Q1)

Show that

$(a)\ \exp(2 \pm 3\pi i) = -e^2; \qquad (b)\ \exp\left(\frac{2+\pi i}{4}\right) = \sqrt{\frac{e}{2}}\,(1+i);$

$(c)\ \exp(z + \pi i) = -\exp z.$

Q2)

State why the function $2z^2 - 3 - ze^z + e^{-z}$ is entire.

Q3) Show that:

$$|\exp(2z + i) + \exp(iz^2)| \leq e^{2x} + e^{-2xy}.$$

Q4)

Show that

$(a)\ \mathrm{Log}(-ei) = 1 - \frac{\pi}{2}i; \qquad (b)\ \mathrm{Log}(1 - i) = \frac{1}{2}\ln 2 - \frac{\pi}{4}i.$

Q5)

Show that

$(a)\ \mathrm{Log}(1+i)^2 = 2\,\mathrm{Log}(1+i); \qquad (b)\ \mathrm{Log}(-1+i)^2 \neq 2\,\mathrm{Log}(-1+i).$