Introduction to Matrices

Evar Lutfalla Sadraddin

Mathematics Department, College of Science, Salahaddin University-Erbil, Erbil, Iraq.

Abstract

In this report, introduction to matrices is presented. Matrix is a rectangular array of elements with many properties and applications. Here we will talk about some of its properties and types of it with only two operations, addition and subtraction. There are different examples to illustrate given definitions and types of matrices.

1. Introduction

Matrices, their singular is matrix, are everywhere. They are used in spreadsheet such as Excel or written numbers in a table. Matrices make presentation of numbers clearer and make calculations easier to program. A matrix is a rectangular array of numbers, symbols, or expressions, organized in rows and columns. Matrices are usually written in box brackets. In matrices, the horizontal and vertical lines of entries are rows and columns.

. Now in more details, Matrix $[A]$ is denoted by

$$[A]=\begin{bmatrix} a_{11} & a_{12} & .... & a_{1n} \\ a_{21} & a_{22} & .... & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & .... & a_{mn} \end{bmatrix}$$

Row i of $[A]$ has n elements and is

$$\begin{bmatrix} a_{i1} & a_{i2}....a_{in} \end{bmatrix}$$

and column j of $[A]$ has m elements and is

$$\begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{bmatrix}$$

Each matrix has rows and columns and this defines the size of the matrix. If a matrix $[A]$ has m rows and n columns, the size of the matrix is denoted by $m \times n$. The matrix $[A]$ may also be denoted by $[A]_{m\times n}$ to show that $[A]$ is a matrix with m rows and n columns. Each entry in the matrix is called the entry or element of the matrix and is denoted by a_{ij} where i is the row number and j is the column number of the element.

If the size of the resulting matrix is 2 × 3 (read "two by three"), it means the matrix has 2 rows and 3 columns; for example, The matrix for the tire sales example could be denoted by the matrix [A] as

$$[A] = \begin{bmatrix} 25 & 20 & 3 & 2 \\ 5 & 10 & 15 & 25 \\ 6 & 16 & 7 & 27 \end{bmatrix}.$$

There are 3 rows and 4 columns, so the size of the matrix is 3×4. In the above $[A]$ matrix, $a_{34} = 27$.

2. Dimensions of Matrices

A matrix of m rows and n columns is called a matrix with dimensions m x n.

2.1 Some Examples

1.) $\begin{bmatrix} 2 & -3 & 4 \\ -1 & \frac{1}{2} & \pi \end{bmatrix}$

It is a matrix with dimensions 2×3.

$$2.)\begin{bmatrix} -3 & 8 & 9 \\ \pi & -2 & 5 \\ -6 & 7 & 8 \end{bmatrix}$$

It is a matrix with dimensions 3×3.

$$3.)\begin{bmatrix} 10 \\ -7 \end{bmatrix}$$

$$4.)\begin{bmatrix} -3 & 4 \end{bmatrix}$$

While in examples 3 and 4, the dimensions are 2×1 and 1×2 respectively.

3. Some Types of Matrices

In this part, we present some important kinds of matrices.

3.1. Row Vector.

If a matrix $[B]$ has one row, it is called a row vector $[B]=[b_1\ b_2 \ldots\ldots b_n]$ and n is the dimension of the row vector.

3.1.1. Example

Give an example of a row vector.

Solution

$$[B]=[25\ \ 20\ \ 3\ \ 2\ \ 0]$$

is an example of a row vector of dimension 5.

3.2. Column vector:

If a matrix $[C]$ has one column, it is called a column vector

$$[C]=\begin{bmatrix} c_1 \\ \vdots \\ \vdots \\ c_m \end{bmatrix}$$

and m is the dimension of the vector.

3.2.1 Example

Give an example of a column vector.

Solution

$$[C]=\begin{bmatrix}25\\5\\6\end{bmatrix}$$

is an example of a column vector of dimension 3.

3.3. Square matrix

If the number of rows m of a matrix is equal to the number of columns n of a matrix $[A]$, that is, $m=n$, then $[A]$ is called a square matrix. The entries $a_{11},a_{22},...,a_{nn}$ are called the *diagonal elements* of a square matrix. Sometimes the diagonal of the matrix is also called the *principal* or *main of the matrix.*

3.3.1. Example

Give an example of a square matrix.

Solution

$$[A]=\begin{bmatrix}25 & 20 & 3\\5 & 10 & 15\\6 & 15 & 7\end{bmatrix}$$

is a square matrix as it has the same number of rows and columns, that is, 3. The diagonal elements of $[A]$ are $a_{11}=25,\ a_{22}=10,\ a_{33}=7$.

3.4. Diagonal matrix

A square matrix with all non-diagonal elements equal to zero is called a diagonal matrix, that is, only the diagonal entries of the square matrix can be non-zero, $(a_{ij}=0,\ i\neq j)$.

3.4.1. Example

Give examples of a diagonal matrix.

Solution

$$[A] = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2.1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

3.4.2. Example

Any or all the diagonal entries of a diagonal matrix can be zero. For example

$$[A] = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2.1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

is also a diagonal matrix.

3.5. Identity matrix

A diagonal matrix with all diagonal elements equal to 1 is called an identity matrix, ($a_{ij} = 0, i \neq j$ for all i, j and $a_{ii} = 1$ for all i).

3.5.1. Example

Give an example of an identity matrix.

Solution

$$[A] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

is an identity matrix.

4. Adding and Subtracting Matrices

We use matrices to list information or to represent systems. Because the entries are numbers, we can apply methods on matrices. We plus or minus matrices by adding or subtracting corresponding entries. To do this, the entries must correspond. Therefore, the plus and minus of matrices are only applicable when the matrices have equal dimensions. Let $A = [a_{ij}]$, $B = [b_{ij}]$ be $m \times n$ matrices. Then:

$A + B = [a_{ij} + b_{ij}]$, and $A - B = [a_{ij} - b_{ij}]$

Adding (subtracting) matrices is very simple. Just add (subtract) each element in the first matrix to the corresponding element in the second matrix. One of the basic methods that can be done on matrices is the addition process. Just as we plus (minus) two or more integers, two or more matrices can also be added (subtracted) similarly. This is identified as the Addition (Subtraction) of Matrices.

4.1. Example

$$\begin{bmatrix} 1 & -1 \\ 3 & 4 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & -4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 4 & 0 \\ 4 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 3 & 4 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 4 \\ 1 & -4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -5 \\ 2 & 8 \\ 0 & -3 \end{bmatrix}$$

5. Conclusion

The report presented some basics on the matrix topic in mathematics. Different types of matrices are given with many examples.

References

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