# Robust Control

## Lecture 1

### Introduction

**Postgraduate Course, PhD**
**Electrical Engineering Department**
**College of Engineering**
**University of Salahaddin**

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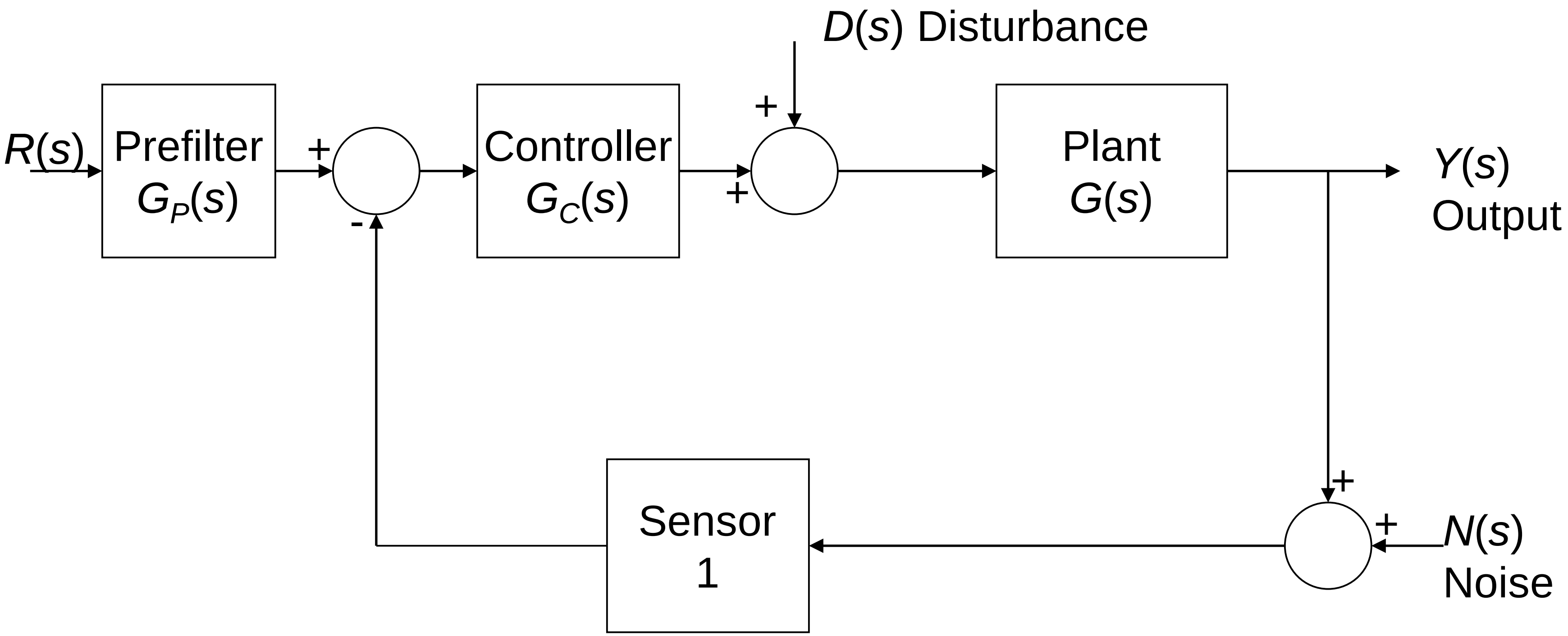
- **When we design a control system, our ultimate goal is to control a particular system in a real environment.**

- **When we design the control system we make numerous assumptions about the system and then we describe the system with some sort of mathematical model.**

- **Using a mathematical model permits us to make predictions about how the system will behave, and we can use any number of simulation tools and analytical techniques to make those predictions.**

- **Any model incorporates two important problems that are often encountered:**
  - **A disturbance signal is added to the control input to the plant. That can account for wind gusts in airplanes, changes in ambient temperature in ovens, etc., and**
  - **A noise that is added to the sensor output.**



D(s) Disturbance
R(s)
Prefilter
G_P(s)
+
-
Controller
G_C(s)
+
+
Plant
G(s)
Y(s)
Output
Sensor
1
+
+
N(s)
Noise

## *Why Robust? Cont.*

- **A robust control system exhibits the desired performance despite the presence of significant plant (process) uncertainty.**
- **The goal of robust design is to retain assurance of system performance in spite of model inaccuracies and changes.**
- **A system is robust when it has acceptable changes in performance due to model changes or inaccuracies.**

**The process of designing a control system usually makes many demands of the engineer or engineering team. These demands often emerge in a step by step design procedure as follows:**

- **Study the system (plant) to be controlled and obtain initial information about the control objectives.**
- **Model the system and simplify the model, if necessary.**
- **Scale the variables and analyze the resulting model; determine its properties.**
- **Decide which variables are to be controlled (controlled outputs).**
- **Decide on the measurements and manipulated variables: what sensors and actuators will be used and where will they be placed?**
- **Select the control configuration.**
- **Decide on the type of controller to be used.**

- **Decide on performance specifications, based on the overall control objectives.**
- **Design a controller.**
- **Analyze the resulting controlled system to see if the specifications are satisfied; and if they are not satisfied modify the specifications or the type of controller.**
- **Simulate the resulting controlled system, either on a computer or a pilot plant.**
- **Repeat from steps, if necessary.**
- **Choose hardware and software and implement the controller.**
- **Test and validate the control system, and tune the controller on-line, if necessary.**

- **Input-output controllability is the ability to achieve acceptable control performance.**
- **It is affected by the location of sensors and actuators, but otherwise it cannot be changed by the control engineer.**
- **Therefore, the process of control system design should in some cases also include a step 0, involving the design of the process equipment itself.**
- **The idea of looking at process equipment design and control system design as an integrated whole is not new, as is clear from the following quote taken from a paper by Ziegler and Nichols (1943):**

- **In the application of automatic controllers, it is important to realize that controller and process form a unit; credit or discredit for results obtained are attributable to one as much as the other.**
- **A poor controller is often able to perform acceptably on a process which is easily controlled.**
- **The finest controller made, when applied to a miserably designed process, may not deliver the desired performance.**

- **On badly designed processes, advanced controllers are able to eke out better results than older models, but on these processes, there is a definite end point which can be approached by instrumentation and it falls short of perfection.**

- **Ziegler and Nichols then proceed to observe that there is a factor in equipment design that is neglected, and state that**

***. . . the missing characteristic can be called the "controllability", the ability of the process to achieve and maintain the desired equilibrium value.***

- **The objective of a control system is to make the output *y* behave in a desired way by manipulating the plant input *u*.**
- **The regulator problem is to manipulate *u* to counteract the effect of a disturbance *d*.**
- **The servo problem is to manipulate *u* to keep the output close to a given reference input *r*.**
- **Thus, in both cases we want the control error $e = y - r$ to be small.**
- **The algorithm for adjusting *u* based on the available information is the controller *K*.**
- **To arrive at a good design for *K* we need a priori information about the expected disturbances and reference inputs, and of the plant model *(G)* and disturbance model *(Gd)*.**
- **Considering a linear models of the form**

$$y = Gu + G_d\, d$$

- **A major source of difficulty is that the models *($G$, $G_d$)* may be inaccurate or may change with time.**

- **In particular, inaccuracy in *$G$* may cause problems because the plant will be part of a feedback loop.**

- **To deal with such a problem we will make use of the concept of model uncertainty.**

- **For example, instead of a single model *$G$* we may study the behavior of a class of models, $G_p = G + E$, where the model "*uncertainty*" or "*perturbation*" *$E$* is bounded, but otherwise unknown.**

- **In most cases weighting functions, *$w(s)$*, are used to express $E = w\,\Delta$ in terms of normalized perturbations, $\Delta$, where the magnitude (norm) of $\Delta$ is less than or equal to 1.**

**The following terms are useful:**

**<u>Nominal stability (NS)</u>**

**The system is stable with no model uncertainty.**

**<u>Nominal Performance (NP)</u>**

**The system satisfies the performance specifications with no model uncertainty.**

**<u>Robust stability (RS)</u>**

**The system is stable for all perturbed plants about the nominal model up to the worst-case model uncertainty.**

**<u>Robust performance (RP)</u>**

**The system satisfies the performance specifications for all perturbed plants about the nominal model up to the worst-case model uncertainty.**

- **A general example of linear, time-invariant systems whose input-output responses are governed by linear ordinary differential equations with constants coefficients can be expressed as:**

$$\begin{aligned}\dot{x}_1(t) &= -a_1 x_1(t) + x_2(t) + \beta_1 u(t)\\ \dot{x}_2(t) &= -a_0 x_1(t) + \beta_0 u(t)\\ y(t) &= x_1(t)\end{aligned}$$

- **where $\dot{x}(t) \equiv dx/dt$ , u(t) represents the input signals, $x_1$(t) and $x_2$(t) are states, and y(t) is the output signal.**

- **The system is time-invariant since the coefficients $a_0$, $a_1$, $\beta_0$ and $\beta_1$ are independent of time.**

- **Applying the Laplace transform, and if u(t), $x_1$(t), $x_2$(t), and y(t) represent variables away from a nominal operating point or trajectory, then we can assume $x_1(t=0) = x_2(t=0) = 0$, these yields the transfer function as;**

$$\frac{y(s)}{u(s)} = G(s) = \frac{\beta_1 s + \beta_0}{s^2 + a_1 s + a_0}$$

- **Importantly, for linear systems, the transfer function is independent of the input signal (forcing function).**
- **Notice that the transfer function may also represent the following system;**

$$\ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) = \beta_1\dot{u}(t) + \beta_0 u(t)$$

**with input u(t) and output y(t).**

- **More generally, the rational transfer function has the following from;**

$$G(s) = \frac{\beta_{n_z} s^{n_z} + \cdots + \beta_1 s + \beta_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

- **For multivariable systems, G(s) is a matrix of transfer functions.**
- **n is the order of the denominator (or pole polynomial) and is also called the order of the system, and $n_z$ is the order of the numerator (or zero polynomial).**
- **Then $n - n_z$ is referred to as the pole excess or relative order.**

- **Definitions**
- **A system G(s) is strictly proper if G(jw) → 0 as w → ∞**
- **A system G(s) is semi-proper or bi-proper if G(jw) → D ≠ 0 as w → ∞**
- **A system G(s) which is strictly proper or semi-proper is proper.**
- **A system G(s) is improper if G(jw) → ∞ as w → ∞**

- **For a proper system, with $n \geq nz$, the state-space description,**

$$\dot{x} = Ax + Bu, \; y = Cx + Du$$

- **The transfer function can be written as**

$$G(s) = C(sI - A)^{-1}B + D$$

- **or**

$$G(s) \stackrel{S}{=} \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

- **All practical systems will have zero gain at a sufficiently high frequency, and are therefore strictly proper.**

- **It is often convenient, however, to model high frequency effects by a non-zero D-term, and hence semi-proper models are frequently used.**

- **Furthermore, certain derived transfer functions, such as $S = (I + GK)^{-1}$ are semi-proper.**

- **Usually we use G(s) to represent the effect of the inputs u on the outputs y, whereas Gd(s) represents the effect on y of the disturbances d.**

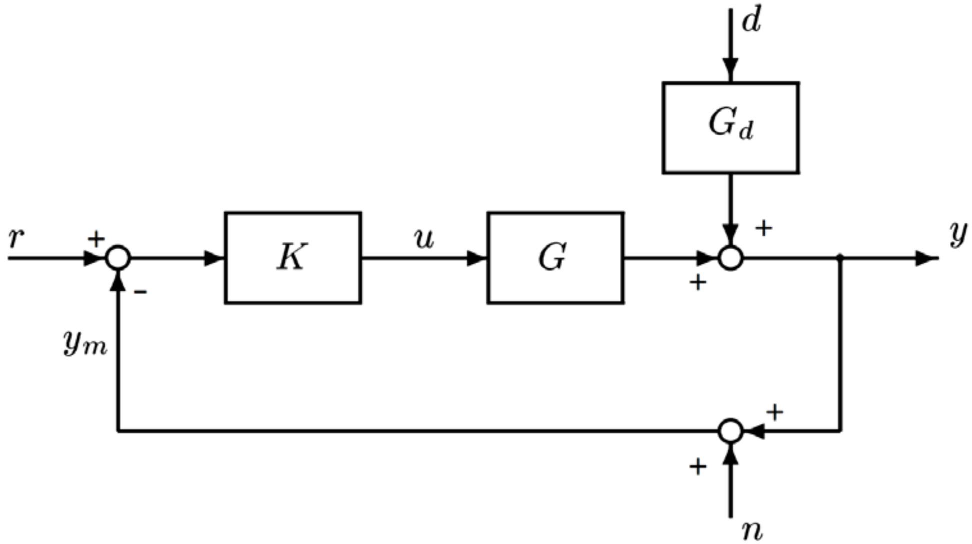
- **We then have the following linear process model in terms of deviation variables**

$$y(s) = G(s)u(s) + G_d(s)d(s)$$

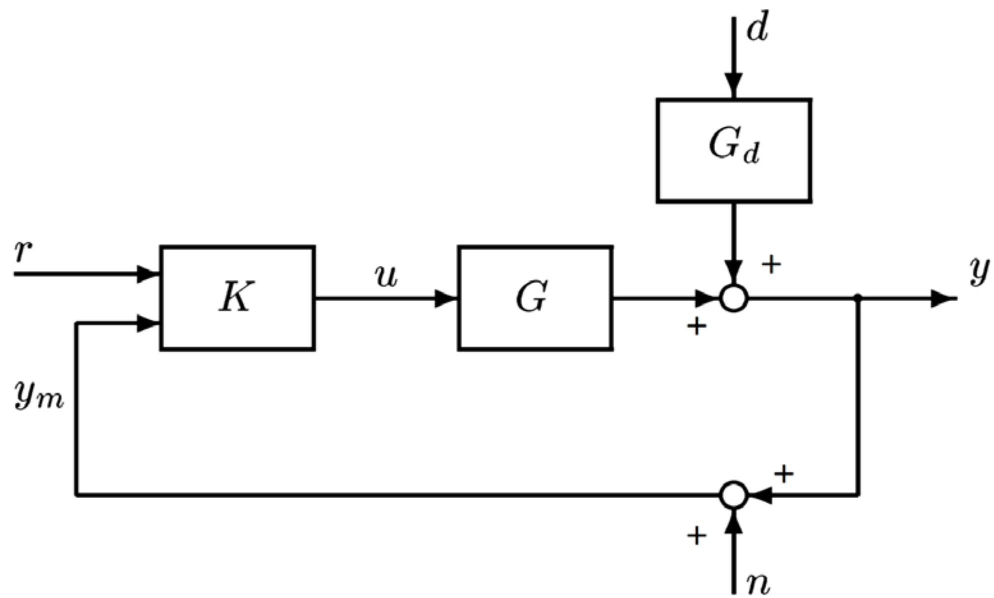
- **We have made use of the superposition principle for linear systems, which implies that a change in a dependent variable (here y) can simply be found by adding together the separate effects resulting from changes in the independent variables (here u and d) considered one at a time.**

- **All the signals u(s), d(s) and y(s) are deviation variables. This is sometimes shown explicitly, for example, by use of the notation δu(s), but since we always use deviation variables when we consider Laplace transforms, the δ is normally omitted.**

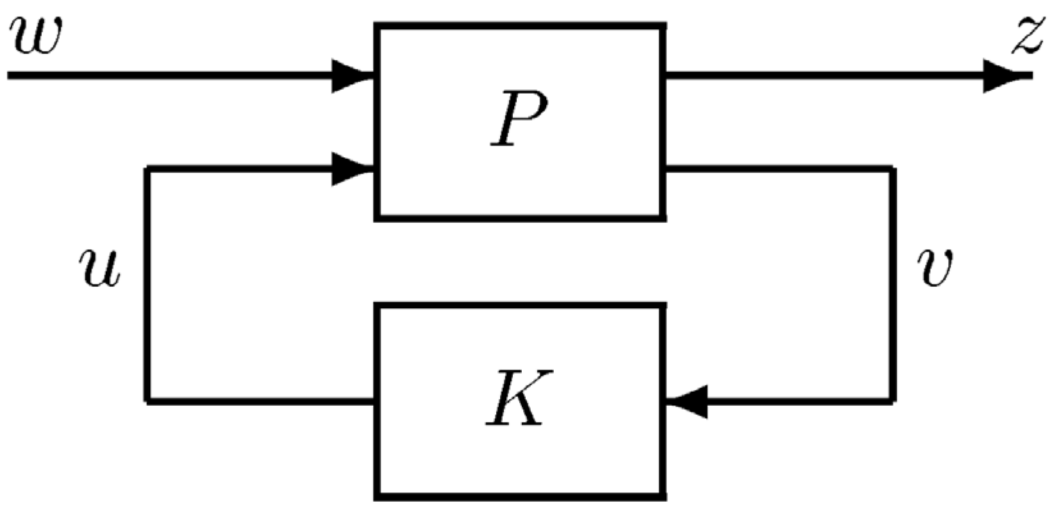
- **One degree-of-freedom control configuration with negative feedback**




- **Two degrees-of-freedom control configuration**



- **General control configuration**




- **The General control configuration can be used to represent a wide class of controllers, including the one and two degrees-of-freedom configurations, as well as feedforward and estimation schemes and many others; and it can also be used to formulate optimization problems for controller design.**

- **Where $K$ is the controller, in whatever configuration. Sometimes the controller is broken down into its constituent parts. For example, in the two degrees-of-freedom controller $K = [K_r\ K_y]^T$ where $K_r$ is a prefilter and $k_y$ is the feedback controller.**

- **The symbols used in all configurations are defined in the following table.**

**For the conventional control configurations**

| | |
|---|---|
| $G$ | plant model |
| $G_d$ | disturbance model |
| $r$ | reference inputs (commands, setpoints) |
| $d$ | disturbances (process noise) |
| $n$ | measurement noise |
| $y$ | plant outputs. These signals include the variables to be controlled ("primary" outputs with reference values $r$) and possibly some additional "secondary" measurements to improve control. Usually the signals $y$ are measurable. |
| $y_m$ | measured $y$ |
| $u$ | control signals (manipulated plant inputs) |

**For the general control configuration**

| | |
|---|---|
| $P$ | generalized plant model. It will include $G$ and $G_d$ and the interconnection structure between the plant and the controller. In addition, if $P$ is being used to formulate a design problem, then it will also include weighting functions. |
| $w$ | exogenous inputs: commands, disturbances and noise |
| $z$ | exogenous outputs; "error" signals to be minimized, e.g. $y - r$ |
| $v$ | controller inputs for the general configuration, e.g. commands, measured plant outputs, measured disturbances, etc. For the special case of a one degree-of-freedom controller with perfect measurements we have $v = r - y$. |
| $u$ | control signals |

***End of Lecture 1!***