Chapter Two

EXERCISES

1) Let the observed value of the mean $\bar{X}$ of a random sample of size 20 from a distribution that is $n(\mu, 80)$ be 81.2. Find a 95 per cent confidence interval for μ.

2) Let $\bar{X}$ be the mean of a random sample of size n from a distribution that is $n(\mu, 9)$. Find n such that $\Pr(\bar{X} - 1 < \mu < \bar{X} + 1) = 0.90$, approximately.

3) Let a random sample of size 17 from the normal distribution $n(\mu, \sigma^2)$ yield $\bar{x} = 4.7$ and $s^2 = 5.76$. Determine a 90 per cent confidence interval for μ.

4) Let $\bar{X}$ denote the mean of a random sample of size n from a distribution that has mean μ, variance $\sigma^2 = 10$, and a moment-generating function. Find n so that the probability is approximately 0.954 that the random interval $(\bar{X} - \frac{1}{2}, \bar{X} + \frac{1}{2})$ includes μ.

5) Let $X_1, X_2, \ldots, X_9$ be a random sample of size 9 from a distribution that is $n(\mu, \sigma^2)$.

(a) If σ is known, find the length of a 95 per cent confidence interval for μ if this interval is based on the random variable $\sqrt{9}(\bar{X} - \mu)/\sigma$.

(b) If σ is unknown, find the expected value of the length of a 95 per cent confidence interval for μ if this interval is based on the random variable $\sqrt{8}(\bar{X} - \mu)/S$.

(c) Compare these two answers. *Hint.* Write $E(S) = (\sigma/\sqrt{n})E[(nS^2/\sigma^2)^{1/2}]$.

6) Let $X_1, X_2, \ldots, X_n, X_{n+1}$ be a random sample of size $n + 1$, $n > 1$, from a distribution that is $n(\mu, \sigma^2)$. Let $\bar{X} = \sum_1^n X_i/n$ and $S^2 = \sum_1^n (X_i - \bar{X})^2/n$. Find the constant c so that the statistic $c(\bar{X} - X_{n+1})/S$ has a t distribution. If $n = 8$, determine k such that $\Pr(\bar{X} - kS < X_9 < \bar{X} + kS) = 0.80$. The observed interval $(\bar{x} - ks, \bar{x} + ks)$ is often called an 80 per cent *prediction interval* for X_9.

EXERCISES

7). Let two independent random samples, each of size 10, from two independent normal distributions $n(\mu_1, \sigma^2)$ and $n(\mu_2, \sigma^2)$ yield $\bar{x} = 4.8$, $s_1^2 = 8.64$, $\bar{y} = 5.6$, $s_2^2 = 7.88$. Find a 95 per cent confidence interval for $\mu_1 - \mu_2$.

8) Let $\bar{X}$ and $\bar{Y}$ be the means of two independent random samples, each of size n, from the respective distributions $n(\mu_1, \sigma^2)$ and $n(\mu_2, \sigma^2)$, where the common variance is known. Find n such that $\Pr(\bar{X} - \bar{Y} - \sigma/5 < \mu_1 - \mu_2 < \bar{X} - \bar{Y} + \sigma/5) = 0.90$.

9) If 8.6, 7.9, 8.3, 6.4, 8.4, 9.8, 7.2, 7.8, 7.5 are the observed values of a random sample of size 9 from a distribution that is $n(8, \sigma^2)$, construct a 90 per cent confidence interval for σ^2.

10) Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution $n(\mu, \sigma^2)$. Let $0 < a < b$. Show that the mathematical expectation of the length of the random interval $\left[\sum_1^n (X_i - \mu)^2/b, \sum_1^n (X_i - \mu)^2/a\right]$ is $(b - a) \times (n\sigma^2/ab)$.

11) A random sample of size 15 from the normal distribution $n(\mu, \sigma^2)$ yields $\bar{x} = 3.2$ and $s^2 = 4.24$. Determine a 90 per cent confidence interval for σ^2.

12) Let two independent random samples of sizes $n = 16$ and $m = 10$, taken from two independent normal distributions $n(\mu_1, \sigma_1^2)$ and $n(\mu_2, \sigma_2^2)$, respectively, yield $\bar{x} = 3.6$, $s_1^2 = 4.14$, $\bar{y} = 13.6$, $s_2^2 = 7.26$. Find a 90 per cent confidence interval for σ_2^2/σ_1^2 when μ_1 and μ_2 are unknown.

Chapter Three

Exercises:

2) Let $X_1, X_2, \ldots, X_{10}$ be a random sample of size 10 from a normal distribution $n(0, \sigma^2)$. Find a best critical region of size $\alpha = 0.05$ for testing H_0: $\sigma^2 = 1$ against H_1: $\sigma^2 = 2$. Is this a best critical region of size 0.05 for testing H_0: $\sigma^2 = 1$ against H_1: $\sigma^2 = 4$? Against H_1: $\sigma^2 = \sigma_1^2 > 1$?

3) If $X_1, X_2, \ldots, X_n$ is a random sample from a distribution having p.d.f. of the form $f(x; \theta) = \theta x^{\theta - 1}$, $0 < x < 1$, zero elsewhere, show that a best critical region for testing H_0: $\theta = 1$ against H_1: $\theta = 2$ is $C = \left\{(x_1, x_2, \ldots, x_n); c \le \prod_{i=1}^{n} x_i\right\}$.

4) Let $X_1, X_2, \ldots, X_{10}$ be a random sample from a distribution that is $n(\theta_1, \theta_2)$. Find a best test of the simple hypothesis H_0: $\theta_1 = \theta_1' = 0$, $\theta_2 = \theta_2' = 1$ against the alternative simple hypothesis H_1: $\theta_1 = \theta_1'' = 1$, $\theta_2 = \theta_2'' = 4$.

5) Let $X_1, X_2, \ldots, X_n$ denote a random sample from a normal distribution $n(\theta, 100)$. Show that $C = \{(x_1, x_2, \ldots, x_n); c \le \bar{x} = \sum_1^n x_i/n\}$ is a best critical region for testing H_0: $\theta = 75$ against H_1: $\theta = 78$. Find n and c so that

$$\Pr[(X_1, X_2, \ldots, X_n) \in C; H_0] = \Pr(\bar{X} \ge c; H_0) = 0.05$$

and

$$\Pr[(X_1, X_2, \ldots, X_n) \in C; H_1] = \Pr(\bar{X} \ge c; H_1) = 0.90, \text{ approximately.}$$

6) Let $X_1, X_2, \ldots, X_n$ denote a random sample from a distribution having the p.d.f. $f(x; p) = p^x(1-p)^{1-x}$, $x = 0, 1$, zero elsewhere. Show that $C = \{(x_1, \ldots, x_n); \sum_1^n x_i \le c\}$ is a best critical region for testing $H_0: p = \frac{1}{2}$ against $H_1: p = \frac{1}{3}$. Use the central limit theorem to find n and c so that approximately $\Pr\left(\sum_1^n X_i \le c; H_0\right) = 0.10$ and $\Pr\left(\sum_1^n X_i \le c; H_1\right) = 0.80$.

7) Let $X_1, X_2, \ldots, X_{10}$ denote a random sample of size 10 from a Poisson distribution with mean θ. Show that the critical region C defined by $\sum_1^{10} x_i \ge 3$ is a best critical region for testing $H_0: \theta = 0.1$ against $H_1: \theta = 0.5$. Determine, for this test, the significance level α and the power at $\theta = 0.5$.