1. What is the general form of second order non-linear partial differential equations (x and y being independent variables and z being a dependent variable)?
   a) $F(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}, \frac{\partial^2 z}{\partial x \partial y}) = 0$
   b) $F(x, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}, \frac{\partial^2 z}{\partial x \partial y}) = 0$
   c) $F(y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}) = 0$
   d) $F(x, y) = 0$
2. What is the reason behind the non-existence of any real function which satisfies the differential equation, $(y')2 + 1 = 0$?
   a) Because for any real function, the left-hand side of the equation will be less than, or equal to one and thus cannot be zero
   b) Because for any real function, the left-hand side of the equation becomes zero
   c) Because for any real function, the left-hand side of the equation will be greater than, or equal to one and thus cannot be zero
   d) Because for any real function, the left-hand side of the equation becomes infinity
3. Which of the following is the condition for a second order partial differential equation to be hyperbolic?
   a) $b2 - ac < 0$
   $b)\ b2 - ac = 0$
   $c)\ b2 - ac > 0$
   $d)\ b2 - ac =< 0$
4. First order partial differential equations arise in the calculus of variations.
   a) True
   b) False
5. Which of the following is an example for first order linear partial differential equation?
   a) Lagrange's Partial Differential Equation
   b) Clairaut's Partial Differential Equation
   c) One-dimensional Wave Equation
   d) One-dimensional Heat Equation

06. Find the complete integral of $p+q=pq$. **Ans:** $z=ax+\left(\frac{a}{a-1}\right)y+c$

07. Find the complete integral of $z=px+qy+\sqrt{pq}$. **Ans:** $z=ax+by+\sqrt{ab}$.

08. Find the complete integral of $pq=xy$. **Ans:** $z=a\frac{x^2}{2}+\frac{1}{a}\frac{y^2}{2}+c$.

09. Find the complete integral of $p+q=x+y$. **Ans:** $z=\frac{(a+x)^2}{2}+\frac{(y-a)^2}{2}+c$.

10. Find the complete integral of $(z-px-qy)(p+q)=1$. **Ans:** $z=ax+by+\frac{1}{a+b}$.

11. Find the complete integral of $\frac{z}{pq}=\frac{x}{q}+\frac{y}{p}+\sqrt{pq}$. **Ans:** $z=ax+by+(ab)^{3/2}$.

12. Find the solution of $px^2+qy^2=z^2$. **Ans:** $\phi\left[\frac{1}{y}-\frac{1}{x},\frac{1}{z}-\frac{1}{y}\right]=0$

13. Find the general solution of $\frac{\partial^2 z}{\partial x^2}=0$. **Ans:** $z=xf(y)+g(y)$.

14. Solve $\frac{\partial^2 z}{\partial y^2}=\sin y$. **Ans:** $z=-\sin y+yf(x)+g(x)$.

Problem Related to Homogeneous Linear PDE

01. Find the particular integral of the equation $\left[D^2+2DD'+D'^2\right]z=e^{x-y}$. **Ans:** $P.I=\frac{x^2}{2}e^{x-y}$

02. Solve $\left[D^2-3DD'+2D'^2\right]z=0$. **Ans:** $z=f_1(y+x)+f_2(y+2x)$.

03. Solve $\left[2D^2+5DD'+2D'^2\right]z=0$. **Ans:** $z=f_1\left(y-\frac{1}{2}x\right)+f_2(y-2x)$.

04. Solve $\left[D^2-2DD'+D'^2\right]z=0$. **Ans:** $z=f_1(y+x)+xf_2(y+x)$.

05. Solve $\left[D^2-7DD'+6D'^2\right]z=0$. **Ans:** $z=f_1(y+x)+f_2(y+6x)$.

06. Solve $[D-D']^3z=0$. **Ans:** $z=f_1(y+x)+x\,f_2(y+x)+x^2f_3(y+x)$

07. Solve $(D-1)[D-D'+1]z=0$. **Ans:** $z=e^xf_1(y)+e^{-x}f_2(y+x)$.

Problem Related to Lagrange's Linear PDE [Method of Multipliers]

01. Solve $x(y^2+z)p - y(x^2+z)q = z(x^2-y^2)$, **Ans:** $\phi(x^2+y^2-2z, xyz)=0.$

02. Solve $x(y^2+z)p + y(x^2+z)q = z(x^2-y^2)$, **Ans:** $\phi\left(x^2-y^2-2z, \frac{xz}{y}\right)=0.$

03. Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$, **Ans:** $\phi\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}, xyz\right)=0.$

04. Solve $x(y-z)p + y(z-x)q = z(x-y)$, **Ans:** $\phi(x+y+z, xyz)=0.$

05. Solve $(mz-ny)p + (nx-lz)q = ly - mx.$ **Ans:** $\phi(lx+my+nz, x^2+y^2+z^2)=0.$

06. Solve $(3z-4y)p + (4x-2z)q = 2y-3x.$ **Ans:** $\phi(2x+3y+4z, x^2+y^2+z^2)=0.$

07. Solve $x(z^2-y^2)p + y(x^2-z^2)q = z(y^2-x^2).$ **Ans:** $\phi(xyz, x^2+y^2+z^2)=0.$

08. Solve $(x^2+y^2+yz)p + y(x^2+y^2-xz)q = z(x+y).$ **Ans:** $\phi\left(x-y-z, \frac{x^2+y^2}{z^2}\right)=0.$

Problem Related to Eliminating Arbitrary Functions and Constant

01. Form the PDE from $z=f(2x-6y)$ **Ans:** $3p-q=0$

02. Form the PDE from $z=ax^3+by^3$ **Ans:** $px+qy=3z$

03. Form the PDE from $z^2\cot^2\partial=(x-a)^2+(y-b)^2$ **Ans:** $p^2+q^2=\tan^2\partial.$

04. Form the PDE by eliminating f from $z=f(y/x).$ **Ans:** $px+qy=0$

05. Form the PDE by eliminating f from $z^2-xy=f(x/z).$ **Ans:** $x^2p+q(2z^2-xy)=zx.$

06. Eliminate the function f from $z=f(x^2+y^2).$ **Ans:** $py=qx$

07. Form the partial differential equation from $(x-a)^2+(y-b)^2+z^2=1.$ **Ans:** $p^2+q^2+1=\frac{1}{z^2}.$

08. Form the partial differential equation from $(x+a)^2+(y+b)^2=z.$ **Ans:** $p^2+q^2=4z$

09. Form the partial differential equation from $(x^2+a^2)(y^2+b^2)=z.$ **Ans:** $pq=4xyz.$

10. Find the PDE of all spheres whose canters lie on the z-axis. **Ans:** $py-qx=0.$

11. Find the PDE of all planes having equal intercepts on the 'x' and 'y' axis. **Ans:** $p=q$