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# **Decision Theory**

**Research Project** 

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#### **Certification of the supervisors:**

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## Abstract

In this project we study decision theory, which is about games with nature: choice under uncertainty. There are two players, the Decision Maker (DM), and nature. Also, we study the basic concepts of game theory. Game theory studies strategic interactions between two of more players. Game theory then studies how rational players will behave in order to maximize their payoff.

## Table of Contents

Abstract	
Introduction	5
Chapter one	6
Decision Theory	6
Prisoner's dilemma	6
Basic concepts	7
Formalization	
Decision rules and consequences	
• Formal decision rules:	
Quantification of consequences:	
Informal description	9
Chapter two	
Normative and Descriptive Decision Theory	
Risk, Ignorance and Uncertaint decision theory:	
Chapter three	
Game Theory	
Basic Concepts and Zero-sum Games	
Applications Of Game Theory on Event Management	
The Prisoner's Dilemma Game	
The Decision Matrix	
References	

## Introduction

The problem of making decision is typical for many areas of human activity, especially in business. A decision maker (DM) has to make a choice among several possibilities that serves best to his or her goals. The best choice may depend on numerous exterior factors and very often the DM has only partial information (if any) about which particular situation has been realized. Statistical decision theory is a branch of mathematics that deals with choice under uncertainty [4].

In this project we shall study a little bit of game theory and a little of statistical decision theory. Game theory studies strategic interactions between two of more players. If we say have 2 players, A and B where each of them has a set of possible actions/strategies  $\{a1, \ldots, ak\}$  and  $\{b1, \ldots, bl\}$ .

For each combination of strategies (ai, bj) there is a payoff for each player

$$p(ai, bj) = (pA(ai, bj), pB(ai, bj)).$$

Game theory then studies how rational players will behave in order to maximize their payoff.

- Will they collaborate or not?
- Will they reward each other or punish each other?

This project includes three chapters. In chapter one, we study some basic concepts of decision theory which is about games with nature: choice under uncertainty. There are two players, the Decision Maker (DM), and nature. The DM has to make a choice among several possibilities that will offer him the highest payoff, or the lowest cost.

In chapter two, I will show a distinguish between descriptive and normative decision theory. Descriptive decision theories seek to explain and predict how people actually make decisions. This is an empirical discipline, stemming from experimental psychology. Normative theories seek to yield prescriptions about what decision makers are rationally required or ought to do.

In chapter three, we talk about the game theory. Game theory is the study of mathematical models of conflict and cooperation between intelligent rational decision- makers.

## **Chapter one**

## **Decision Theory**

Decision theory is about games with nature: choice under uncertainty. There are two players, the Decision Maker (DM), and nature.

The DM has to make a choice among several possibilities that will offer him the highest payoff, or the lowest cost.

The best choice will typically depend on many factors. Also, in most cases the DM will only have partial information about the overall situation.

• Simple example: should you take an umbrella with you today?

• You can look outside the window, read the weather forecasts, but you can never be too sure.

• The weather will be the action of nature. Your choices are to take an umbrella, wear a raincoat etc.

#### Prisoner's dilemma

#### Example:-

• The police have arrested two bank robbers-but they have no evidence to prove they were the robbers which would put them in jail for ten years.

• The robbers are split and each put in a different cell.

• An inspector tells each one of them: If you both confess you get 5 years each. If just one of you confesses, he goes free while the other gets the full 10 years. If none of you confess you get 2 years each.

- Player  $1 \setminus 2$ ConfessRefuseConfess(-5, -5)(0, -10)Refuse(-10, 0)(-2, -2).
- We encode this information in the payoff matrix, Table1

## **Basic concepts**

The problem of making decision is typical for many areas of human activity, especially in business. A decision maker (DM) has to make a choice among several possibilities that serves best to his or her goals. The best choice may depend on numerous exterior factors and very often the DM has only partial information about which particular situation has been realized. Statistical decision theory is a branch of mathematics that deals with choice under uncertainty.

We formalize the situation using the following objects:

- 1) A non-empty set  $\Theta$  of possible states/actions of nature—referred to as the parameter space;
- 2) A non-empty set  $\Delta$  of possible decisions available to the DM—referred to as the decision space;
- 3) A loss function  $L : \Delta \times \Theta \rightarrow R$  representing the loss incurred by the DM—also referred to as the cost function.
- Typically nature does not have preferences-it chooses at random.
- The DM is trying to optimize—minimize the cost.

• uncertainty: the state  $\theta$  of nature is not known—the DM has to resort to probabilistic/statistical techniques.

Nature may be considered as an impersonal player with no preferences, while the main objective of the DM is to choose an 'optimal' decision from so as to minimize the loss resulted from the particular state of nature and the decision chosen.

Typically, there is an uncertainty about the state in which nature finds itself, hence the DM has to resort to probabilistic assessment of possible outcomes, which is the subject of statistical decision making. The fundamental conceptual elements supporting this theory comprise the following:

#### **Formalization**

**Parameter space:** this is modelled as a measurable space ( $\Theta$ , T). The state of nature encoding all information relevant to the DM is an element  $\theta \in \Theta$ .

#### Observations

• The DM is allowed to make one or more observations on which he will base his decision.

• Observations are random and depend on the state of nature  $\theta$ .

• To make this rigorous, each observation is a sample x of a random variable X taking values in a

sample space (X, B).

• The distribution function  $FX(x|\theta)$  depends on the true state of nature  $\theta$ .

• The observations are fully specified by a parametric family of distribution functions.

$$\{F(\cdot | \theta), \theta \in \Theta\}.$$

#### Decision rules and consequences

• Formal decision rules: A decision rule will tell you which action to to choose among the possible options  $\Delta$ .

This is modelled as a decision function  $\delta : X \to \Delta$ —the same observations must always lead to the same decisions! Write S for the set of all decision rules.

• Quantification of consequences: This is expressed via a loss function

$$L(\delta, \theta) : \Delta \times \Theta \to R,$$

which specifies the cost when the decision  $\delta$  is made when the true state of nature is  $\theta$ .

## **Informal description**

- The DM wants to minimize his loss, maximize his gain.
- This will of course depend on the state of nature— $L(d, \theta)$ .
- The DM does not know  $\theta$ , but he is allowed to make one observation x.
- x is a sample from a random variable  $X \sim f_X(\cdot | \theta)$ .

• From this observation the DM tries to extract as much info as possible to make the optimal decision.

## **Chapter two**

## Normative and Descriptive Decision Theory

Decision theory is an interdisciplinary project to which philosophers, economists, psychologists, computer scientists and statisticians contribute their expertise. However, decision theorists from all disciplines share a number of basic concepts and distinctions. In this work I will show a distinguish between descriptive and normative decision theory. Descriptive decision theories seek to explain and predict how people actually make decisions. This is an empirical discipline, stemming from experimental psychology. Normative theories seek to yield prescriptions about what decision makers are rationally required or ought to do. Descriptive and normative decision theory are, thus, two separate fields of inquiry, which may be studied independently of each other.

*For example*, from a normative point of view it seems interesting to question whether people visiting casinos in Las Vegas ought to gamble as much as they do. In addition, no matter whether this behavior is rational or not, it seems worthwhile to explain why people gamble (even though they know they will almost certainly lose money in the long run).

In this chapter I focus on normative decision theory. There are two reasons for this. First, normative decision theory is of significant philosophical interest. Anyone wishing to know what makes a rational decision rational should study normative decision theory. How people actually behave is likely to change over time and across cultures, but a sufficiently general normative theory can be expected to withstand time and cultural differences.

The second reason for focusing on normative decision theory is a pragmatic one. A reasonable point of departure when formulating descriptive hypotheses is that people behave rationally, at least most of the time. It would be difficult to reconcile the thought that most people most of the time make irrational decisions with the observation that they are in fact alive and seem to lead fairly good lives – in general, most of us seem to do pretty well. Moreover, if we were to discover that people actually behave irrationally, either occasionally or frequently, we would not be able

to advise them how to change their behavior unless we had some knowledge about normative decision theory.

It seems that normative decision theory is better dealt with before we develop descriptive hypotheses. That said, normative and descriptive decision theory share some common ground. A joint point of departure is that decisions are somehow triggered by the decision maker's beliefs and desires. This idea stems from the work of the Scottish eighteenth-century philosopher David Hume.

#### **Risk, Ignorance and Uncertaint decision theory:**

In decision theory, everyday terms such as risk, ignorance and uncertainty are used as technical terms with precise meanings. In decisions under risk the decision maker knows the probability of the possible outcomes, whereas in decisions under ignorance the probabilities are either unknown or nonexistent. Uncertainty is used either as a synonym for ignorance, or as a broader term referring to both risk and ignorance. Although decisions under ignorance are based on less information than decisions under risk, it does not follow that decisions under ignorance must therefore be more difficult to make. In the 1960s, Dr. Christiaan Barnard in Cape Town experimented on animals to develop a method for transplanting hearts. In 1967 he offered 55-year-old Louis Washkansky the chance to become the first human to undergo a heart transplant. Mr. Washkansky was dying of severe heart disease and was in desperate need of a new heart. Dr. Barnard explained to Mr. Washkansky that no one had ever before attempted to transplant a heart from one human to another.

It would therefore be meaningless to estimate the chance of success. All Dr. Barnard knew was that his surgical method seemed to work fairly well on animals. Naturally, because Mr. Washkansky knew he would not survive long without a new heart, he accepted Dr. Barnard's offer. The donor was a 25-year-old woman who had died in a car accident the same day. Mr. Washkansky's decision problem is illustrated in **Table 2.1**. The operation was successful and Dr. Barnard's surgical method worked quite well. Unfortunately, Mr. Washkansky died 18 days later from pneumonia, so he did not gain as much as he might have hoped. The decision made by Mr. Washkansky was a decision under ignorance. This is because it was virtually impossible for him (and Dr. Barnard) to assign meaningful probabilities

to the possible outcomes. No one knew anything about the probability that the surgical method would work. However, it was nevertheless easy for Mr. Washkansky to decide what to do. Because no matter whether the new surgical method was going to work on humans or not, the outcome for Mr. Washkansky was certain to be at least as good as if he decided to reject the operation. He had nothing to lose. Decision theorists say that in a case like this the first alternative(to have the operation) dominates the second alternative.

#### Table 2

Method works	Method fails	
Operation	Live on for some time	Death
No operation	Death	Death

Ever since Mr. Washkansky underwent Dr. Barnard's pioneering operation, thousands of patients all over the world have had their lives prolonged by heart transplants. The outcomes of nearly all of these operations have been carefully monitored. Interestingly enough, the decision to undergo a heart transplant is no longer a decision under ignorance. Increased medical knowledge has turned this kind of decision into a decision under risk. Recent statistics show that 71.2% of all patients who undergo a heart transplant survive on average 14.8 years, 13.9% survive for 3.9 years, and 7.8% for 2.1 years. However, 7.1% die shortly after the operation. To simplify the example, we will make the somewhat unrealistic assumption that the patient's life expectancy after a heart transplant is determined entirely by his genes. We will furthermore suppose that there are four types of genes.

**Group I:** People with this gene die on average 18 days after the operation (0.05 years).

Group II: People with this gene die on average 2.1 years after the operation.

Group III: People with this gene die on average 3.9 years after the operation.

Group IV: People with this gene die on average 14.8 years after the operation.

Because heart diseases can nowadays be diagnosed at a very early stage, and because there are several quite sophisticated drugs available, patients who decline transplantation can expect to survive for about 1.5 years.

The decision problem faced by the patient is summarized in Table 2.2.

	Group I: 7.1%	Group II: 7.8%	Group III: 13.9%	Group IV: 71.2%
Operation	0.05 years	2.1 years	3.9 years	14.8 years
No operation	1.5 years	1.5 years	1.5 years	1.5 years

#### Problems

In each of the following problems you are asked to choose between two lotteries. A "lottery" gives you certain monetary prizes with given probabilities.

For instance:

A: \$0 0.5

\$1,000 0.5

A is a lottery that gives you \$0 with probability 50%, and \$1,000

otherwise.

A "sure" prize will be represented as a lottery with probability 1,

say:

B: \$500 1

B is a "lottery" that gives you \$500 for sure.

Please denote your preferences between the lotteries by

 $A \prec B \text{ or } A \succ B$ 

(or  $A \sim B$  if you are indifferent between the two).

## **Risk Aversion**

Imagine you have just won your first professional golf tournament. The chairman of the United States Golf Association offers you to either collect the \$1,000,000 prize check right away or toss a fair coin. If the coin lands heads he will double the prize check, but if it lands tails you lose the money and have to walk away with \$0. As a relatively poor newcomer to the world of professional golf you strictly prefer \$1,000,000 to the actuarially fair gamble in which you either double or lose your prize money. This is hardly controversial. It may take years before you win your next tournament. Few would question that decision makers are sometimes rationally permitted to be risk averse, especially if the stakes are high. That said, it remains to explain how risk aversion should be defined and analyzed, and to determine whether it is always rationally permissible to be risk averse.

The term "risk aversion" has several different but interconnected meanings in decision theory. Some of these are compatible with the principle of maximizing expected utility, while others are not. In what follows we shall discuss three of the most influential notions of risk aversion discussed in the literature:

- 1. Aversion against actuarial risks
- 2. Aversion against utility risks
- 3. Aversion against epistemic risks

Each of these notions of risk aversion can be characterized formally. As we will see shortly, this makes it easier to assess their applicability and limitations.

## **Chapter three**

#### **Game Theory**

Statistical decision theory fits in an even larger mathematical construct of game theory. The basic components of a game with N players are

1. A nonempty set, Ai, of possible actions available to Player i.

2. A set of payoff or utility functions,  $ui: A^N := A_1 \times \ldots \times A_N \rightarrow R$ 

The function  $u_i$  represents the gain of Player *i* depending on the actions chosen by all players. Typically, Li = -ui may be thought of as loss incurred by Player i. The objective of each player is to choose an action which minimizes maximizes his/her gain or equivalently minimizes the loss. Normally the objectives of players are conflicting, and that is obvious in the zero-sum case  $\sum_{i=1}^{N} u_i = 0$ .

Let us start our consideration with the simplest case of two-person zero-sum game, where the gain of one player equals the loss of the other. Here is a simple example:

Example 4.3. Two contestants simultaneously put up either one or two fingers. Player 1 wins if the sum of the digits is odd, and Player 2 wins if the sum of the digits is even. The winner in all cases receives in pounds the sum of the digits showing, this being paid to him by the loser. To create a triple (A1, A2, L) for this game we define  $A1 = A2 = \{1, 2\}$  and define payoff function of Player 1 by

$$u(1,1) = -2$$
,  $L(l,2) = 3$ ,  $L(2,1) = 3$ ,  $L(2,2) = -4$ .

It is customary to arrange the payoff function into a cost matrix as depicted below (payoff to Player 1).

A1/A2	1	2
1	-2	3
2	3	-4

So a two-player zero-sum game is represented by a cost matrix ||uij|| whose entries  $uij = u_1(ai, bj)$  are the payoffs to Player 1 from the strategies  $(ai, bj) \in A_1 \times A_2$ . If the number of players is greater then 2, it will be a more general object called a tensor. Having relaxed the zero sum condition, the entries of the payoff matrix would be vectors rather then real numbers. Every com- ponent of such a vector will be 'gain' of the corresponding player. Gain is understood as relative utility for the player, which, of course, may be measured in monetary units.

The matrix presentation of game is often called its normal form, in contrast to extensive form, which reflects a 'fine structure' underlying the game. That fine structure involves more elementary moves made by the players so that every action from Ai is in fact a sequence of moves. Usually in the extensive form actions are called strategies, which reflects the intuitive perception of a strategy as something complex, non-elementary. Under a mild restriction (perfect memory condition) the normal and extensive forms provide equivalent description of the game.

The basic ingredients of the game theory resembles those of the decision theory if one puts  $\Delta = A_1$  and  $\theta = A_1$ . Here and further on we use notation  $A_1 = A_1 \times \ldots \times A_i - 1 \times A_i + 1 \times \ldots A_N$ . Although, to a certain extent, the decision problem can be viewed as a game against nature. Here are some important differences between the two theories.

- In a two-person game, it is assumed that the players are simultaneously trying to maximize their gains, whereas decision theory assigns a neutral role to nature and only the DM is trying to find the optimum decision. Of course, a paranoid player might want to consider nature as opponent, but most people feel content to think of nature as being neutral.
- In a game, it is usually assumed that each player makes its decision based on exactly the same information (cheating is not allowed), whereas in decision theory, the DM may have available additional information, via observations, that may be used to gain an advantage on nature. This difference is more apparent than real, because there is nothing about game theory that says a game has to be fair. From this perspective, decision problems can be viewed as more complex games, however there are enough special issues and structure involved in decision making to warrant its being a theory on its own.

#### **Basic Concepts and Zero-sum Games**

Game theory studies decisions in which the outcome depends partly on what other people do. Chess is a paradigmatic example. Before I make a move, I always carefully consider what my opponent's best response will be, and if the opponent can respond by doing something that will force a checkmate, she can be fairly certain that I will do my best to avoid that move. Both I and my opponent know all this, and this assumption of common knowledge of rationality (CKR) determines which move I will eventually choose, as well as how my opponent will respond. I do not consider the move to be made by my opponent to be a state of nature that occurs with a fixed probability independently of what I do. On the contrary, the move I make effectively decides my opponent's next move. Chess is, however, not the best game to study for newcomers to game theory. This is because it is such a complex game with many possible moves. Like other parlor games, such as bridge, monopoly and poker, chess is also of limited practical significance. In this chapter we will focus on other games, which are easier to analyze but nevertheless of significant practical importance. Consider, for example, two hypothetical supermarket chains, Row and Col.

Both have to decide whether to set prices high and thereby try to make a good profit from every item sold, or go for low prices and make their profits from selling much larger quantities. Naturally, each company's profit depends on whether the other company decides to set its prices high or low. If both companies sell their goods at high prices, they will both make a healthy profit of \$100,000. However, if one company goes for low prices and the other for high prices, the company retailing for high prices will sell just enough to cover their expenses and thus make no profit at all (\$0), whereas the other company will sell much larger quantities and make

## **Applications Of Game Theory on Event Management.**

Game theory is "the study of mathematical models of conflict and cooperation between intelligent rational decision- makers." Game theory is mainly used in economics, political science, and psychology, as well as logic, computer science and biology. Originally, it addressed zero-sum games, in which one person's gains result in losses for the other participants.

#### The Prisoner's Dilemma Game

• One classic type of game is the prisoner's dilemma game.

• Prisoner's dilemma games are games in which each player has a dominant strategy and when both players play the dominant strategy, the payoffs are smaller than if each player played the dominated strategy.

• The dilemma is how to avoid this bad outcome.

#### The basics of the prisoner's dilemma game are as follows:

- Two prisoners have the option to confess or not confess to a crime they committed.
- The prosecutor has only enough information to convict both criminals of a minor offense and is, therefore, relying on a confession.
- The minor offense carries one year in jail.
- The prisoners are questioned in different cells, without the ability to communicate.
- They are told that if one prisoner confesses while the other remains silent, the prisoner confessing will go free and the prisoner remaining silent will serve 20 years in jail.
- If both prisoners confess, both prisoners will serve three years in jail.

• If a player goes free, the payoff is O. If a player serves one year in jail, the payoff is -1.

• If a player spends 20 years in jail, the payoff is -20.

• Use these numbers in your payoff matrix. Note that the negative numbers come from losing years of freedom.

#### i. Determine the three basic elements of the game.

- The players: Prisoner 1 and Prisoner 2
- The strategies for each player: Confess or Not Confess
- The payoffs for each player: If one confesses, he or she goes free, and the other gets 20 years in jail.

#### ii. Create a payoff matrix for the prisoner's dilemma game.

Prisoner2

		Confess	Not confess
nerl	confess	-3	-20 0
	Not confess	-20	-1

Prisoner

- iii. dentify any dominant strategies. Prisoner 1: Confess. Prisoner 2: Confess
- iv. Identify any dominated strategies. Prisoner 1: Not Confess. Prisoner 2: Not Confess



Case 1- Prisoner 1: Confess, Prisoner 2: Not Confess

Prisoner2

		Confess	Not confess
Prisoner1	confess	-3	-20 0
	Not confess	0 -20	-1 -1

onclusion-Prisoner 1 confesses, he or she goes free, and the Prisoner 2 gets 20 years in jail

#### Case 2- Prisoner 1: Not Confess, Prisoner 2: Confess

Prisoner2



Conclusion-Prisoner 1 not confesses ,he or she gets 20 years in jail and the Prisoner 2 confesses, he or she goes free

#### Case3- prisoner 1: not confess, prisoner 2 : not confess

Prisoner 2

Prisone1

	confess	Not confess
confess	-3	-20 0
Not confess	0 -20	-1 -1

Conclusion-Prisoner 1 and Prisoner 2 both not confesses both gets 1 years in jail.

## Case 4- Prisoner 1: Confess, Prisoner 2: Confess

Prisoner 2



Conclusion-Prisoner 1 and Prisoner 2 both confesses, both gets 3 years in jai

#### **The Decision Matrix**

Before you make a decision you have to determine what to decide about. Or, to put it differently, you have to specify what the relevant acts, states and outcomes are. Suppose, for instance, that you are thinking of taking out fire insurance on your home. Perhaps it costs \$100 to take out insurance on a house worth \$100,000 and you ask, "Is it worth it?" Before you decide, you have to get the formalization of the decision problem right. In this case, it seems that you face a decision problem with two acts, two states, and four outcomes. It is helpful to visualize this information in a decision matrix; see **Table 3.1**.

To model one's decision problem in a formal representation is essential in decision theory, because decision rules are only defined relative to a formal representation.

For example, it makes no sense to say that the principle of maximizing expected value recommends one act rather than another unless there is a formal listing of the available acts, the possible states of the world and the corresponding outcomes. However, instead of visualizing information in a decision matrix it is sometimes more convenient to use a decision tree. The decision tree in Figure 1 is equivalent to the matrix in Table 3.1. The square represents a choice node, and the circles represent chance nodes. At the choice node the decision maker decides whether to go up or down in the tree. If there are more than two acts to choose from, we simply adds more lines. At the chance nodes nature decides which line to follow. The rightmost boxes represent the possible outcomes. Decision trees are often used for representing sequential decisions, i.e. decisions that are divided into several separate steps. (Example: In a restaurant, you can either order all three courses before you start to eat, or divide the decision-making process into three separate decisions taken at three points in time. If you opt for the latter approach, you face a sequential decision problem.)

#### Table 3.1





#### Figur 1

represent a sequential decision problem in a tree, one simply adds new choice and chance nodes to the right of the existing leaves.

Many decision theorists distinguish only between decision problems and a corresponding decision matrix or tree. However, it is worth emphasizing that we are actually dealing with three levels of abstraction:

- 1. The decision problem
- 2. A formalization of the decision problem
- 3. A visualization of the formalization

A decision problem is constituted by the entities of the world that prompt the decision maker to make a choice, or are otherwise relevant to that choice. By definition, a formalization of a decision problem is made up of information about the decision to be made, irrespective of how that information is visualized. Formalizations thus comprise information about acts, states and outcomes, and sometimes also information about probabilities. Of course, one and the same decision problem can be formalized in different ways, not all of which are likely to be equally good. For example, some decision problems can be formalized either as decisions under risk or as decisions under ignorance. However, if one knows the probabilities of the relevant states, it is surely preferable to choose the

#### Table 3.2

```
{
  [a1 = take out insurance,
  a2 = do not];
  [s1 = fire,
  s2 = no fire];
  [(a1, s1) = No house and $100,000,
  (a1, s2) = House and $0,
  (a2, s1) = No house and $100,
  (a2, s2) = House and $100
}
```

former type of formalization (since one would otherwise overlook relevant information). Naturally, any given set of information can be visualized in different

ways. We have already demonstrated this by drawing a matrix and a tree visualizing the same formalization.

**Table 3.2** is another example of how the same information could be presented, which is more suitable to computers. In **Table 3.2** information is stored in a vector, i.e. in an ordered list of mathematical objects. The vector is comprised of three new vectors, the first of which represents acts. The second vector represents states, and the third represents outcomes defined by those acts and states. From a theoretical perspective, the problem of how to formalize decision problems is arguably more interesting than questions about how to visualize a given formalization.

Once it has been decided what pieces of information ought to be taken into account, it hardly matters for the decision theorist whether this information is visualized in a matrix, a tree or a vector

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Devore, J.L., Berk, K.N. and Carlton, M.A., 2012. *Modern mathematical statistics with applications* (Vol. 285). New York: Springer. لمم پرۆژەيەدا ئيمە لەتيۆرى بريار دەكۆلىنەو، كەتايبەتە بەيارىيەكان لەگەل سروشت: و سروشت. ھەروەھا، (DM) ھەلبژاردن لە ژير نادلنياييدا. دوو ياريزان ھەيە، برياردەر ئيمە لەچەمكە بنەرەتىيەكانى تيۆرى يارىيەكان دەكۆلىنەوە. تيۆرى يارىيەكان لەكارليكە ستراتيژييەكانى نيوان دوو ياريزانى زياتر دەكۆلىتەوە. پاشان تيۆرى يارىيەكان لىكۆلىنەوە لەوە دەكات كەياريزانە عەقلانيەكان چۆن ھەلسوكەوت دەكەن بۆ ئەوەى زۆرترين