Chapter Seven

Root Locus Technique

Fourth Academic Year

Power, E/C, and C/C

Electrical Engineering Department

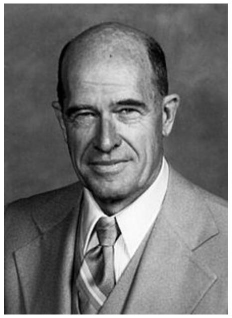
College of Engineering

Salahaddin University - Erbil

January 2021

Introduction

- **Root locus, a graphical presentation of the closed-loop poles as a system parameter is varied, is a powerful method of analysis and design for stability and transient response (Walter Evans, 1920-1999).**



- **In the design of control systems it is often necessary to investigate the performance of a system when one or more parameters of the system vary over a given range. Since the characteristic equation plays an important role in the dynamics behavior of linear systems, an important problem in linear control systems theory is the investigation of the trajectories of the roots of the characteristic, or simply, the root loci, when a certain system parameter varies.**

- **The root locus can be used to describe qualitatively the performance of a system as various parameters are changed. For example, the effect of varying gain upon percent overshoot, settling time, and peak time can be vividly displayed. The qualitative description can then be verified with quantitative analysis. Besides transient response, the root locus also gives a graphical representation of a system's stability. We can clearly see ranges of stability, ranges of instability, and the conditions that cause a system to break into oscillation.**

- **By using the root locus technique the designer can predict the effects on the location of the closed loop poles of varying the gain value or adding open loop poles and / or open loop zeros. Therefore, it is desired the designer have a good understanding of the technique for generating the root loci of the closed loop system, both by hand and by use of computer software like MATLAB.**

General Rules for Sketching the Root Locus

1. **Obtain the characteristic equation** ***$1 + G(s)H(s) = 0$***

2. **Rearranging the characteristic equation so that the parameter of interest appears as the multiplying factor in the form**

$$1 + \frac{K(s+z_1)(s+z_2)\,.......(s+z_m)}{(s+p_1)(s+p_2).......(s+p_n)} = 0$$

Note, here the parameter of interest is assumed to be the gain K, where K > 0, if K < 0 then K corresponds to positive feedback case in which the angle condition must be modified.

3. **Locate the poles and zeros of G(s)H(s) on the *s-plane*. The root locus branches starts from open loop poles and terminate at zeros (finite zeros or zeros at infinity). Note that the root loci are symmetrical about the real axis of the s plane, because the complex poles and complex zeros occur only in conjugate pairs.**

4. **The number of branches of the root loci equal to the number of poles of open loop transfer function (n).**

5. **The number of branches terminate at infinity is (n - m) which is number of asymptotes. Where n - m implicit zeros at infinity.**

6. **In case where poles and zeros included at infinity, the number of open loop poles are equal to the open loop zeros. Hence its state that, the root loci start at the poles and end at the zeros of G(s)H(s), as K increases from *zero to infinity*.**

7. **Determining the root loci on the real axis.**

   i. Root loci on the real axis are determined by open loop poles and zeros lying on it.
   ii. The complex conjugate poles and zeros of the open loop transfer function have no effect on the location of the root loci on the real axis.
   iii. The angle contribution of a pair of complex conjugate poles or zeros is 360° on the real axis.
   iv. Each portion of the root locus on the real axis extends over a range from a pole or zero to another pole or zero.

v. **Choose a test point on the constructed root locus; if the total number of real poles and real zeros to the right of this test point is odd, then this point lies on a root locus.**

vi. **Every real zero or pole to the right of the point contribute on angle 180°.**

vii. **Every real zero or pole to the left of the point contribute on angle 0°.**

viii. **If the poles and zeros of open loop are simple poles and simple zeros, then the root locus and its complement from alternate segments along the real axis.**

8. Determine the asymptotes of root loci.

i. **If the test point s is located far from the origin, then the angle of each complex quantity may be considered the same. Therefore, the root loci for very large values of s must be asymptotic to straight lines whose angle are given by**

$$Angle\ of\ Asymptotes, \theta_a = \frac{\pm 180°(2q+1)}{n-m}$$

where q = 0, 1 , 2, …
n = number of finite poles of G(s)H(s)
m = number of finite zeros of G(s)H(s)

Here, q = 0 corresponds to the asymptotes with the smallest angle with the real axis. Although q assumes an infinite number of values, as q is increased the angle repeats itself, and the number of distinct asymptotes is n-m.

ii. **The asymptotes intersect on the real axis at a point called centroid and given by**

$$\sigma_a = \frac{\sum_{i=1}^{n} P_i - \sum_{j=1}^{m} Z_j}{n-m}$$

9. Find the breakaway and break in points. Because of the conjugate symmetry of the root loci, the breakaway points and break-in points either lie on the real axis or occur in complex conjugate pairs. If a root locus lies between two adjacent open-loop poles on the real axis, then there exists at least one breakaway point between the two poles. Similarly, if the root locus lies between two adjacent zeros (one zero may be located at –q) on the real axis, then there always exists at least one break-in point between the two zeros. If the root locus lies between an open-loop pole and a zero (finite or infinite) on the real axis, then there may exist no breakaway or break-in points or there may exist both breakaway and break-in points.

Suppose that the characteristic equation is given by B(s) + KA(s) = 0

The breakaway points and break-in points correspond to multiple roots of the characteristic equation. Hence the breakaway and break-in points can be determined form the roots of

$$\frac{dK}{ds} = \frac{B'(s)A(s) \; - \; B(s)A'(s)}{A^2(s)} = 0$$

Also it can be found by taken the following

$$\sum_{i=1}^{n} \frac{1}{\sigma_b + P_i} = \sum_{j=1}^{m} \frac{1}{\sigma_b + Z_j}$$

where Pi and Zj are negative value of poles and zeros, respectively.

The root locus branches must approach or leaver the breakaway point on real axis at angle ±180/r , where r is the number of branches approaching or leaving the breakaway point.

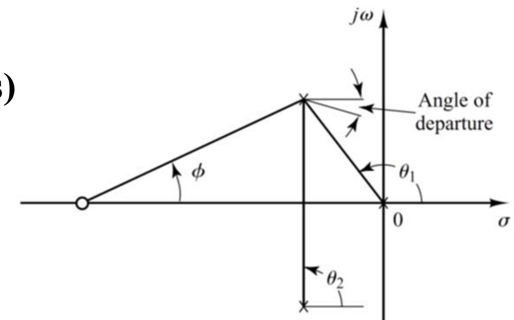
10. Determine the angle of departure (angle of arrival) of the root locus from a complex pole (at a complex zero). The angle of departure (or angle of arrival) of the root locus from a complex pole (or at a complex zero) can be found by subtracting from 180° the sum of all the angles of vectors from all other poles and zeros to the complex pole (or complex zero) in question, with appropriate signs included.

Angle of departure from a complex pole=180°

– (sum of the angles of vectors to a complex pole in question from other poles)

± (sum of the angles of vectors to a complex pole in question from zeros)

Angle of arrival at a complex zero=180°
– (sum of the angles of vectors to a complex zero in question from other zeros)
± (sum of the angles of vectors to a complex zero in question from poles)



11. Find the points where the root loci may cross the imaginary axis. The points where the root loci intersect the jw can be found by:

a. Use the Routh's stability criterion, or

b. Letting s = jw in the characteristic equation, equating both the real part and the imaginary part to zero, and solving for w and K. The values of w thus found gives the frequency at which roots loci cross the imaginary axis. The K value corresponding to each crossing frequency gives the gain at the crossing point.

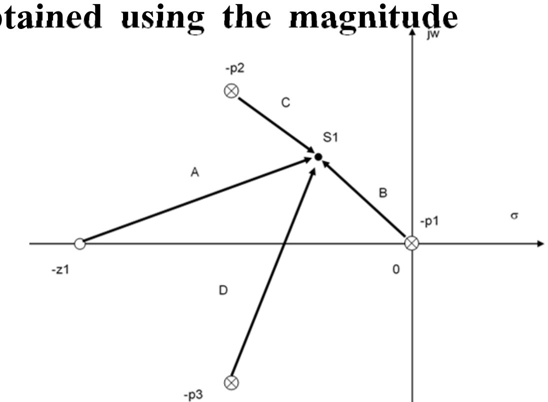
12. The value of K corresponding to any points s on a root locus can be obtained using the magnitude condition as;

$$K = \frac{product\ of\ lengths\ between\ point\ s\ to\ poles}{product\ of\ lengths\ between\ point\ s\ to\ zeros}$$

The value of K at point s is $|K| = \dfrac{|s_1||s_1+p_2||s_1+p_3|}{|s_1+z_1|}$

Where |s1+ z1| is the length of the vector drawn from zero z1 to the point s1.

If the vector lengths are represented by A, B , C and D thus $|K| = \dfrac{BCD}{A}$



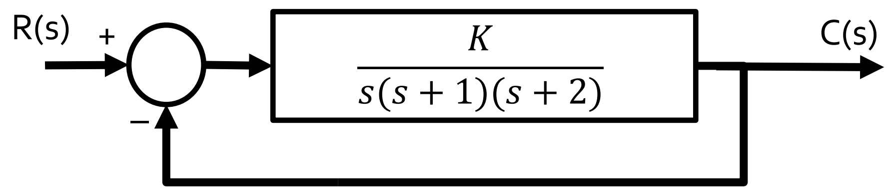
The sign of K, of course, depends on whether s1 is on the root loci or the complementary root loci. Consequently, given the pole - zero configuration of G(s)H(s), the construction of the complete root diagram involves the following two steps:

1. A search of all the s1 points in the s-plane that satisfy angle condition equation

2. The determination of the value of K at points on the root loci and the complementary root loci by use the magnitude condition equation.

Example 1

Sketch the root-locus plot of the following control system
Where $k \geq 0$



Answer :

Step1: the characteristic equation is $1 + G(s)H(s) = 0$

where $G(s) = \frac{K}{s(s+1)(s+2)}$ **and $H(s) = 1$ (unity feedback system)**

thus $$1 + \frac{K}{s(s+1)(s+2)} = 0$$

or $$K = -s(s+1)(s+2)$$

Step2: Locate poles and zeros of G(s)H(s) on s-plane

No. of poles = n = 2 0, -1 , -2

No. of zeros = m = 0

Step3: Show root-loci on the real axis of s-plane

According to the locations of poles and zeros on s-plane

there are 4 areas

area 1: [0 ∞] n = 0 and m = 0

|n-m|=0 (even) no root loci

area 2: [-1 0] n=1 and m=0

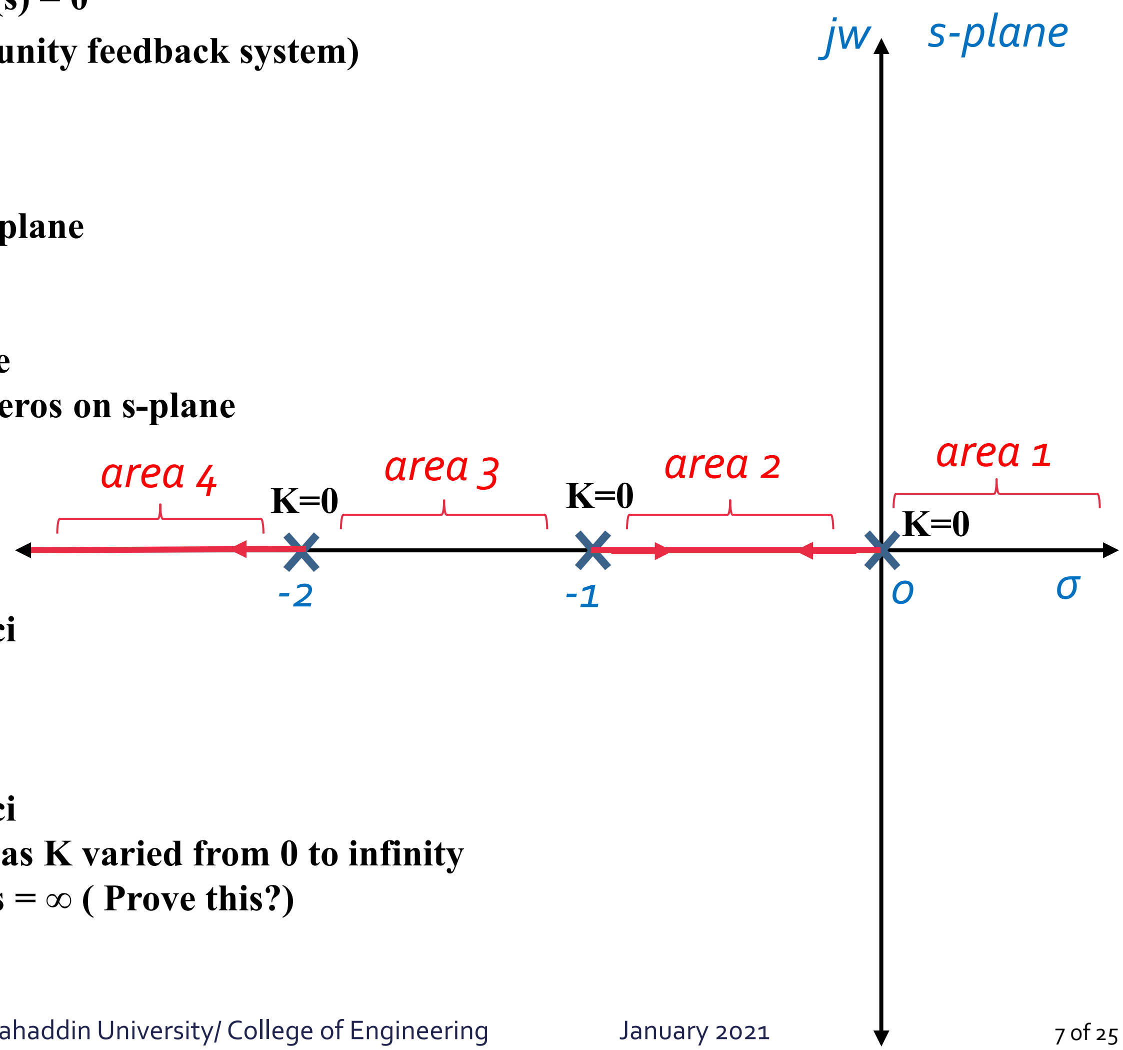
[1 – 0] = 1 (odd) there is a root loci

area 3: [-2 -1] n=2 and m=0

[2 – 0] = 2 (even) no root loci

area 4: [-∞ -2] n=3 and m=0

[3 – 0] = 3 (odd) there is a root loci



Step4: Root loci start in poles and end in zeros as K varied from 0 to infinity

Knowing that K at poles = 0 and at zeros = ∞ (Prove this?)

Example 1 Cont'd

Step4: No. of branches = No. of poles = n

here n = 3 so there are three branches

No. of branches terminate at infinity = |n-m| = |3 - 0| = 3

Step5: No. of asymptotes = |n – m| = |3 – 0| = 3

Step6: Asymptotes' angles are

$\theta_a = \frac{\pm 180°(2q+1)}{n-m}$ **q ≡|n-m| → q = 0,1,2**

$\theta_{a1} = \frac{180°(2x0+1)}{3-0} = 60^0$

$\theta_{a2} = \frac{180°(2x1+1)}{3-0} = 180^0$

$\theta_{a3} = \frac{180°(2x2+1)}{3-0} = 300^0$

Centroid point of asymptotes is

$\sigma_a = \frac{\sum_{i=1}^{n} P_i - \sum_{j=1}^{m} Z_j}{n-m} = \frac{(0-1-2)-0}{3-0} = -1$

Step7: Breakaway points (if exists)

from the characteristic equation

K = - s(s+1)(s+2) = - (s3 + 3s² + 2s)

taken dK/ds = 0

$\frac{dK}{ds} = -\left(3s^2 + 6s + 2\right) = 0$

solving second order polynomial equations:

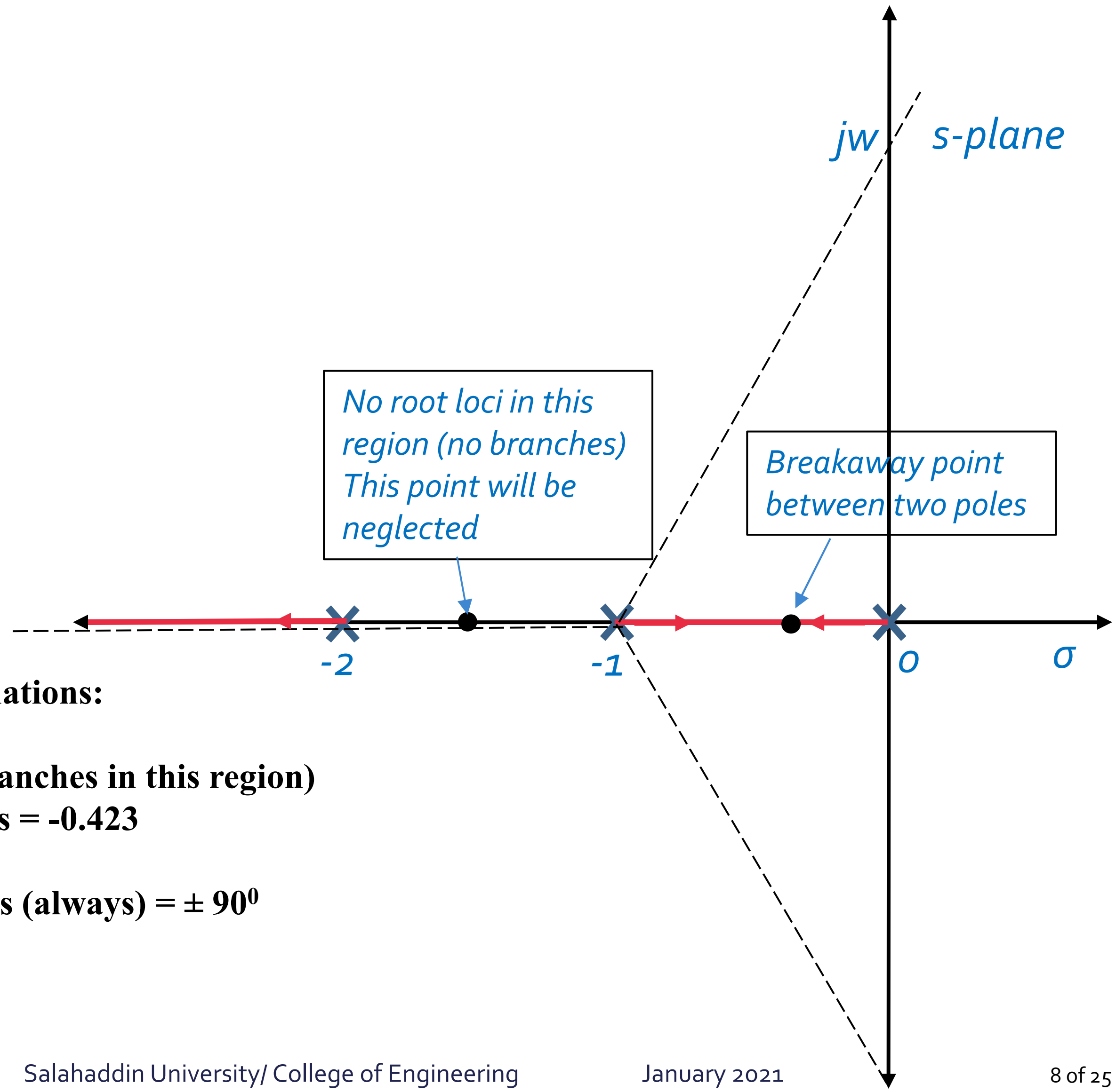
$s_{1,2}$ **= -0.423 and -1.577**

since no root loci at s = - 1.577 (no branches in this region)

and there is root loci in the region of s = -0.423

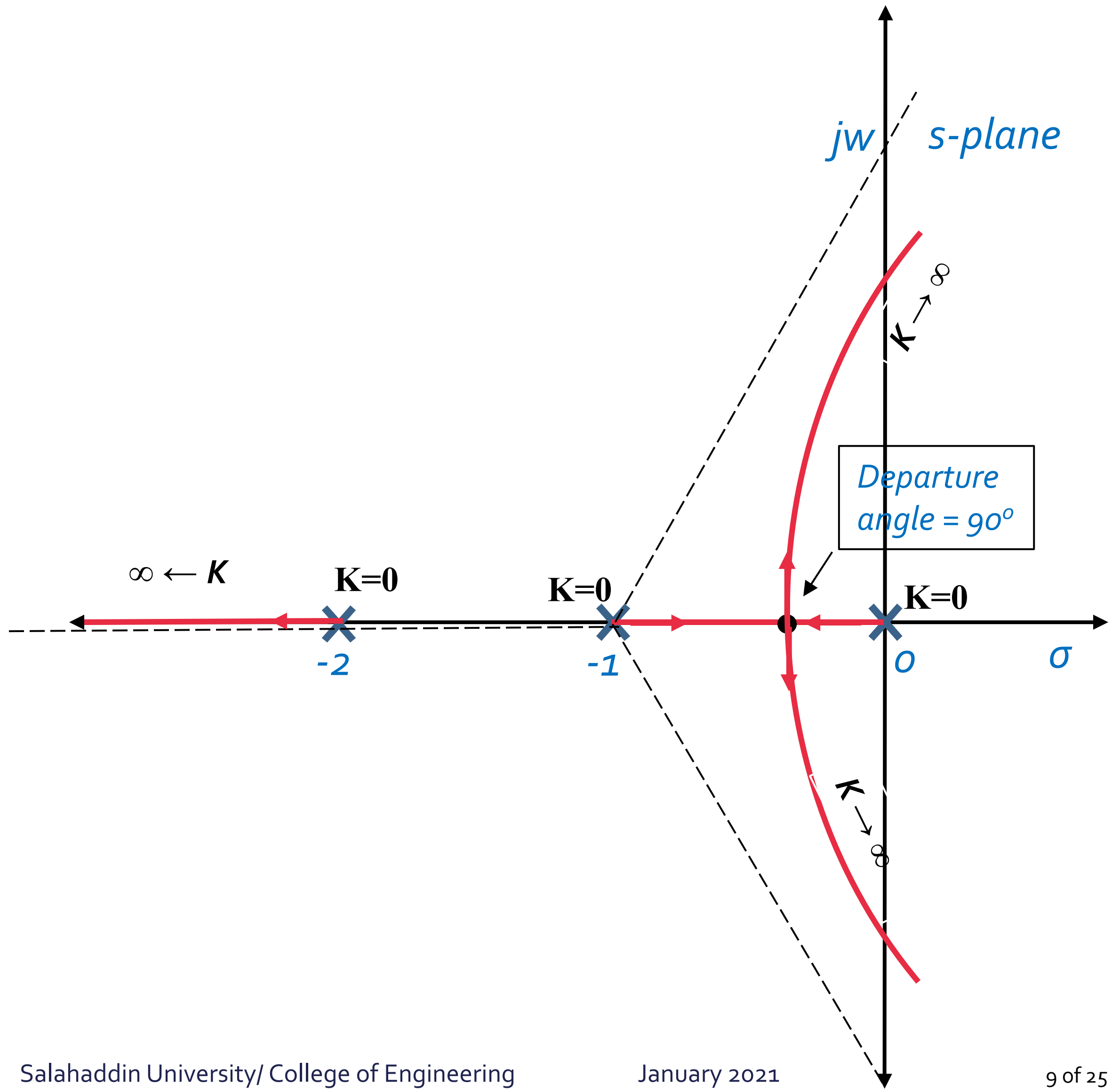
Thus: Breakaway point = -0.4233

Step8: departure angle at breakaway points (always) = ± 90⁰



Example 1 Cont'd

Step9: Thera are no complex poles and no complex zeros in this system.
Step10: Completing root locus of the system at poles extending to zeros



Example 2

In Example 1, find the root locus intersection points with imaginary axis.

Answer

Using Routh stability criterion method

the characteristic equation is $1 + \frac{K}{s(s+1)(s+2)} = 0$

re-arranging it: $s(s+1)(s+2) + K = 0$

$s^3 + 3s^2 + 2s + K = 0$

Routh Table is

s^3	1	2
s^2	3	K
s^1	(6 – k)/3	0
s^0	K	

For imaginary roots (intersection with imaginary axis) the first column should be zeros, thus

$(6 - K)/3 = 0$ **or** $K = 6$

Substituting K in auxiliary equation

$3s^2 + K = 0$

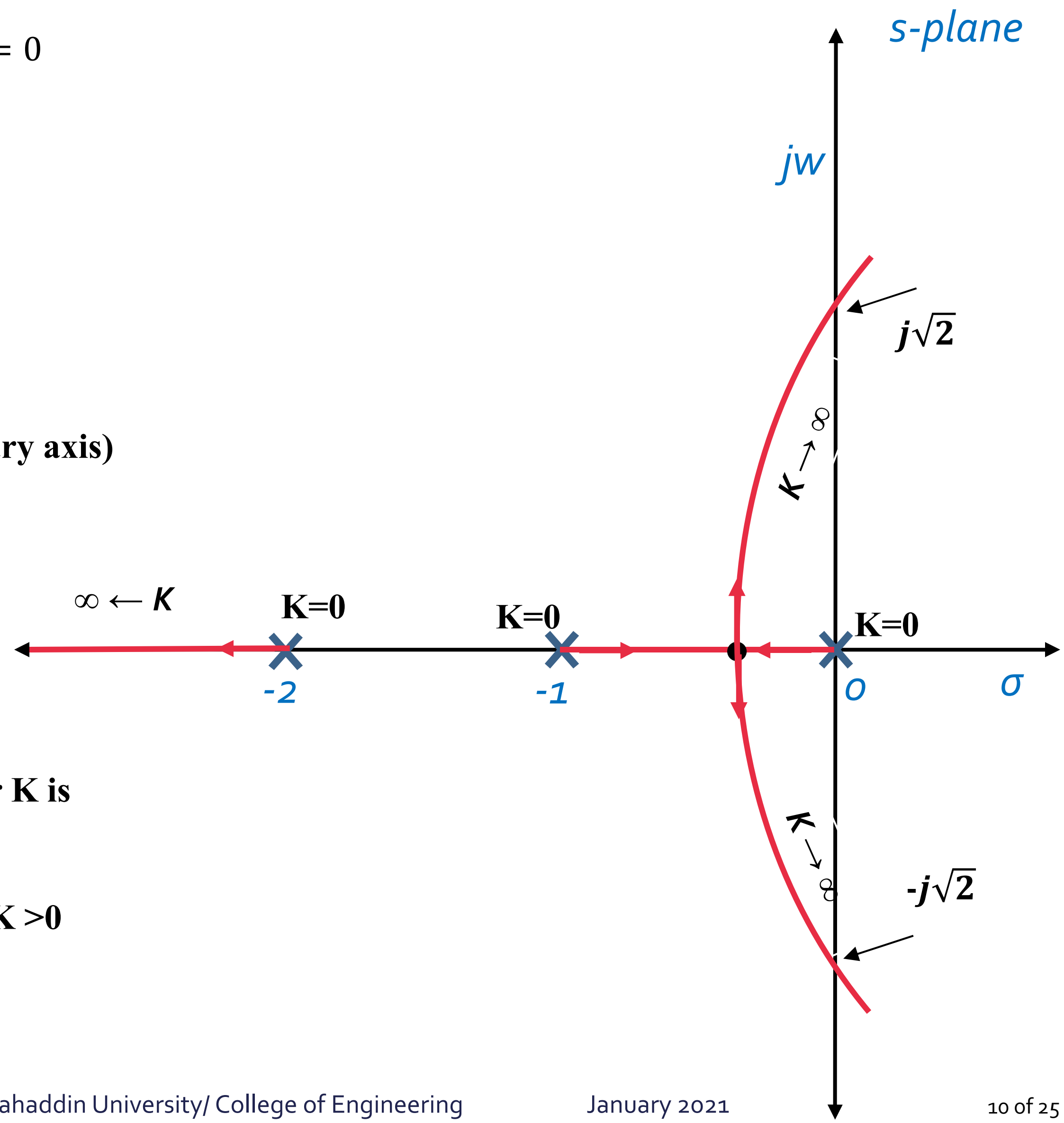
$3s^2 + 6 = 0$

$s^2 = -2$

$s = \pm j\sqrt{2}$

From the root locus plot, the stability range for K is

$0 < K < 6$



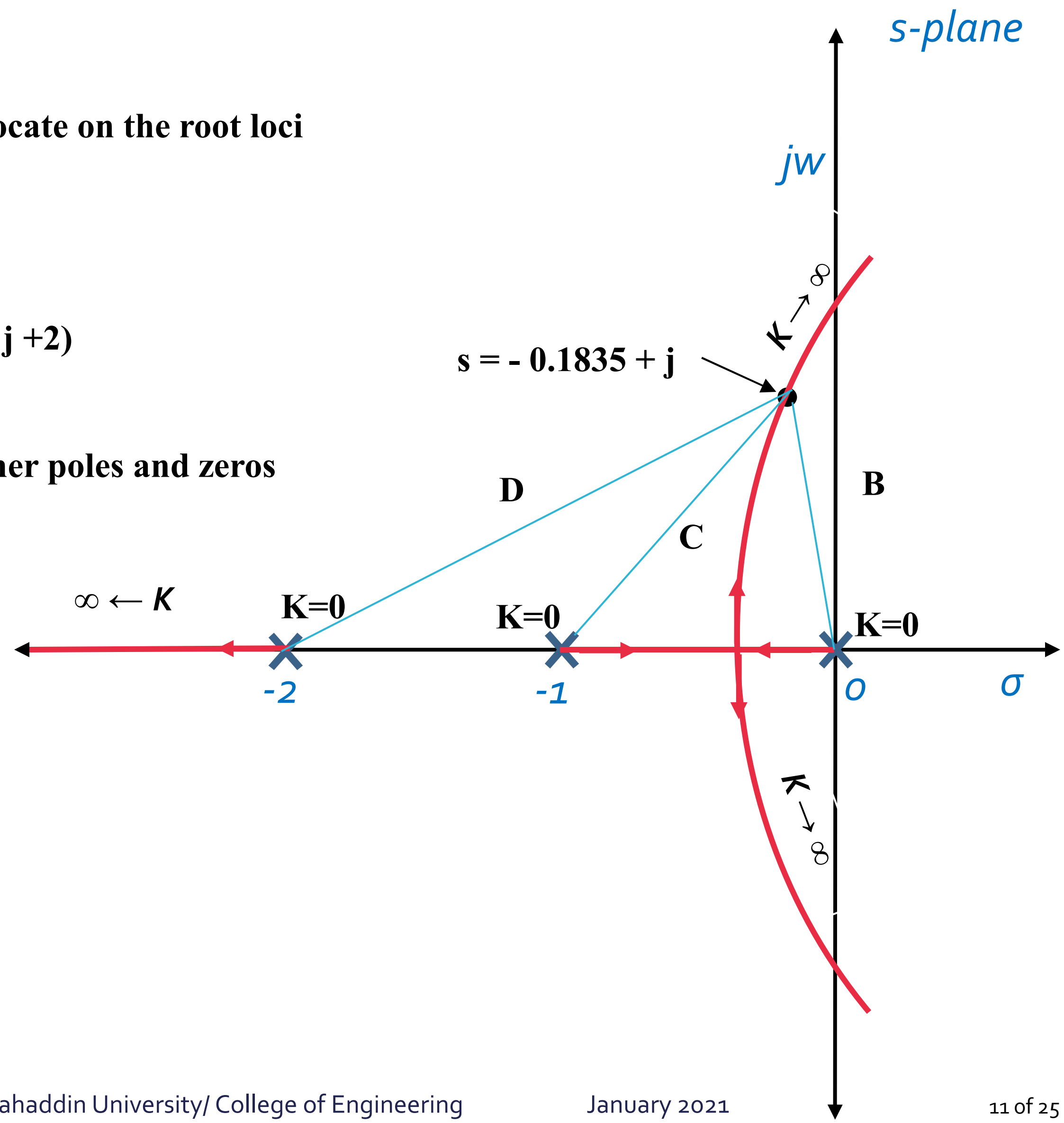
(From Routh table $(6 - K)/3 > 0$ **or** $K > 6$ **&** $K > 0$

thus for stability $0 < K < 6$ **)**

Example 3

In Example 1, at (s = - 0.1835 + j) find

1. K at this point
2. Damping ratio ζ, undamped natural frequency w_n, damped natural frequency w_d, maximum overshoot %OS, peak time t_p at this point



Answer

First we need to verify that (s = - 0.1835 + j) locate on the root loci

It is clear s is on root loci

1. Value of K can be fount in two ways

Method 1: from the characteristic equation

K = - s(s+1)(s+2)   at s = - 0.1835 + j

K = - (- 0.1835 +j)(- 0.1835 + j + 1)(- 0.1835 + j +2)

= 2.7217

Method 2: measuring distance from s with other poles and zeros

Line B = 1

Line C = 1.3

Line D =2.1

K = (B*C*D)/A

= (1 * 1.3 * 2.1)/ 1   (A =1 for no zeros)

= 2.73

2. From the root locus

$w_d = 1$ rad/sec

$w_n = \sqrt{(-0.1835)^2 + (1)^2} = 1.0091$ rad/sec

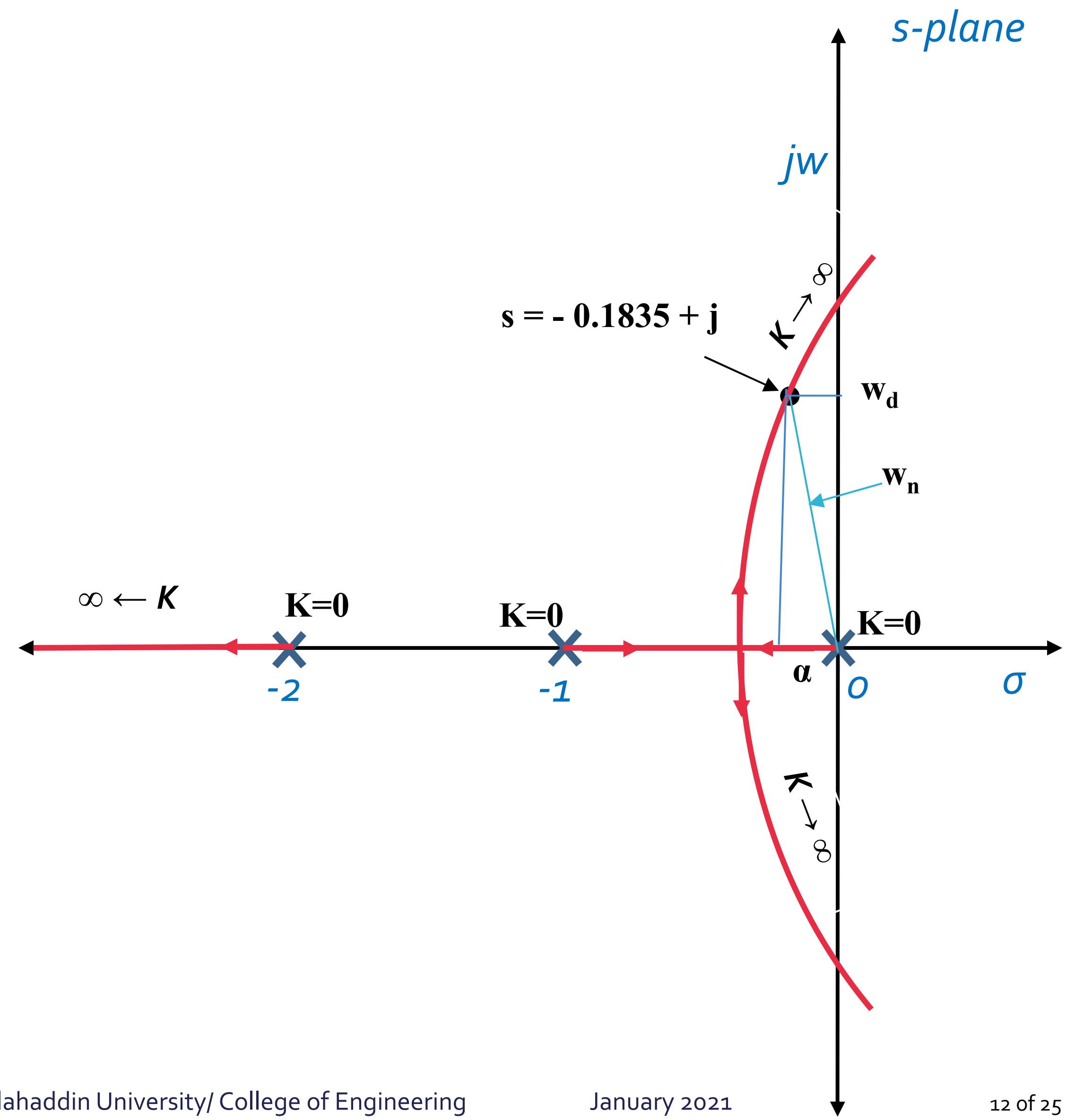
$w_d = w_n\sqrt{1 - \zeta^2}$

or $\alpha = w_n\zeta$ where $\alpha = 0.1835$

$$\zeta = \frac{\alpha}{w_n} = \frac{0.1835}{1.0091} = 0.1818$$

$$\%OS = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi} x100 = \%56.1$$

$$t_p = \frac{\pi}{w_d} = \frac{\pi}{1} = 3.14\ sec$$



Example 4

Sketch the root locus of the following system:

$$G(s)H(s) = \frac{K}{s(s+4)(s^2+4s+20)}$$

Answer

Step1: the characteristic equation is : $1 + \frac{K}{s(s+4)(s^2+4s+20)} = 0$

Re-arranging the characteristic equation will be:

$s^4 + 8s^3 + 36s^2 + 80s + K = 0$

Step2: No. of poles: n=4 **0, -4 , - 2 ± j4**

No. of zeros: m=0

Step3: Locate poles and zeros on s-plane and showing root loci region on real axis

area 1: [0 ∞] n = 0 and m = 0

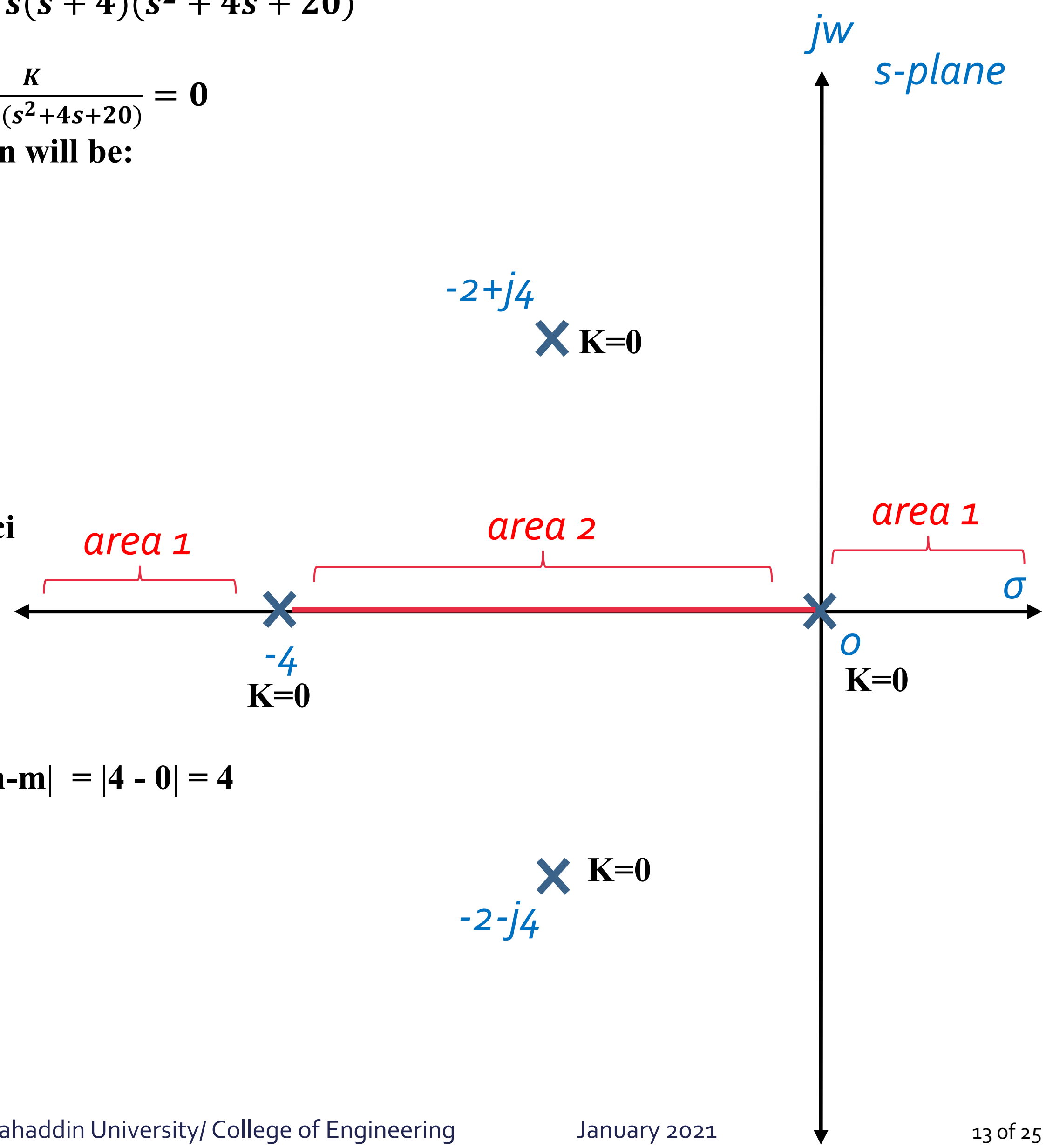
|n-m]=0 (even) no root loci

area 2: [-4 0] n=1 and m=0

[1 – 0] = 1 (odd) there is a root loci

area 3: [-∞ -4] n=2 and m=0

[2 – 0] = 2 (even) no root loci



Step4: No. of branches = No. of poles = n

here n = 4 so there are Four branches

No. of branches terminate at infinity = |n-m| = |4 - 0| = 4

Step5: No. of asymptotes = |n – m| = |4 – 0| = 4

Step6: Asymptotes' angles are

$\theta_a = \frac{\pm 180°(2q+1)}{n-m}$ **q ≡|n-m| → q = 0,1,2,3**

$\theta_{a1} = \frac{180°(2x0+1)}{4-0} = 45^0$

$$\theta_{a2} = \frac{180°(2x1+1)}{4-0} = 135^0$$
$$\theta_{a3} = \frac{180°(2x2+1)}{4-0} = 225^0$$
$$\theta_{a4} = \frac{180°(2x3+1)}{4-0} = 315^0$$

Centroid point of asymptotes is

$$\sigma_a = \frac{\sum_{i=1}^{n} P_i - \sum_{j=1}^{m} Z_j}{n-m} = \frac{(0 -1 -4 -2+j4 -2-j4)-0}{4-0} = -2$$

Step7: Breakaway points (if exists)

from the characteristic equation

K = - ($s^4 + 8s^3 + 36s^2 + 80s$)

taken dK/ds = 0

$$\frac{dK}{ds} = -(4s^3 + 24s^2 + 72s + 80) = 0$$

solving second order polynomial equations:

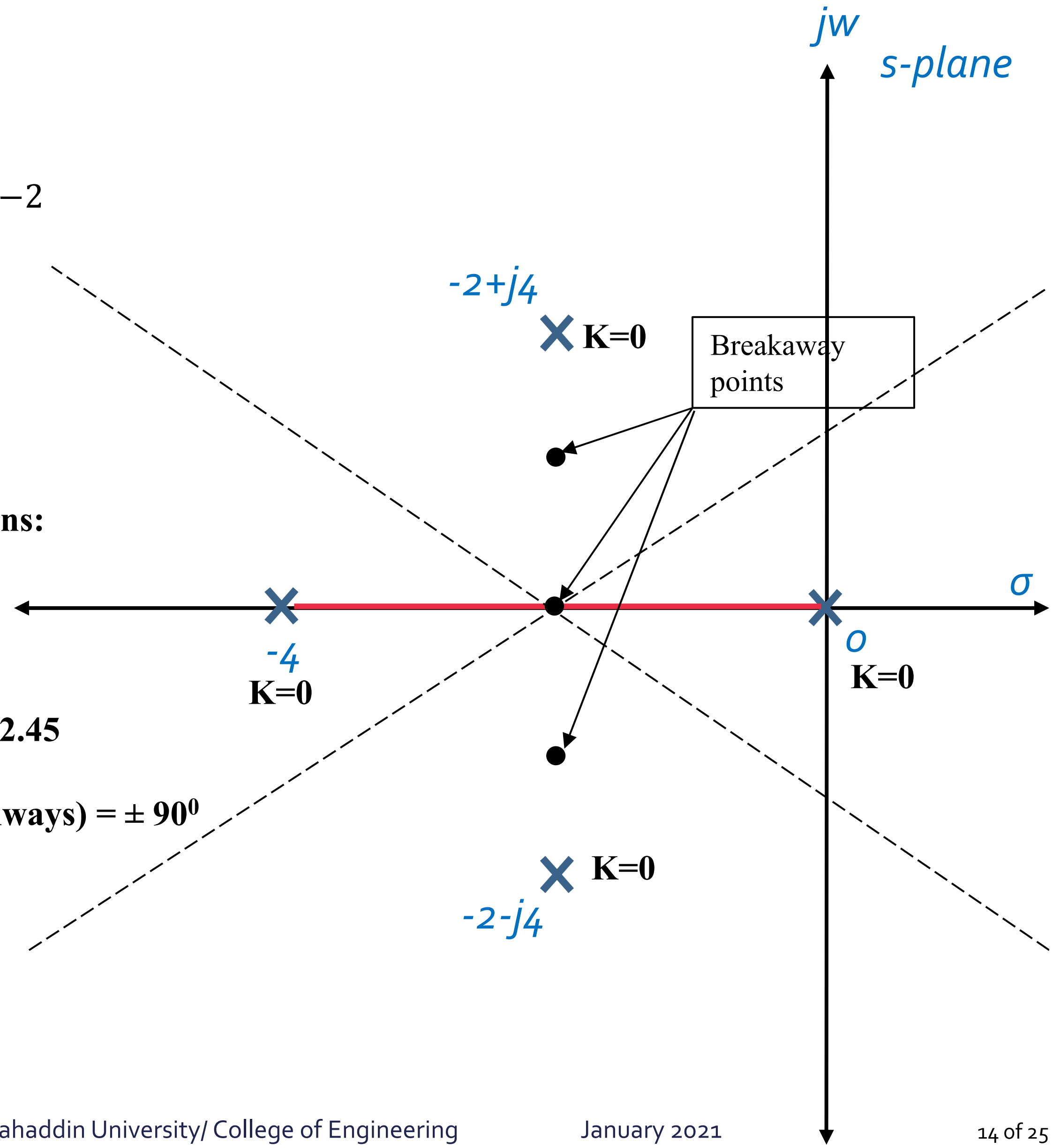
$s_{1,2,3}$ = -2 , -2 ±j2.45

since there is root loci at s = - 2

Thus: Breakaway at point = -2

and two other breakaways points at -2 ±j2.45

Step8: departure angle at breakaway points (always) = ± 90^0



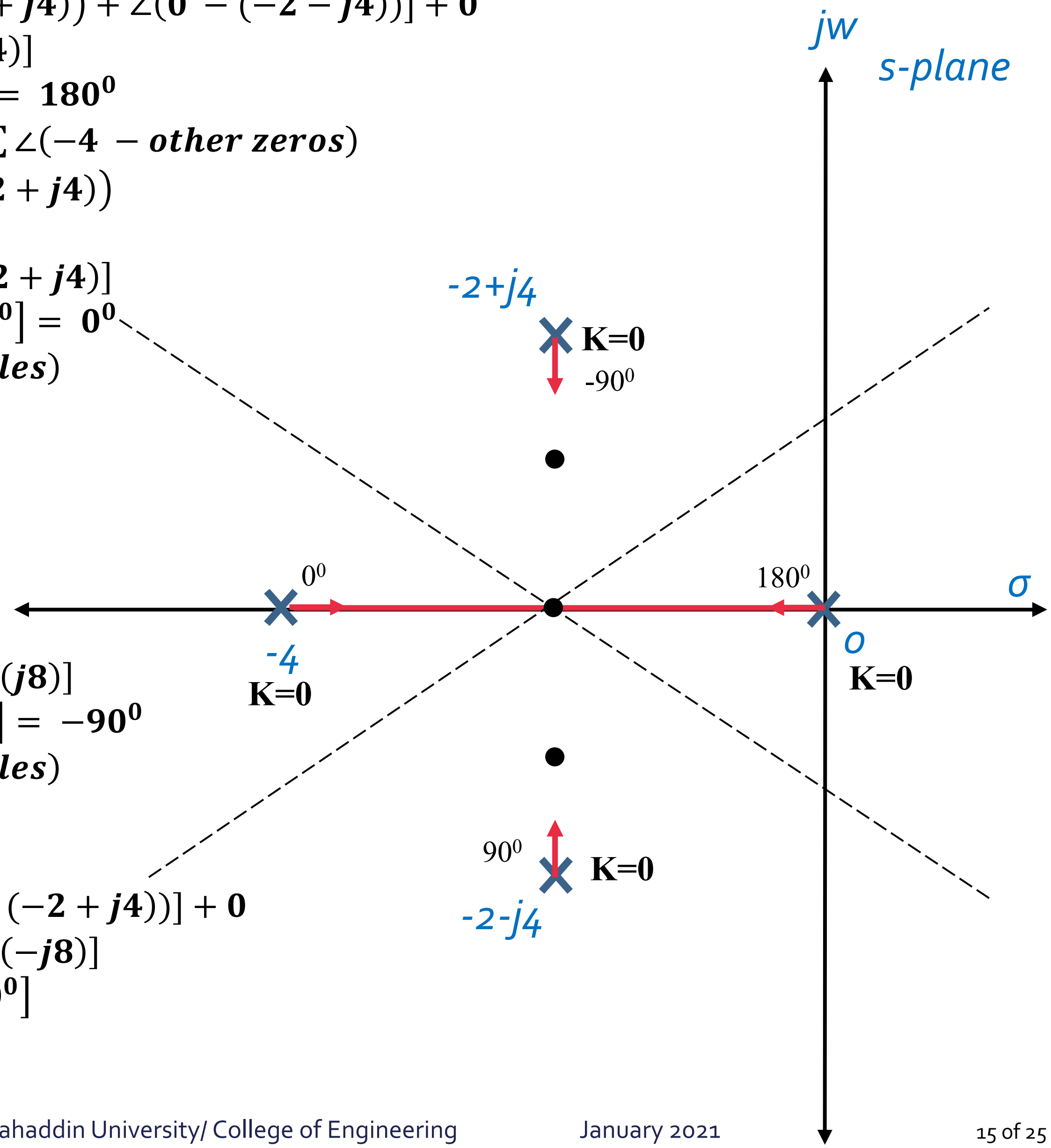
Step9: departure angles at poles

$$\phi_p|_0 = 180^0 - \sum\angle(0 - other\ poles) + \sum\angle(0 - other\ zeros)$$
$$= 180^0 - [\angle(0 - (-4)) + \angle(0 - (-2 + j4)) + \angle(0 - (-2 - j4))] + 0$$
$$= 180^0 - [\angle(4) + \angle(2 - j4) + \angle(2 + j4)]$$
$$= 180^0 - [0^0 - 63.435^0 + 63.435^0] = 180^0$$

$$\phi_p|_{-4} = 180^0 - \sum\angle(-4 - other\ poles) + \sum\angle(-4 - other\ zeros)$$
$$= 180^0 - [\angle(-4 - (0)) + \angle(-4 - (-2 + j4)) + \angle(-4 - (-2 - j4))] + 0$$
$$= 180^0 - [\angle(-4) + \angle(-2 - j4) + \angle(-2 + j4)]$$
$$= 180^0 - [180^0 - 116.56^0 + 116.56^0] = 0^0$$

$$\phi_p|_{-2+j4} = 180^0 - \sum\angle(-2 + j4 - other\ poles) + \sum\angle(-2 + j4 - other\ zeros)$$
$$= 180^0 - [\angle(-2 + j4 - (0)) + \angle(-2 + j4 - (-4)) + \angle(-2 + j4 - (-2 - j4))] + 0$$
$$= 180^0 - [\angle(-2 + j4) + \angle(2 + j4) + \angle(j8)]$$
$$= 180^0 - [116.56^0 + 63.435^0 + 90^0] = -90^0$$

$$\phi_p|_{-2-j4} = 180^0 - \sum\angle(-2 - j4 - other\ poles) + \sum\angle(-2 - j4 - other\ zeros)$$
$$= 180^0 - [\angle(-2 - j4 - (0)) + \angle(-2 - j4 - (-4)) + \angle(-2 - j4 - (-2 + j4))] + 0$$
$$= 180^0 - [\angle(-2 - j4) + \angle(2 - j4) + \angle(-j8)]$$
$$= 180^0 - [-116.56^0 - 63.435^0 - 90^0]$$
$$= 450^0 \ (= 90^0)$$



Step10: Intersection with imaginary axis

From the characteristic equation $s^4 + 8s^3 + 36s^2 + 80s + K = 0$

Creating Routh Table

s^4	1	36	K	
s^3	8	80	0	(÷8)
s^3	1	10	0	
s^2	26	K	0	
s^1	(260-K)/26	0		
s^0	K			

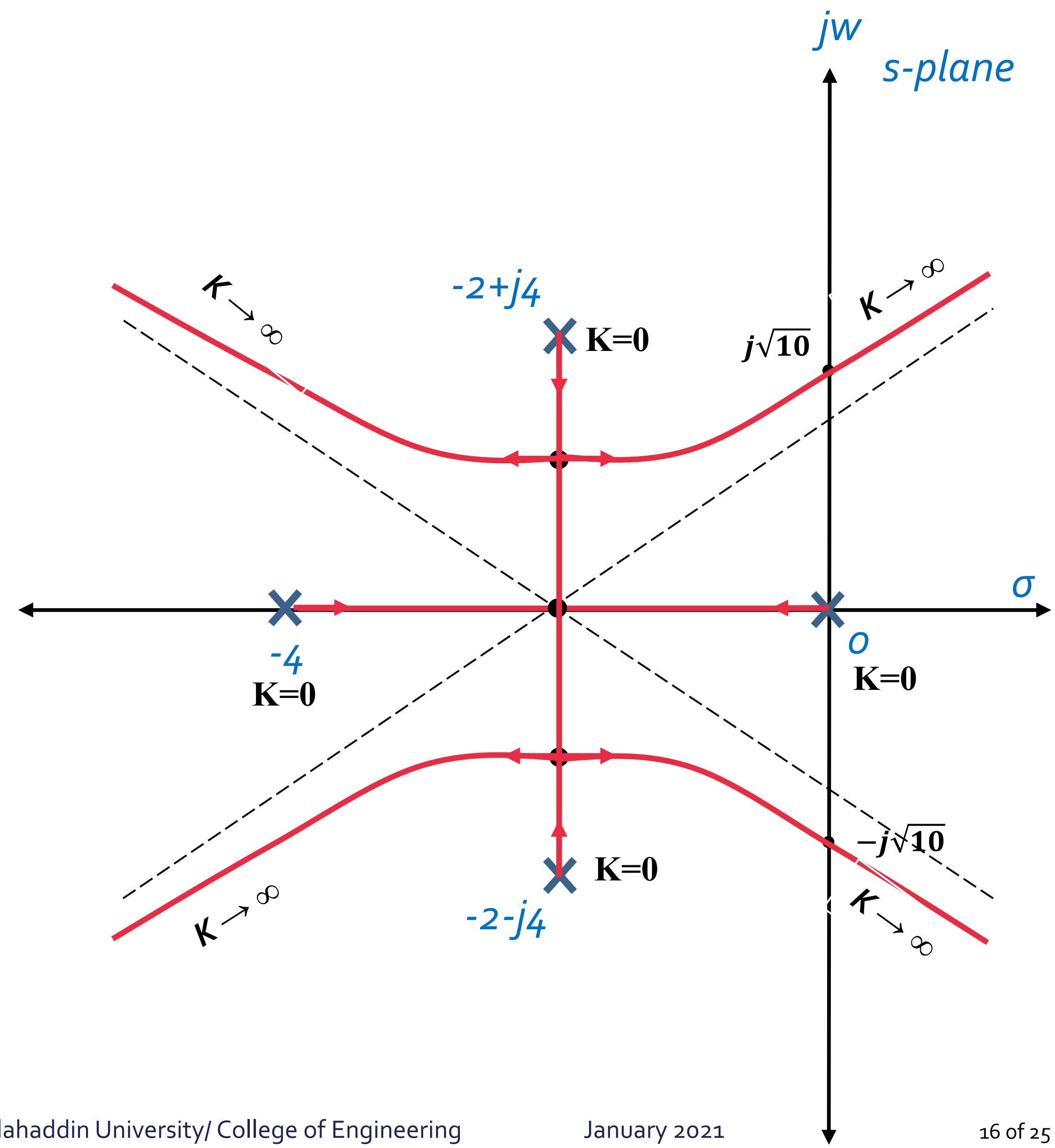
$\frac{260-K}{26} = 0 \;\rightarrow\; K = 260$

Substituting K in auxiliary equation

$26s^2 + K = 0$

$26s^2 + 260 = 0$

$s = \pm j\sqrt{10}$

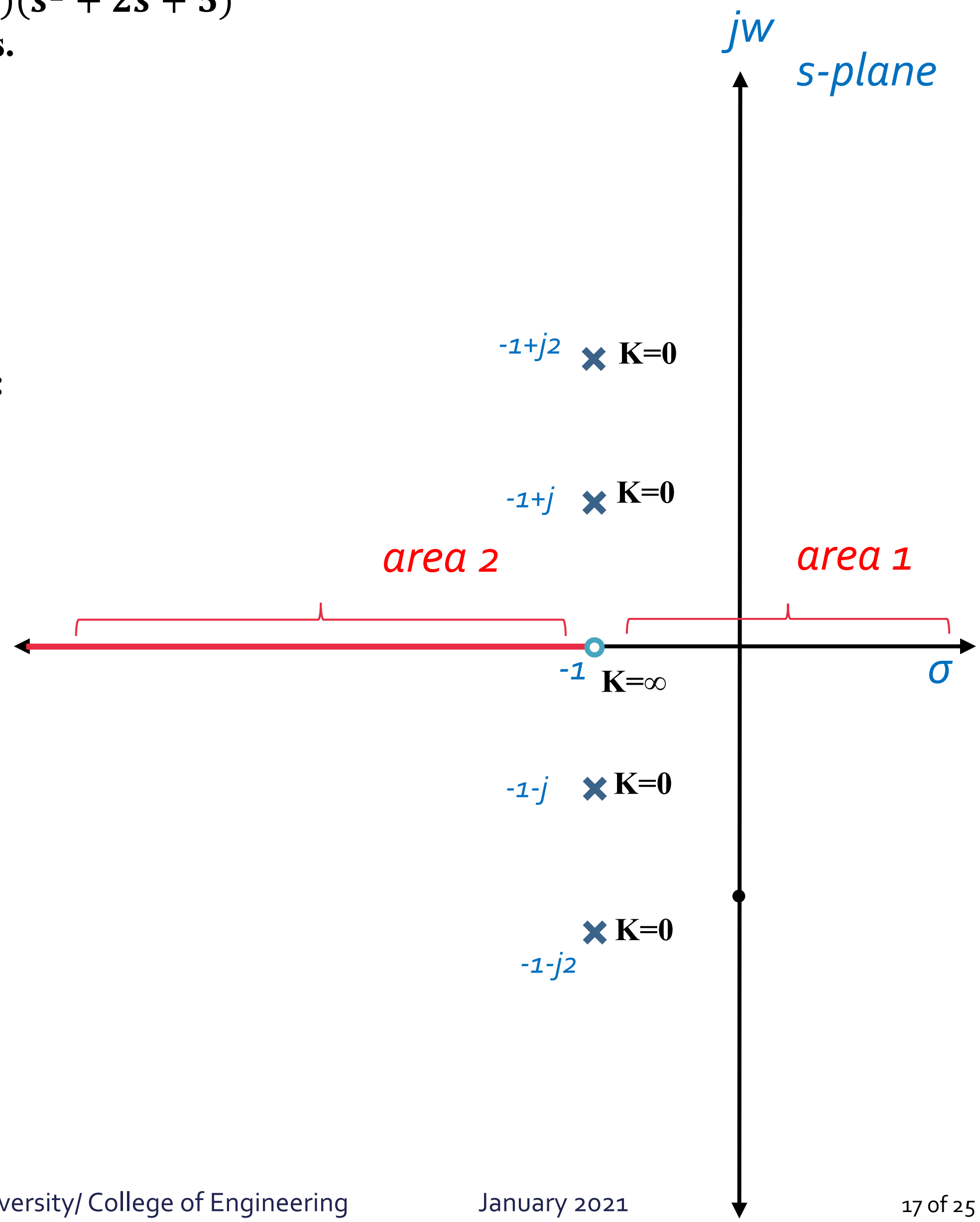


Example 5

Sketch the root locus of the following unity-feedback control system:

$$G(s) = \frac{K(s+1)}{(s^2+2s+2)(s^2+2s+5)}$$

Also find the intersection points with the imaginary axis.



Answer

Step1: the characteristic equation is : 1 + G(s)H(s) = 0

for unity feedback system, H(s) = 1

$$1 + \frac{K(s+1)}{(s^2+2s+2)(s^2+2s+5)} . 1 = 0$$

Re-arranging the characteristic equation will be:

$$K = -\frac{(s^2+2s+2)(s^2+2s+5)}{(s+1)}$$

$$s^4 + 4s^3 + 11s^2 + (14+K)s + (10+K) = 0$$

Step2: No. of poles: n = 4 $s_{1,2} = -1 \pm j2$

$s_{3,4} = -1 \pm j$

No. of zeros: m = 1 $s_0 = -1$

Step3: Locate poles and zeros on s-plane and showing root loci region on real axis

area 1: [-1 ∞] n = 0 and m = 0

|n-m]=0 (even) no root loci

area 2: [- ∞ -1] n=0 and m=1

[0 – 1] = 1 (odd) there is a root loci

At zero (s = -1)

$$K = -\frac{((-1)^2+2(-1)+2)((-1)^2+2(-1)+5)}{((-1)+1)} = \infty$$

At poles: K = 0

Step4: No. of branches = n = 4

No. of branches terminate at infinity = |n – m| = |4-1| = 3

Therefore there are 4 branches three of them end at infinity and one complete in zero = -1

Step5: No. of asymptotes = |n – m| = |4 – 1| = 3

Step6: Asymptotes' angles are

$$\theta_a = \frac{\pm 180°(2q+1)}{n-m}$$ **q ≡|n-m| ≡ 4-1 ≡3 → q = 0,1,2**

$$\theta_{a1} = \frac{180°(2x0+1)}{4-1} = 60^0$$

$$\theta_{a1} = \frac{180°(2x1+1)}{4-1} = 180^0$$

$$\theta_{a1} = \frac{180°(2x2+1)}{4-1} = 300^0$$

Centroid point of asymptotes is

$$\sigma_a = \frac{\sum_{i=1}^{n} P_i - \sum_{j=1}^{m} Z_j}{n-m} = \frac{(-1+j2-1-j2-1+j-1-j)-(-1\_}{4-1} = -1$$

Step7: Breakaway and breakin points (if exists)

$$\mathbf{K = -\frac{(s^2+2s+2)(s^2+2s+5)}{(s+1)} = -\frac{(s^4+4s^3+11s^2+14s+10)}{(s+1)}}$$

taken dK/ds = 0

$$\frac{dK}{ds} = -\left[\frac{(4s^3+12s^2+22s+14)(s+1)-(s^4+4s^3+11s^2+14s+10)}{(s+1)^2}\right] = 0$$

$$3s^4 + 12s^3 + 23s^2 + 22s + 4 = 0$$

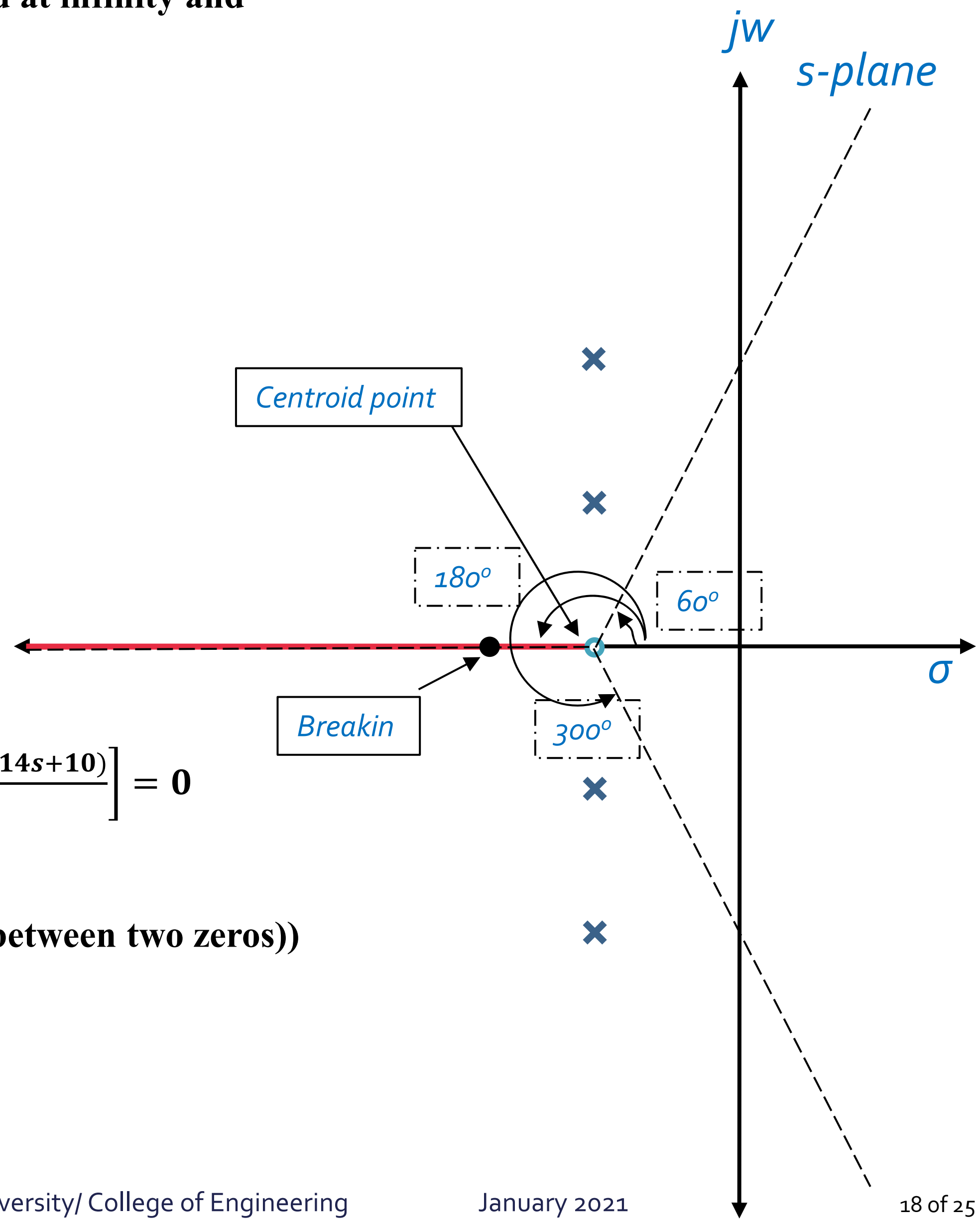
$s_{1,2}$ = -1 ± j1.5024 (no root loci – neglect)

s_3 = -1.7685 (root loci – breakin point (between two zeros))

s_4 = -0.2315 (no root loci - neglect

No breakaway points in this system

Arrival angle at breakin point = ± 90^0



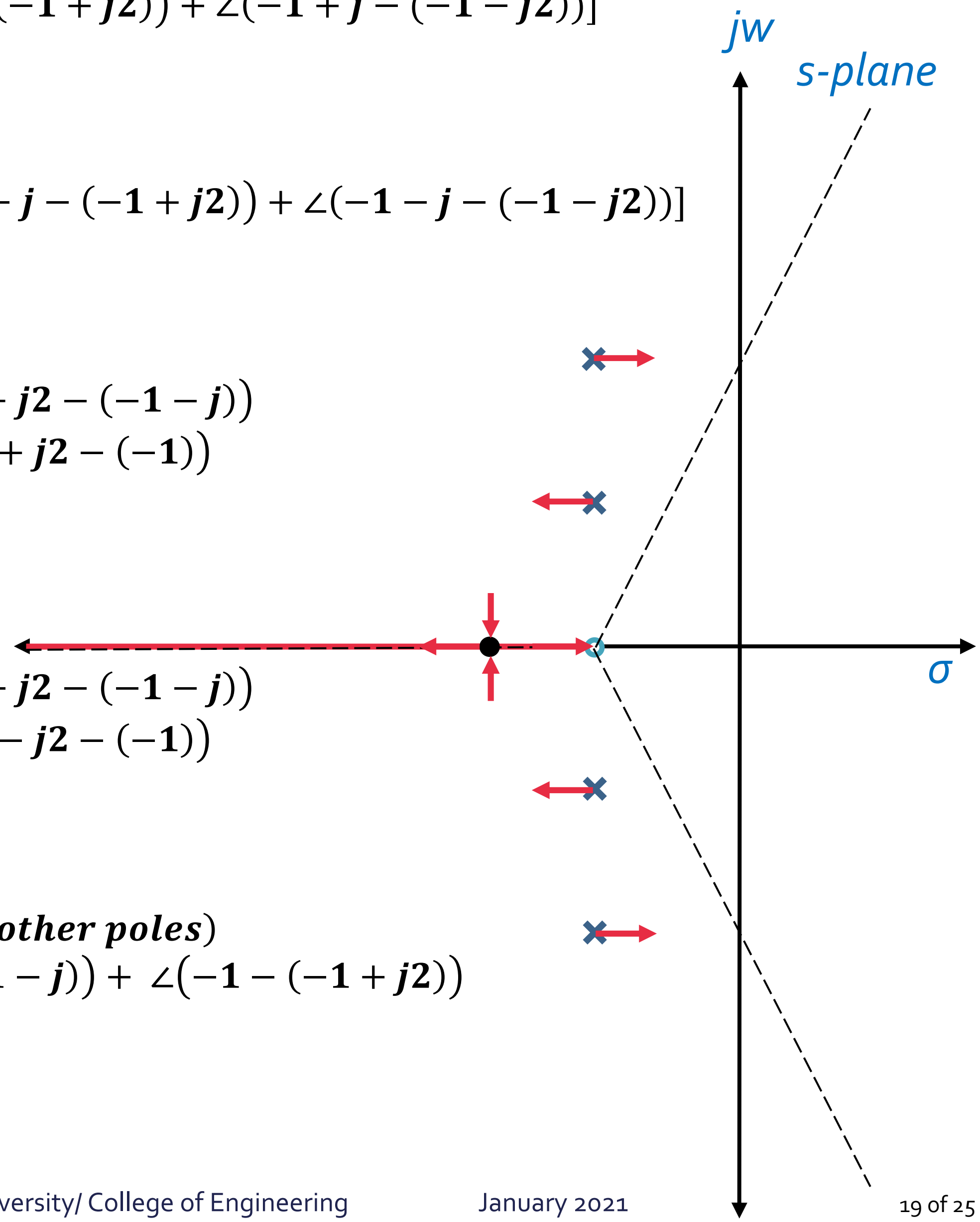
Step8: Departure and arrival angles at poles and zeros

$$\boldsymbol{\phi_p|_{-1+j} = 180^0 - \sum\angle(-1+j\ - other\ poles) + \sum\angle(-1+j\ - other\ zeros)}$$

$$= 180^0 - [\angle(-1+j-(-1-j)) + \angle(-1+j-(-1+j2)) + \angle(-1+j-(-1-j2))]$$
$$+\angle(-1+j-(-1))$$
$$= 180^0 - [\angle(j2) + \angle(-j) + \angle(j3)] + \angle(j)$$
$$= 180^0 - [90^0 - 90^0 + 90^0] + 90^0 = 180^0$$

$$\phi_p|_{-1-j} = 180^0 - [\angle(-1-j-(-1+j)) + \angle(-1-j-(-1+j2)) + \angle(-1-j-(-1-j2))]$$
$$+\angle(-1-j-(-1))$$
$$= 180^0 - [\angle(-j2) + \angle(-j3) + \angle(j)] + \angle(-j)$$
$$= 180^0 - [-90^0 + 90^0 - 90^0] - 90^0 = 180^0$$

$$\phi_p|_{-1+j2} = 180^0 - [\angle(-1+j2-(-1+j)) + \angle(-1+j2-(-1-j))$$
$$+\angle(-1+j2-(-1-j2))] + \angle(-1+j2-(-1))$$
$$= 180^0 - [\angle(j) + \angle(3) + \angle(j4)] + \angle(j2)$$
$$= 180^0 - [90^0 + 90^0 + 90^0] + 90^0 = 0^0$$



$$\phi_p|_{-1-j2} = 180^0 - [\angle(-1-j2-(-1+j)) + \angle(-1-j2-(-1-j))$$
$$+\angle(-1-j2-(-1+j2))] + \angle(-1-j2-(-1))$$
$$= 180^0 - [\angle(-j3) + \angle(-j) + \angle(-j4)] + \angle(-j2)$$
$$= 180^0 - [-90^0 - 90^0 - 90^0] - 90^0 = 360^0$$

$$\phi_z|_{-1} = 180^0 - \sum\angle(-1\ - other\ zeros) + \sum\angle(-1\ - other\ poles)$$
$$= 180^0 - 0 + [\angle(-1-(-1+j)) + \angle(-1-(-1-j)) + \angle(-1-(-1+j2))$$
$$+\angle(-1-(-1-j2))]$$
$$= 180^0 + [\angle(-j) + \angle(j) + \angle(-j2) + \angle(j2)]$$
$$= 180^0 + [-90^0 + 90^0 - 90^0 + 90^0] = 180^0$$

Step9: Intersection points between root locus and imaginary axis
Using Roth table from characteristic equation

$$s^4 + 4s^3 + 11s^2 + (14+K)s + (10+K) = 0$$

s^4	1	11	(10+K)
s^3	4	(14+K)	0
s^2	$\frac{44-(14+K)}{4}$	$(10+K)$	0
s^1	$\frac{\frac{(30-K)(14+K)}{4}-(40+4K)}{\frac{30-K}{4}}$	0	
s^0	(10+K)		

1) $\frac{30-K}{4} = 0 \quad \Rightarrow \quad K = 30$

2) $\frac{\frac{(30-K)(14+K)}{4}-(40+4K)}{\frac{30-K}{4}} = \frac{\frac{420+16K-K^2-4(40+4K)}{4}}{\frac{30-K}{4}} = \frac{420+16K-K^2-160-16K}{30+K}$

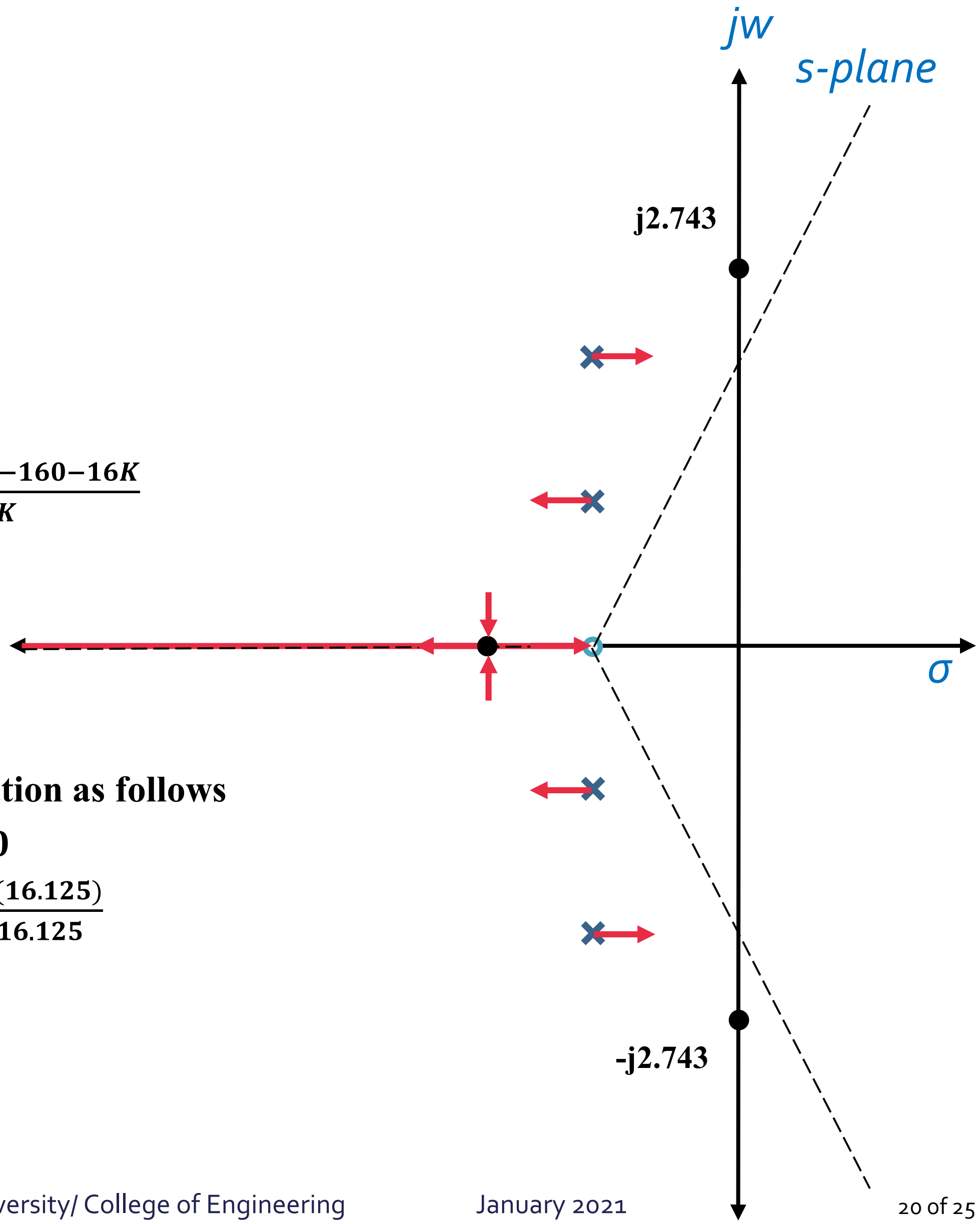
$260 - K^2 = 0 \Rightarrow \quad K = \pm 16.125$

3) $10 + K = 0 \; K = -10$

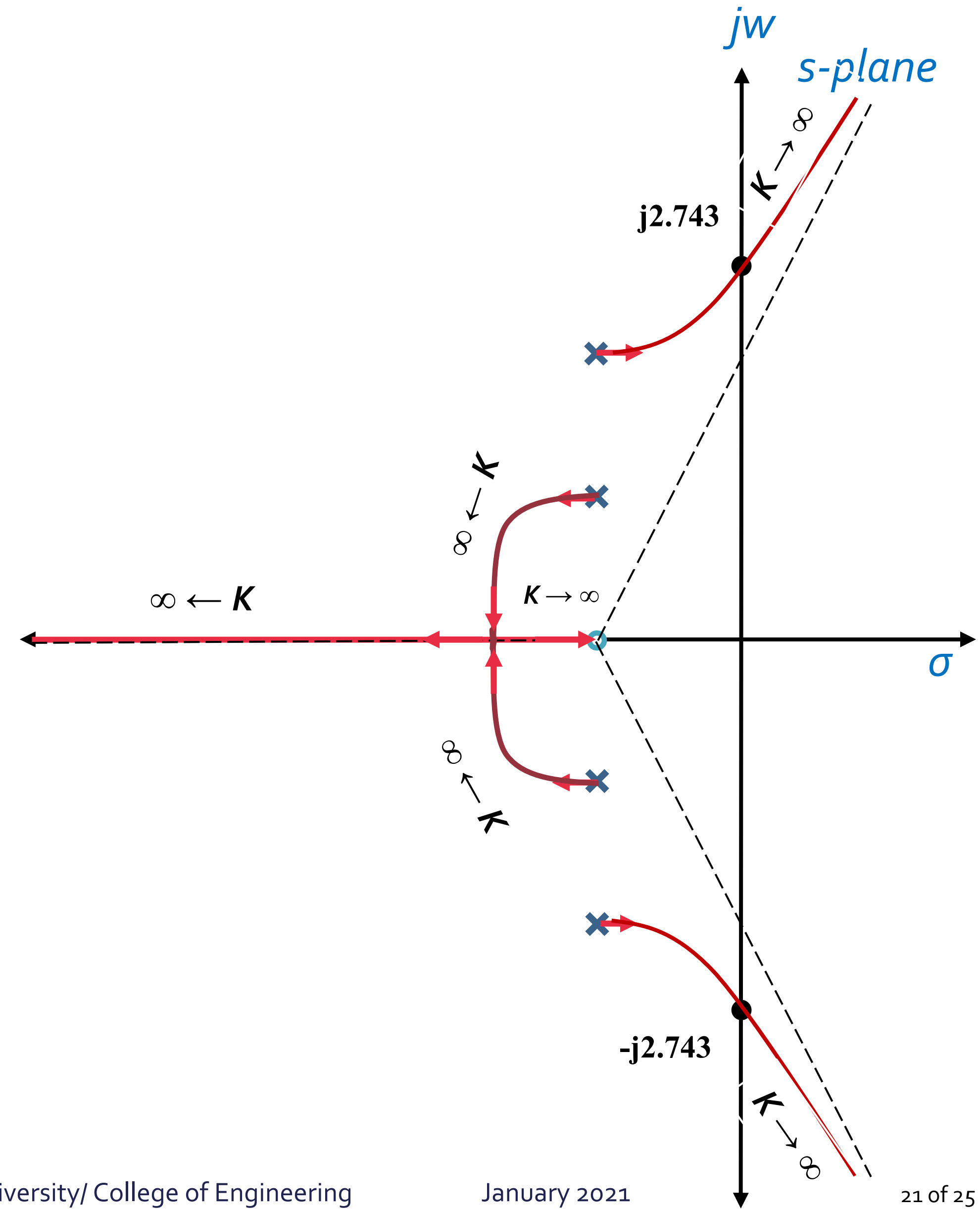
K cannot be negative and K is between 30 and 16.125
But K = 30 cannot be used because of (3) above, thus
K = 16.125 at intersection points
For find intersection points, we are using auxiliary equation as follows
in s^2 row, the auxiliary equation is $\frac{30-K}{4}s^2 + 10 + K = 0$

$$(30-K)s^2 + 40 + 4K = 0 \Rightarrow s^2 = -\frac{40+4K}{30-K} = -\frac{40+4(16.125)}{30-16.125}$$

$$s = \pm j2.743$$

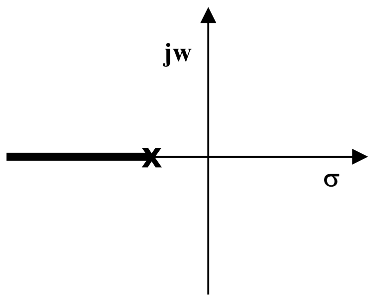


Step10: Completing the root locus

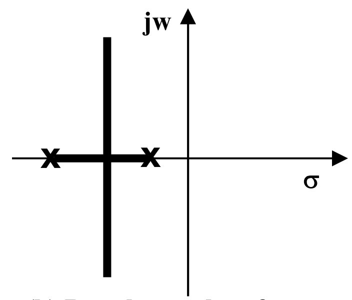


Effect of the Addition of Poles

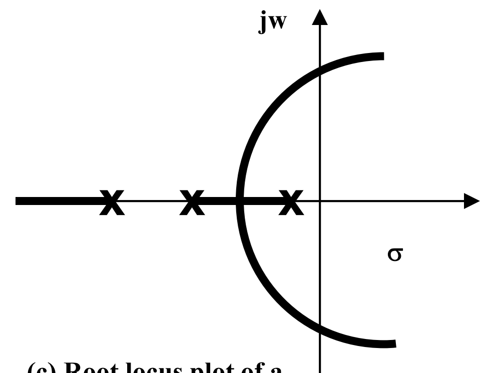
The addition of a pole to the open loop transfer function has the effect of pulling the root locus to the right, tending to lower the system's relative stability and to slow down the settling of the response. The following figure shows examples of root loci illustrating the effects of the addition of a pole to a single pole system and the addition of two poles to a single pole system.



(a) Root locus plot of a single pole system

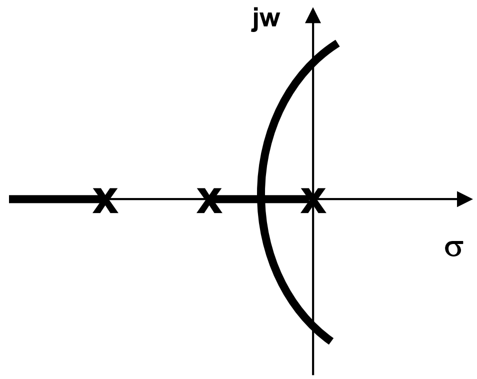


(b) Root locus plot of a tow pole system

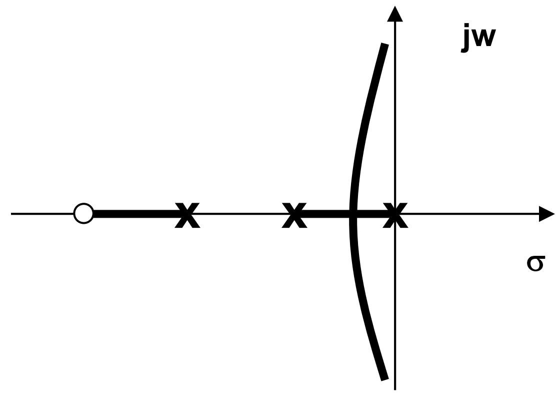


(c) Root locus plot of a three -pole system

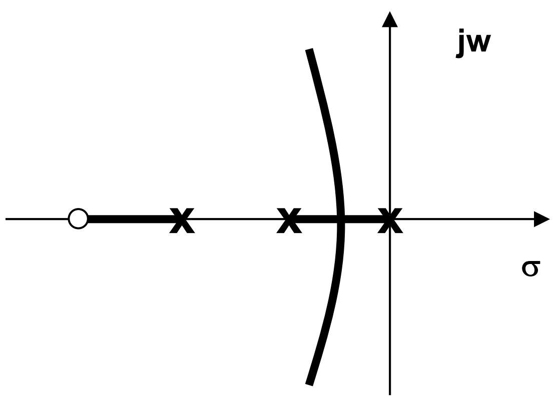
The addition of a zero to the open loop transfer function has the effect of pulling the root locus to the left, tending to make the system more stable and to speed up the settling of the response. The following figure shows examples of root loci illustrating the effect of the addition of zero(s) to system.



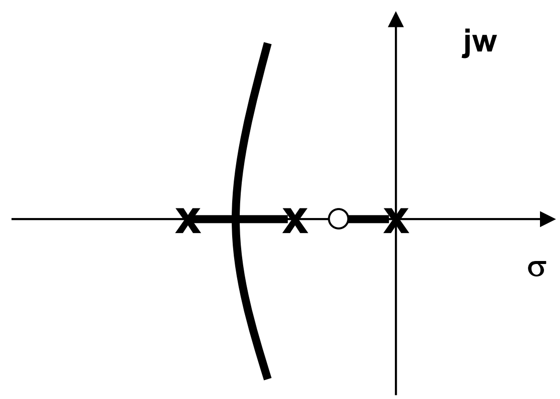
(a) Root locus plot of a three -pole system



(b)



(c)



(d)

(b),(c) and (d) Root locus plots showing effects of addition of a zero to the three pol system.

Assignment 7.1

Use root locus technique to find the values of K and poles at intersection points with the imaginary axis for the following control system:

$$G(s)H(s) = \frac{K(s+5)}{s(s+1)(s+2)(s+3)}$$

End of Chapter Seven!