Q1) Show that each of the following functions is entire:

(a) $f(z) = 3x + y + i(3y - x)$;   (b) $f(z) = \sin x \cosh y + i \cos x \sinh y$;

(c) $f(z) = e^{-y} \sin x - ie^{-y} \cos x$;   (d) $f(z) = (z^2 - 2)e^{-x}e^{-iy}$.

Q2) Show that each of these functions is nowhere analytic:

(a) $f(z) = xy + iy$;   (b) $f(z) = 2xy + i(x^2 - y^2)$;   (c) $f(z) = e^y e^{ix}$.

Q3) Determine the singular points of the following functions:

(a) $f(z) = \dfrac{2z+1}{z(z^2+1)}$;   (b) $f(z) = \dfrac{z^3+i}{z^2-3z+2}$;   (c) $f(z) = \dfrac{z^2+1}{(z+2)(z^2+2z+2)}$.

Q4)

Show that $u(x, y)$ is harmonic in some domain and find a harmonic conjugate $v(x, y)$ when

(a) $u(x, y) = 2x(1 - y)$;   (b) $u(x, y) = 2x - x^3 + 3xy^2$;

(c) $u(x, y) = \sinh x \sin y$;   (d) $u(x, y) = y/(x^2 + y^2)$.