

Q1/ Define joint mgf.

If the random variables X and Y have the joint pdf

$$f(x, y) = \begin{cases} e^{-y} & ; 0 < x < y < \infty \\ 0 & elsewhere \end{cases}, \text{ then}$$

1) Show that $mgf = \frac{1}{[(1-t_1-t_2)(1-t_2)]}$ provided that $t_1 + t_2 < 1$ and $t_2 < 1$.

2) Are X any Y independent? 3) Find $Cov(x, y)$. (12 marks)

Q2/ A number X is picked uniformly at random from 1 to 100. Find a formula for the probability mas

function of X. What is the expected value of X^2 ? (8 marks)

Q3) Consider two random variables X and Y with joint pmf given in the table.

	Y=0	Y=1	Y=2
X=0	1/6	1/4	1/8
X=1	1/8	1/6	1/6

- Find $P(X = 0, Y \leq 1)$.
- Find the marginal pdfs of X and Y.
- Find $P(Y = 1|X = 0)$.

Q1// A) Complete the following frequency distribution table: - (12 Marks)

Classes	Class mark (x_i)	frequency (f_i)
10 ---	13	2
		4
		1
		5
		6
		2

- Then find: 1) Mean Deviation 2) Median 3) Relative frequency
 4) The coefficient of skewness 5) Draw the histogram for the data.

B) If numbers of x_1, x_2, \dots, x_n have deviations from any number k given respectively $y_1 = x_1 - k, y_2 = x_2 - k, \dots, y_n = x_n - k$, then prove that $\bar{Y} = \bar{X} - k$. (3 marks)

Q2//A) There are 3 arrangements of the word DAD, namely DAD, ADD, and DDA.

How many arrangements are there of the word PROBABILITY? (4 marks)

B) What is conditional probability? Let two dice are rolled. Then

A = 'sum of two dice equals 3' and B = 'at least one of the dice shows a 1'

(a) What is $P(A|B)$? (b) Are A and B independent? (6 marks)

C) Define probability function. Let C and D be two events with $P(C) = 0.25, P(D) = 0.45$, and $P(C \cap D) = 0.1$. What is $P(C^c \cap D)$? (5 marks)

Probability and statistics

Second Stage\Mathematics

Time: 90 minutes

28/3/2021

Q1/ If the random variables X and Y have the joint pdf $f(x, y) = \begin{cases} 2e^{-x-y} & ; 0 < x < y < \infty \\ 0 & elsewhere \end{cases}$,

then answer the following questions:

- 1) Are X and Y independent? (10 marks) 2) Find $E(Y|X = x)$ 3) Find $P(1 < Y < 2 | X = \frac{1}{2})$.

Q2/ A) Suppose the joint mgf $M(t_1, t_2)$ exist for random variables X and Y. Prove that if $M(t_1, t_2) = M(t_1)M(t_2)$, then X and Y are independent variables. (8 marks)

B) Let X and Y have joint pdf, defined by

$$f(x, y) = \begin{cases} 4xy & ; 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & elsewhere \end{cases}$$

Then find: 1) $F(x, y)$ 2) $P(X \leq \frac{1}{2}, Y \leq \frac{3}{4})$ 3) $Cov(X, Y)$ 4) Correlation coefficient. (12 marks)

Q3//A) How many 5-digit number can be formed from 3, 4, 6, 8, 5, 1, 7, 9? (3 marks)

B) What is conditional probability? Let two dice are rolled. Then

A = 'sum of two dice equals 7' and B = 'at least one of the dice shows a 2'

(a) Find $P(A \cap B)$ (b) What is $P(A|B)$? (6 marks)

C) Define probability function. If that A and B be two non-mutually events in S. Suppose $P(A) = 0.4, P(B) = 0.3$ and $P((A \cup B)^c) = 0.42$. Are A and B independent? (6 marks)