400	men	400	70
12	10/8	210	women	190	60
13	20/9	120	men	90	44
14	12/7	120	women	480	35
15	11/8	90	men	90	39
16	15/10	320	men	80	25
17	9/6	88	men	80	63
18	18/9	400	women	240	45
19	12/8	170	men	400	90
20	9/6	112	women	100	49

From above table we can get those information's that

P(VITAMIN-D, normal)=5/20 and P(VITAMIN-D, deficiency)=15/20

P(BP)=8/20, and P(not-BP)=12/20

P(Sugary)=10/20 and P(not Sugary)=10/20

P(men)=9/20 and P(women)=9/20

P(Cholesterol)=11/20 and P(no Cholesterol)=9/20

Now, we will take some experiment on the dataset in table 2.1

Experiments on Bayes Theorem

Experiment 1: From Table 2.1, if we select one man, what is the probability that this man has normal VITAMIN-D?

Solution: By using Bayes theorem, we will find

P(vitamin-D-normal|men) =? the probability that a person has vitamin-D, given that the person is men?

$$\text{We have} \quad P(A_k|B) = \frac{P(B|A_k).\mathrm{P}(A_k)}{\sum_{i=1}^{n} P(B|A_i).\mathrm{P}(A_i)}, 1 \le k \le 2$$

Let $A_i = probability\ having\ vitamin - D$

$B = prbability\ men$, then

$$P(vitD - normal|men) = \frac{p[(men|vitD - normal) * p(vitD - normal)]}{p(men|vitD - normal) * p(vitD - normal) + p(men|vitD - def.) * p(vitD - def.)}$$

$$= \frac{\left[\frac{7}{9} * \frac{15}{20}\right]}{\left(\frac{7}{9} * \frac{15}{20}\right) + \left(\frac{2}{9} * \frac{5}{20}\right)} = 0.913$$

Experiment 2: From Table 2.1, if we select one women, what is the probability that this women have normal VITAMIN-D?

Solution: By using Bayes theorem, we will find

P(vitamin-D-normal| women)=? the probability that a person has normal vitamin-D, given that the person is women? We have $P(A_k|B) = \frac{P(B|A_k).\mathrm{P}(A_k)}{\sum_{i=1}^{n} P(B|A_i).\mathrm{P}(A_i)}, 1 \le k \le 2$

Let $A_i = probability\ having\ vitamin - D$

$B = prbability$ women , then

$$P(vitamin - D - normal|\text{women}) = \frac{p[(\text{women}\ |vitD - normal) * p(vitD - normal)]}{p(\text{women}\ |vitD - normal) * p(vitD - normal) + p(\text{women}\ |vitD - def.) * p(vitD - def.)}$$

$$= \frac{\frac{8}{11} * \frac{15}{20}}{8/11 * 15/20 + 3/11 * 5/20} = 0.889$$

Experiment 3: From Table 2.1, if we select person who have Cholesterol, what is the probability that this person have VITAMIN-D?

Solution: we should find p(vitamin-D-normal|cholesterol). the probability that a person has normal vitamin-D, given that the person has cholesterol?

Based on Bayes theorem probability

$$P(vitamin-D-normal|cholesterol) = [p(cholesterol|vitamin-D-normal)*p(vitamin-D-normal)]/p(cholesterol|vitamin-D-normal)*p(vitamin-D+p(cholesterol|vitamin-D-not\ normal)*p(vitamin-D-not\ normal)$$

$$= 9/11*15/20/9/11*15/20+2/11*5/20$$

$$= 0.931$$

Experiment 4: From Table 2.1, if we select person who have Blood pressure, what is the probability that this person have VITAMIN-D?

Solution: we will find p(vitamin-D-normal|BP). the probability that a person has normal vitamin-D, given that the person has blood pressure?

Based on Bayes theorem probability

We have $P(A_k|B) = \frac{P(B|A_k).\mathrm{P}(A_k)}{\sum_{i=1}^{n} P(B|A_i).\mathrm{P}(A_i)}, 1 \leq k \leq n,\ then$

$$P(vitamin-D-normal|BP) = p(BP|vitamin-D-normal)*p(vitamin-D-normal)/p(BP|vitamin-D-normal)*p(vitamin-D-normal)+p(BP|vitamin-D-defeciancy)*p(vitamin-D-defeciancy)$$

$$= 5/8*15/20/5/8*15/20+3/8*5/20$$

$$= 0.833$$

Experiment 5: From Table 2.1, if we select person who have Sugary, what is the probability that this person has VITAMIN-D?

Solution: we should find p(vitamin-D-normal| Sugary). the probability that a person has normal vitamin-D, given that the person has sugary?

Based on Bayes theorem probability,

We have $P(A_k|B) = \frac{P(B|A_k).\mathrm{P}(A_k)}{\sum_{i=1}^{n} P(B|A_i).\mathrm{P}(A_i)}, 1 \leq k \leq 2$

we should find p(vitamin-D-normal| Sugary)=?

$$P(vitamin - D - normal|sugery = p(sugery|vitamin - D - normal) * p(vitamin - D - normal)/p(sugery|vitamin - D - normal) * p(vitamin - D - normal) + p(sugery|vitamin - D - defeciancy) * p(vitamin - D - defeciancy)$$

$$= \frac{\left[\frac{8}{10} * \frac{15}{20}\right]}{\left(\frac{8}{10} * \frac{15}{20}\right) + \left(\frac{2}{10} * \frac{5}{20}\right)} = 0.923.$$

Experiment 6: From Table 2.1, if we select person who have Sugary, what is the probability that this person has no VITAMIN-D?

Solution: we should find p(vitamin-D-deficiency| Sugary). the probability that a person has deficiency (not normal) vitamin-D, given that the person has sugary? Based on Bayes theorem probability, we should find p(vitamin-D-deficiency| Sugary)=?

$$P(vitamin - D - defeciancy|sugery = p(sugery|vitamin - D - defeciancy) * p(vitamin - D - defeciancy)/p(sugery|vitamin - D - normal) * p(vitamin - D - normal) + p(sugery|vitamin - D - deficiency) * p(vitamin - D - defesiaqncy)$$

$$= \frac{\left[\frac{2}{10} * \frac{5}{20}\right]}{\left(\frac{2}{10} * \frac{5}{20}\right) + \left(\frac{8}{10} * \frac{15}{20}\right)} = 0.0769$$

Experiment 7: From Table 2.1, if we select person who have women, what is the probability that this person has deficiency VITAMIN-D?

Solution: we should find p(vitamin-D-deficiency|women). the probability that a person has deficiency (not normal) vitamin-D, given that the person has sugary? Based on Bayes theorem probability,

We have $P(A_k|B) = \frac{P(B|A_k).\mathrm{P}(A_k)}{\sum_{i=1}^{n} P(B|A_i).\mathrm{P}(A_i)}, 1 \leq k \leq 2$

we should find p(vitamin-D-deficiency| women)=?

$$P(vitamin - D - defeciancy|women = p(women|vitamin - D - defeciancy) * p(vitamin - D - defeciancy)/p(women|vitamin - D - defeciancy) * p(vitamin - D - defeciancy) + p(women|vitamin - D - normal) * p(vitamin - D - normal)$$

$$= \frac{\left[\frac{3}{11} * \frac{5}{20}\right]}{\left(\frac{3}{11} * \frac{5}{20}\right) + \left(\frac{8}{11} * \frac{15}{20}\right)} = 0.11.$$

Experiment 8: From Table 2.1, if we select person who have Men, what is the probability that this person has deficiency VITAMIN-D?

Solution: we should find p(vitamin-D-deficiency| Men). the probability that a person has deficiency (not normal) vitamin-D, given that the person has men?

Based on Bayes theorem probability,

We have $P(A_k|B) = \frac{P(B|A_k).\mathrm{P}(A_k)}{\sum_{i=1}^{n} P(B|A_i).\mathrm{P}(A_i)}, 1 \leq k \leq 2$

we should find p(vitamin-D-deficiency| Men)=?

$$P(vitamin - D - defeciancy|men = p(men|vitamin - D - defeciancy) * p(vitamin - D - defeciancy)/p(men|vitamin - D - deficiancy) * p(vitamin - D - defeciancy) + p(men|vitamin - D - normal) * p(vitamin - D - normal)$$

$$= \frac{\left[\frac{2}{9} * \frac{5}{20}\right]}{\left(\frac{2}{9} * \frac{5}{20}\right) + \left(\frac{7}{9} * \frac{15}{20}\right)} = 0.0869$$

Experiment 9: From Table 2.1, if we select person who have Cholesterol, what is the probability that this person has deficiency VITAMIN-D?

Solution: we should find p(vitamin-D-defeciancy|Cholesterol). the probability that a person has deficiency (not normal) vitamin-D, given that the person has cholesterol? Based on Bayes theorem probability,

We have $P(A_k|B) = \frac{P(B|A_k).\mathrm{P}(A_k)}{\sum_{i=1}^{n} P(B|A_i).\mathrm{P}(A_i)}, 1 \leq k \leq 2$

we should find p(vitamin-D-deficiency| Cholesterol)=?

$$P(vitD - def.|cholistrol = p(cholistrol|vitamin - D - defeciancy) * p(vitD - def.)/p(cholistrol|vitD - def.) * p(vitD - def.) + p(cholistrol|vitD - normal) * p(vitD - normal)$$

$$= \frac{\left[\frac{2}{11} * \frac{5}{20}\right]}{\left(\frac{2}{11} * \frac{5}{20}\right) + \left(\frac{9}{11} * \frac{15}{20}\right)} = 0.0689$$

Experiment 10: From Table 2.1, if we select person who have Blood pressure, what is the probability that this person has deficiency VITAMIN-D?

Solution: we should find p(vitamin-D-deficiency| BP). the probability that a person has deficiency (not normal) vitamin-D, given that the person has Blood pressure?

Based on Bayes theorem probability,

We have $P(A_k|B) = \frac{P(B|A_k).\mathrm{P}(A_k)}{\sum_{i=1}^{n} P(B|A_i).\mathrm{P}(A_i)}, 1 \leq k \leq 2$

we should find p(vitamin-D-deficiency| BP)=?

$$P(vitamin - D - defeciancy|BP = p(BP|vitamin - D - defeciancy) * p(vitamin - D - defeciancy)/p(BP|vitamin - D - defeciancy) * p(vitamin - D - defeciancy) + p(BP|vitamin - D - normal) * p(vitamin - D - normal)$$

$$= \frac{\left[\frac{8}{10} * \frac{5}{20}\right]}{\left(\frac{8}{10} * \frac{5}{20}\right) + \left(\frac{2}{10} * \frac{15}{20}\right)} = 0.571$$

Experiment 11: From Table 2.1, if we select person who have Sugary, what is the probability that this person has Cholesterol?

Solution: we should find p(Cholesterol| Sugary). the probability that a person has cholesterol-yes, given that the person has sugary?

Based on Bayes theorem probability,

We have $P(A_k|B) = \frac{P(B|A_k).\mathrm{P}(A_k)}{\sum_{i=1}^{n} P(B|A_i).\mathrm{P}(A_i)}, 1 \leq k \leq 2$

we should find p(cholesterol-yes| Sugary)=?

$$P(cholistrol - yes|sugery = p(sugery|cholistrol - yes) * p(cholistrol - yes)/p(sugery|cholistrol - yes) * p(cholistrol - \ yes) + p(sugery|cholistrol - no) * p(vitamin - D - not\ )$$

$$= \frac{\left[\frac{8}{10}*\frac{11}{20}\right]}{\left(\frac{8}{10}*\frac{11}{20}\right)+(\frac{2}{10}*\frac{9}{20})} = 0.830$$

Experiment 12: From Table 2.1, if we select person who have Blood pressure, what is the probability that this person has Cholesterol?

Solution: we should find p(cholesterol-yes| BP). the probability that a person has cholesterol-yes, given that the person has Blood pressure?

Based on Bayes theorem probability,

We have $P(A_k|B) = \frac{P(B|A_k).\mathrm{P}(A_k)}{\sum_{i=1}^{n} P(B|A_i).\mathrm{P}(A_i)}, 1 \leq k \leq 2$

we should find p(cholesterol-yes| BP)=?

$$P(cholistrol|BP = p(BP|cholistrol - yes) * p(cholistrol - yes\ )/p(BP|cholistrol - yes) * p(cholistrol - yes) + p(BP|cholistrol - no) * p(cholistrol - no)$$

$$= \frac{\left[\frac{6}{8}*\frac{11}{20}\right]}{\left(\frac{6}{8}*\frac{11}{20}\right)+(\frac{2}{8}*\frac{9}{20})} = 0.785$$

Experiment 13: From Table 2.1, if we select person one men , what is the probability that this man has Cholesterol?

Solution: we should find p(Cholesterol|Men). the probability that a person has cholesterol, given that the person is men?

Based on Bayes theorem probability,

We have $P(A_k|B) = \frac{P(B|A_k).\mathrm{P}(A_k)}{\sum_{i=1}^{n} P(B|A_i).\mathrm{P}(A_i)}, 1 \leq k \leq 2$

we should find p(cholesterol| men)=?

$$P(cholistrol - yes|men = p(men|cholistrol - yes) * p(cholistrol$$

$$)/p(men|cholistrol - yes) * p(cholistrol - yes) + p(men|cholistrol - no) * p(cholistrol - no)$$

$= \frac{\left[\frac{5}{9} * \frac{11}{20}\right]}{\left(\frac{5}{9} * \frac{11}{20}\right) + \left(\frac{4}{9} * \frac{9}{20}\right)}$ =0.604

Experiment 14: From Table 2.1, if we select person one women , what is the probability that this woman has Cholesterol?

Solution: we should find p(Cholesterol|Women). the probability that a person has cholesterol, given that the person is women?

Based on Bayes theorem probability,

We have $P(A_k|B) = \frac{P(B|A_k).\mathrm{P}(A_k)}{\sum_{i=1}^{n} P(B|A_i).\mathrm{P}(A_i)}, 1 \leq k \leq 2$

we should find p(cholesterol| women)=?

$$P(cholistrol - yes|women = p(women|cholistrol - yes) * p(cholistrol$$

$$)/p(women|cholistrol - yes) * p(cholistrol - yes) + p(women|cholistrol - no) * p(cholistrol - no)$$

$= \frac{\left[\frac{6}{11} * \frac{11}{20}\right]}{\left(\frac{6}{11} * \frac{11}{20}\right) + \left(\frac{5}{11} * \frac{9}{20}\right)}$ =0.595.

Conclusion:

Bayes' Theorem is a way of calculating conditional probabilities. A theorem describing how the condition probability of each of a set of possible causes for a given observed outcome can be computed from knowledge of the probability of each cause. And the condition probability of the outcome of each cause.

The preceding case examples show how Bayes' theorem can be used to determine the probability of VITAMIN-D disease in patients undergoing diagnostic exercise testing. Today, in many noninvasive laboratories, probability analysis is used to broadly categorize patients into general classes of post-test probability.

In this work, we reviewed on Probability Theory and Bayes theorem and explain what is it? We explain how Bayes theorem calculate a conditional probability?
We take several experiment on the dataset which is collected by myself. The result show that probability of one person who have vitamin-D is 0.913 if you know the person who chosen is men. We found that the probability of one person who have vitamin-D is 0.931 if you know the person who selected has cholesterol disease. Moreover, the probability of person who have vitamin-D is 0.923 if you know the person who selected has Sugary.

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پوختە

لەم پڕۆژەیەدا ئێمە لە تیۆری ئەگەرەکان دەکۆڵینەوە. ئێمە پێداچوونەوە بە چەمکە بنەڕەتییەکان دەکەین لەسەر تیۆرمی بایس و تیۆرمی ئەگەر. تیۆرمی بایس ڕێگایەکە بۆ حیسابکردنی ئەگەرە مەرجدارەکان.

لەم پڕۆژەیەدا، من تیۆری ئەگەری بایسم لەسەر داتا سێت بەکارهێنا کە لەلایەن خۆمەوە کۆکرایەوە. من هەندێک تاقیکردنەوەم لەم کۆمەڵە داتایە وەرگرت.