Topic in Mathematical Application

Subject

Subject	mathematical application
Period	2nd semester

Q1/ reduce the following equations to canonical form and solve it

1) $ys - p = xy^2\sin(xy)$
2) $r + s = 3y^2$
3) $2yt - xs + 2q = x^2y$
4) $xr + ys + p = 8xy^2 + 9x^2$
5) $6r - s - t = 18y - 4x$
6) $x(xy-1)r - (x^2y^2-1)s + y(y-x)t + (y+x)(p-q) = 2(x+y+1)$
7) $xyr + x^2s - yp = x^3y^2$

Q2) if $f(x)$ is periodic function, then $\int_0^t f(x)dx$ is also periodic of the same period f

Q3) if $f(x)$ is periodic function of period P, then $f'(x)$ is also periodic of the same period P

Q4) Expand $f(x) = sinx \quad 0 < x < \pi$ in Fourier cosine

Q5) $f(x) = x(10 - x) \quad 0 < x < 10$

Q6) $f(x) = \begin{cases} cosx & 0 < x < \pi \\ 0 & \pi < x < 2\pi \end{cases}$

Q7) $f(x) = \begin{cases} 2 & 0 < x < 3 \\ -2 & -3 < x < 0 \end{cases} \quad period = 6$

Q8) Expand $f(x) = x^2 \quad 0 < x < 2\pi$

Q9) a} find the Fourier coefficients corresponding to the function

$$f(x) = \begin{cases} 0 & -5 < x < 0 \\ 3 & 0 < x < 5 \end{cases} \quad period = 10$$

b} write the corresponding Fourier series

c} How should $f(x)$ be defined at $x = -5, x = 0, and\ x = 5$ in order that Fourier series will converge to $f(x) \quad for -5 < x < 5$

Q10) a}expand $f(x) = x^2 \quad 0 < x < 2\pi$ in a Fourier series if the period 2π

b}prove that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$

Q11) a}find a Fourier series for $f(x) = x^2 \quad 0 < x < 2$ by integrating the series of sine series

b} use{a} to evaluate the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

Q12) show that term by term differentiation of the series in $f(x) = x$ is not valid.

Q13) If $f(x) and\ f'(x)$ are piecewise continuous in $(-\pi, \pi)$ prove that

$$\lim_{m\to\infty} S_m(x) = \frac{f(x+0)+f(x-0)}{2}$$

Q14) find corresponding Fourier series, using properties of even and odd function

a]$f(x) = \begin{cases} -x & -4 \le x < 0 \\ x & 0 \le x \le 4 \end{cases}$ b]

$$f(x) = \begin{cases} 8 & 0 < x < 2 \\ -8 & 2 < x < 4 \end{cases}$$

Q15)a]show that for $-\pi < x < \pi$

$$x = 2\left(\frac{sinx}{1} - \frac{sin2x}{2} + \frac{sin3x}{3} - \cdots.\right)$$

b] integrating the result of [a] show that

$$x^2 = \frac{\pi^2}{3} - 4\left(\frac{cosx}{1^2} - \frac{cos2x}{2^2} + \frac{cos3x}{3^2} - \cdots\right)$$

C]show that the series on the right in part [b] converge unformly to the functions on left [a]

Q16) show that for $-\pi < x < \pi$

$$xcosx = -\frac{1}{2}sinx + 2(\frac{2sin2x}{1.3} - \frac{2sin3x}{2.4} + \frac{4sin4x}{3.5} - \cdots.)$$

Q17)use [13] show that for $-\pi < x < \pi$

$$xsinx = 1 - \frac{1}{2}cosx - 2(\frac{cos2x}{1.3} - \frac{cos3x}{2.4} + \frac{cos4x}{3.5} - \cdots.)$$

Q18) by using Parseval's equality show that

$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{20}$

Q19) by using Parseval's equality show that

$\sum_{n=1}^{\infty} \frac{1}{n^6} = \frac{\pi^6}{945}$

Q20) Solve B.V.P $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$ $u(0,t) = 10, u(3,t) = 40, u(x,0) = 25$, $|u(x,t)| < m$

Q21) Solve B.V.P $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ $u_x(0,t) = 0, u_x(L,t) = 0, u(x,0) = f(x)$, $|u(x,t)| < m$

Q22) Solve $\frac{\partial^2 u}{\partial p^2} + \frac{1}{p}\frac{\partial u}{\partial p} + \frac{1}{p^2}\frac{\partial^2 u}{\partial \emptyset^2} = 0$ with B.C $u(1,\emptyset) = \begin{cases} u_1 & 0 < \emptyset < \pi \\ u_2 & \pi < \emptyset < 2\pi \end{cases}$

$|u(p,\emptyset)| < m$

Q23) Solve B.V.P $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$ $u(0,t) = u(4,t) = 0, u(x,0) = 25x$, $where\ 0 < x < 4, t > 0$

Q24) Solve B.V.P $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ $u_x(0,t) = u_x(\pi,t) = 0, u(x,0) = f(x),$ $where\ 0 < x < \pi, t > 0$ is given by

$$u(x,t) = \frac{1}{\pi}\int_0^{\pi} f(x)dx + \frac{2}{\pi}\sum_{m=1}^{\infty} e^{-m^2 t} cosmx \int_0^{\pi} f(x) cosmxdx$$

Q25) Solve B.V.P $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \alpha^2 u$ $u(0,t) = u_1, u(L,t) = u_2, u(x,0) = 0$, $where\ \ 0 < x < L, t > 0$, α, L are constant.

Q26) Solve B.V.P $\frac{\partial^2 y}{\partial t^2} + b^2 \frac{\partial^4 y}{\partial x^4} = 0$ where

$$y(0,t) = 0, y(L,t) = 0, y(x,0) = f(x), \qquad |y(x,t)| < m$$

$$y_t(x,0) = 0, y_{xx}(0,t) = 0, y_{xx}(L,t) = 0$$