

# Young Women in PDEs and Applications

20th-22nd September, 2021

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**Webpage:**

<https://www.iam.uni-bonn.de/afa/guests-and-events/young-women-in-pdes-and-applications/>

**Schedule:** Times are according to Central European Summer Time (CEST) (current time in Bonn)

	Monday 20th	Tuesday 21st	Wednesday 22nd
9:30 - 10:00	Welcome*		
10:00 - 11:00	Rocca	Rocca	Schlömerkemper
11:00 - 11:30	<b>Break</b>	<b>Break</b>	<b>Break</b>
11:30 - 12:30	Majumdar	Schlömerkemper	Majumdar
12:30 - 14:00	<b>Lunch Break</b>	<b>Lunch Break</b>	<b>Lunch Break</b>
14:00 - 14:35	Bruna	Lienstromberg	Scardia
14:35 - 15:10	Lamacz-Keymling	Kreisbeck	Simon
15:10 - 15:30	<b>Break</b>	<b>Break</b>	<b>Break</b>
15:30 - 16:30	Contributed talks: Session 1	Contributed talks: Session 2	Contributed talks: Session 3
16:30 - 18:00	Discussions**	Discussions**	Discussions**

**Contributed talks:**

**Session 1 :** Ahmadova - Wehbe - Zhang - Holzinger - Lahbiri

**Session 2 :** Ritorto - Pacchiano - Marveggio - Salih - Sportelli - Nastasi

**Session 3 :** Schmeller - Conte - Maltsi - Pelen - Khot - Dharmatti

\*: In the first day, we will start the session at 9:30 so that there is the chance to briefly talk and make some announcements. The other days, we will open the session at 9:45.

\*\* : After the talks, we will keep the session (and the breakout rooms open) until 18:00. Everyone who wants to set up discussions is very welcome to use them.

## Minicourses

**Apala Majumdar** (University of Strathclyde, UK)

Nematic Liquid Crystals: A Playground for Mathematics and Applications

Nematic liquid crystals are paradigm examples of soft materials and complex fluids. Nematics combine the fluidity of liquids with the orientational order of conventional solids i.e. they have distinguished special directions, referred to as directors. Consequently, they have a direction-dependent response to external fields and light, making them the working material of choice for the multi-billion liquid crystal display industry, along with new applications in microfluidics, actuators and the health sciences etc. The mathematics of nematic liquid crystals is broad and rich, spanning multiple branches of mathematics such as the calculus of variations, nonlinear partial differential equations, numerical analysis, topology, stochastic analysis and scientific computation. In the first lecture, we will survey the powerful Landau-de Gennes theory for nematic liquid crystals, the governing systems of nonlinear coupled partial differential equations and the theory of isotropic-nematic phase transitions. In the second lecture, we will discuss the applications of the Landau-de Gennes theory to recent experiments on self-assembled bacterial structures and to multistable liquid crystal devices. All collaborations will be acknowledged during the lectures.

**Elisabetta Rocca** (University of Pavia, Italy)

Optimal control problems in engineering and biology

Optimal control and topological optimization with applications to biomedical problems (e.g. models of tumor growth) and engineering problems (e.g. topological optimization in additive manufacturing) will be introduced. In both cases the phase-field method will be used in the modeling approach.

**Anja Schlömerkemper** (Würzburg University, Germany)

Modeling and analysis of magneto-viscoelastic materials

Materials with magnetic, elastic or viscoelastic properties have a huge variety of technological applications. In this lecture series I will present mathematical results on materials that show a coupling of magnetic and elastic behavior. We will focus on the derivation of related systems of partial differential equations by means of variational techniques as well as on the analysis of such evolutionary systems.

## Invited Talks

**Maria Bruna** (University of Cambridge, UK)

Phase separation in active Brownian particles

In this talk, I will discuss models for active matter systems consisting of many self-propelled particles. These can be used to describe biological systems such as bird flocks, fish schools, and bacterial suspensions. In contrast to passive particles, these systems can undergo phase separation without any attractive interactions, a mechanism known as motility-induced phase separation. Starting with a microscopic model for active Brownian particles with repulsive interactions, I will discuss four possible macroscopic PDEs (ranging from a nonlocal model to a local cross-diffusion system). I will then present recent work concerning the stability and analysis of such models.

**Carolin C. Kreisbeck** (KU Eichstätt-Ingolstadt, Germany)

Nonlocal variational problems: Structure-preservation during relaxation?

Nonlocal variational problems arise in various applications, such as continuum mechanics, the theory of phase transitions, or image processing. Naturally, the presence of nonlocalities leads to new effects, and the standard methods in the calculus of variations, which tend to rely intrinsically on localization arguments, do not apply. This talk addresses relaxation of two classes of functionals - double-integrals and nonlocal supremals. Our focus lies on the question of whether the resulting relaxed functionals preserve their structure. We give an affirmative answer for nonlocal supremals in the scalar setting, along with a closed representation formula in terms of separate level convexification of a suitably diagonalized supremal, and discuss results in the vectorial case. Regarding double-integrals, a full understanding of the problem is still missing. We present the first counterexample showing that weak lower semicontinuous envelopes fail to be double-integrals in general. On a technical level, both findings rely on a characterization of the asymptotic behavior of (approximate) nonlocal inclusions, a theoretical result of independent interest.

This is based on joint work with Elvira Zappale (Sapienza University of Rome) and Antonella Ritorto (KU Eichstätt-Ingolstadt).

**Agnes Lamacz-Keymli** (Duisburg-Essen University, Germany)

Overdamped limit of the Vlasov-Fokker-Planck equation: a variational approach

Coarse-graining is the procedure of approximating a complex system by a simpler or lower-dimensional one, often in some limiting regime. Rigorous proofs of such limits typically hinge on exploiting certain structural features of the equations such as variational-evolution structures, which, for instance, are present in gradient flows. In this talk we introduce and discuss such a variational structure arising from the theory of large deviations for stochastic processes. We show how in systems, which are characterized by a large deviation rate functional, passing to a limit is facilitated by the dual formulation of the rate functional, in a way that interacts particularly well with coarse-graining. Being closely related to classical variational methods for gradient flows, our approach is also applicable to systems with non-dissipative effects. As an example we use the technique to derive the large friction (overdamped) limit of the Vlasov-Fokker-Planck equation. The talk is based on a joint work with M. Hong Duong, Mark A. Peletier, Andre Schlichting and Upanshu Sharma.

**Christina Lienstromberg** (University of Bonn, Germany)

Local strong solutions to a quasilinear degenerate fourth-order non-Newtonian thin-film equation

This talk is concerned with questions for existence and uniqueness of strong solutions to a degenerate quasilinear fourth-order non-Newtonian thin-film equation. Originating from a non-Newtonian Navier-Stokes system, the equation is derived by lubrication theory and under the assumption that capillarity is the only driving force. The fluid's shear-thinning rheology is described by the Ellis constitutive law. We prove an abstract existence result for quasilinear parabolic problems of fourth order with Hölder-continuous dependence. This result provides existence of strong solutions to the non-Newtonian thin-film problem in the setting of fractional Sobolev spaces and little Hölder spaces. Uniqueness is proved by energy methods and by using the particular structure of the equation. The talk is based on a joint work with Stefan Müller, published in NoDEA 27 (2020).

**Lucia Scardia** (Heriot-Watt University, UK)

Nonlocal anisotropic energy-driven pattern formation

Nonlocal energies are continuum models for large systems of particles with long-range interactions. Under the assumption that the interaction potential is radially symmetric, several authors have investigated qualitative properties of energy minimisers. But what can be said in the case of anisotropic kernels? Motivated by the example of dislocation interactions in materials science, we pushed the methods developed for nonlocal energies beyond the case of radially symmetric potentials, and discovered surprising connections with random matrices, fluid dynamics, and Calderon-Zygmund operators. This is based on work in collaboration with José Antonio Carrillo, Joan Mateu, Maria Giovanna Mora, Luca Rondi and Joan Verdera.

**Theresa Simon** (University of Bonn, Germany)

Skyrmions and stability of degree 1 harmonic maps from the plane to the two-dimensional sphere

Skyrmions are topologically nontrivial patterns in the magnetization of extremely thin ferromagnets. Typically thought of as stabilized by the so-called Dzyaloshinskii-Moriya interaction (DMI), or antisymmetric exchange interaction, arising in such materials, they are of great interest in the physics community due to possible applications in memory devices. In this talk, we will characterize skyrmions as local minimizers of a two-dimensional limit of the full micromagnetic energy, augmented by DMI and retaining the nonlocal character of the stray field energy. In the regime of dominating Dirichlet energy, we will provide rigorous predictions for their size and “wall angles”. The main tool is a quantitative stability result for harmonic maps of degree 1 from the plane to the two-dimensional sphere, relating the energy excess of any competitor to the homogeneous  $H^2$ -distance to the closest harmonic map.

## Contributed Talks:

**Arzu Ahmadova** (Eastern Mediterranean University, North Cyprus, Turkey)

Approximate controllability of fractional stochastic degenerate evolution equations

We study a class of dynamic control systems described by nonlinear fractional stochastic degenerate evolution equations in Hilbert spaces. Using fixed point technique, fractional calculations, stochastic analysis technique and methods adopted directly from deterministic control problems, a new set of sufficient conditions for approximate controllability of fractional stochastic degenerate evolution equations is formulated and proved. In particular, we discuss the approximate controllability of nonlinear fractional stochastic degenerate control system under the assumptions that the corresponding linear system is approximately controllable. The results in this paper are generalization and continuation of the recent results on this issue. As an application of main results we consider fractional partial differential equations and prove approximate controllability results by verifying main assumptions.

**Martina Conte** (Universidad de Granada, Spain)

Emergence of invasion patterns in glioblastoma growth and progression

Glioblastoma (GB) growth and migration inside the brain tissue is a highly complex phenomenon driven by specific signaling pathways and microenvironmental interactions. Recent studies on tumor cell activity have highlighted the important role that cell membrane protrusions, located at the tumor front, have in cancer progression.

In this work, we analyze the dynamics of GB cell protrusions at the invasion front and some bio-

chemical and biomechanical mechanisms related to it and leading cell migration. Starting from biological experiments in a *Drosophila* model of GB, we formulate a nonlinear macroscopic model based on flux-saturated mechanisms. We exploit the ability of our mathematical model to partially guide the biological experiments on protein signaling distribution. We numerically analyze the evolution of the tumor propagation front in several scenarios and the emergence of a coordination between the collective processes characterizing different agents involved in the dynamics. Joint work with J. Soler (Universidad de Granada) and S. Casas Tintò (Istituto Cajal - CSIC).

**Sheetal Dharmatti** (IISER TVM, India)

Existence, uniqueness of viscosity solutions of value function of local Cahn-Hilliard-NS equation

In this work, we consider the local Cahn-Hilliard-Navier-Stokes equation with regular potential in a two dimensional bounded domain. We formulate a distributed optimal control problem as the minimization of a suitable cost functional subject to the controlled local Cahn-Hilliard-Navier-Stokes system and define the associated value function. We prove the Dynamic Programming Principle satisfied by the value function. Due to the lack of smoothness properties for the value function, we use the method of viscosity solutions to obtain the corresponding solution of the infinite dimensional Hamilton-Jacobi-Bellman equation. We show that the value function is the unique viscosity solution of the Hamilton-Jacobi-Bellman equation. The uniqueness of the viscosity solution is established via the comparison principle.

**Alexandra Holzinger** (TU Wien, Austria)

Mean-field derivation of cross-diffusion population models

In the field of population dynamics, cross-diffusion partial differential equations have gained more impact, see for instance the SKT-model by Shigesada-Kawasaki-Teramoto. However, a rigorous derivation starting from a stochastic many-particle system was still missing in the literature.

In this talk, I will show how the approach of moderately interacting particles in the mean-field limit can be used in order to derive a class of cross-diffusion models starting from a stochastic interacting many-particle system. As a byproduct of the mean-field derivation, we also study a non-local version of the underlying PDE models. These non-local PDEs represent an intermediate level between particle dynamics and the final cross-diffusion partial differential equation. This talk is based on the joint work with Li Chen, Esther Daus and Ansgar Jüngel *Rigorous derivation of population cross-diffusion systems from moderately interacting particle systems* which is currently submitted for publication.

**Rekha Khot** (IIT Bombay, India)

Error analysis of the lowest order NCVEM for second order linear indefinite elliptic problems

The virtual element method (VEM) is one of the well-received polygonal methods for approximating the solutions to partial differential equations (PDEs). On the account of its versatility in shape of polygonal domains, the local finite-dimensional space (the space of shape functions) comprises non-polynomial functions. The novelty of this approach lies in the fact that it does not demand for the explicit construction of non-polynomial functions and the knowledge of degrees of freedom along with projections onto polynomials is sufficient to implement the method. I would like to discuss the nonconforming virtual element method (NCVEM) for the approximation of the weak solution to a general linear second-order non-selfadjoint indefinite elliptic PDE in a polygonal domain under reduced elliptic regularity. Enrichment operators play a vital role in the analysis of nonconforming finite element methods. The main tool in the a priori error analysis is the connection between the nonconforming virtual element space and the Sobolev space  $H_0^1(\Omega)$  by a right-inverse of the interpolation operator. The stability of

the discrete solution allows for the proof of existence of a unique discrete solution, of a discrete inf-sup estimate and, consequently, for optimal error estimates in the  $H^1$  and  $L^2$  norms. The explicit residual-based *a posteriori* error estimate for the NCVEM is reliable and efficient up to the stabilization and oscillation terms. Numerical experiments on different types of polygonal meshes illustrate the robustness of an error estimator and support the improved convergence rate of an adaptive mesh-refinement in comparison to the uniform mesh-refinement.

**Fatima Zahra Lahbiri** (Ibn Zohr University, Morocco)

A semigroup approach to the well-posed stochastic linear systems

We extend the concept of admissible observation operator developed in the deterministic systems to stochastic observed linear systems. We investigate also the perturbed theory for stochastic evolution equations. This allows us to give a new representation of solution.

**Anieza Maltzi** (Weierstrass Institute, Germany)

On the Darwin-Howie-Whelan equations in TEM imaging

The growth of semiconductor quantum dots (QDs) with desired electronic properties would highly benefit from the assessment of QD geometry, distribution and strain profile in a feedback loop between epitaxial growth and analysis of their properties. To assist the optimization of QDs transmission electron microscopy (TEM) imaging can be applied.

The Darwin-Howie-Whelan equations are commonly used to describe and simulate the scattering of electrons in TEM. They are a system of infinitely many envelope functions, derived from the Schrödinger equation. However, for the numerical simulation of TEM images only a finite set of envelope functions is necessary, leading to a system of ordinary differential equations in thickness direction of the specimen. Usually heuristic rules are employed to select the relevant beams, e.g. by thresholds for the excitation error. We studied the mathematical structure of this system and provide error estimates to evaluate the accuracy of special approximations, like the two-beam and the systematic-row approximation.

Furthermore, we used these equations to simulate TEM images of realistic QDs with various shapes, e.g. pyramidal and lens-shaped, which can now be used to achieve a model based geometry reconstruction of the QDs from experimental data.

**Alice Marveggio** (IST Austria, Austria)

On a non-isothermal Cahn-Hilliard model based on a microforce balance

We consider a non-isothermal Cahn-Hilliard model based on a microforce balance. The model was derived by A. Miranville and G. Schimperna starting from the two fundamental laws of Thermodynamics, following M. Gurtins two-scale approach. The main working assumptions are on the choice of the Ginzburg-Landau free energy, and on the behaviour of the heat flux as the absolute temperature tends to zero and to infinity. By deriving suitable a priori estimates and by showing weak sequential stability of families of approximating solutions, we prove global-in-time existence for the initial-boundary value problem associated to the entropy formulation and, in a subcase, also to the weak formulation of the model. (Joint work with G. Schimperna).

**Antonella Nastasi** (University of Palermo, Italy)

Boundary value  $p$ -Laplacian problems on metric spaces

We use a variational approach to study existence and regularity of solutions for a Neumann  $p$ -Laplacian problem with a reaction term on metric spaces equipped with a doubling measure and supporting a Poincaré inequality. Trace theorems for functions with bounded variation are applied in the definition of the variational functional and minimizers are shown to satisfy De Giorgi type conditions.

**Cintia Pacchiano** (Aalto University, Finland)

Existence of parabolic minimizers to the total variation flow on metric measure spaces

In this talk we discuss some fine properties and existence of the variational solutions to the Total Variation Flow. Instead of the classical Euclidean setting, we intend to work in the general setting of metric measure spaces. During the past two decades, a theory of Sobolev functions and BV functions has been developed in this abstract setting. A central motivation for developing such a theory has been the desire to unify the assumptions and methods employed in various specific spaces, such as weighted Euclidean spaces, Riemannian manifolds, Heisenberg groups, graphs, etc.

The total variation flow can be understood as a process diminishing the total variation using the gradient descent method. This idea can be reformulated using parabolic minimizers, and it also gives rise to a definition of variational solutions. The advantages of the approach using a minimization formulation include much better convergence and stability properties. This is a very essential advantage as the solutions naturally lie only in the space of BV functions.

We give an existence proof for variational solutions  $u$  associated to the total variation flow. Here, the functions being considered are defined on a metric measure space  $(X, d, \mu)$ . For such parabolic minimizers that coincide with a time-independent Cauchy-Dirichlet datum  $u_0$  on the parabolic boundary of a space-time-cylinder  $\Omega \times (0, T)$  with  $\Omega \subset X$  an open set and  $T > 0$ , we prove existence in the weak parabolic function space  $L_w^1(0, T; BV(\Omega))$ . In this paper, we generalize results from a previous work by Bögelein, Duzaar and Marcellini and argue completely on a variational level. This is a joint project with Vito Buffa and Michael Collins, from Friedrich-Alexander-Universität Erlangen-Nürnberg.

**Neslihan Nesliye Pelen** (Ondokuz Mayıs University, Turkey)

Buruli Ulcer Modeling, Its Analysis and Numerical Simulations

Buruli ulcer is a skin and soft tissue disease which when untreated in due time can lead to severe disabilities. The affliction is caused by the bacterium *M. Ulcerans* which secretes mycolactone, a toxin causing necrosis of soft tissues. BU lesions typically begin as painless and feverless subcutaneous nodules, prevalently on limbs. If untreated (sometimes, however, also under antibiotic treatment) the swellings evolve into necrotic skin ulcers with deeply undermined edges often resulting in scarring and contractual deformities, occasionally also affecting bones, thus causing amputations and severe disabilities. Advanced stages of BU require surgery, however the true extent of the lesions cannot be assessed by the common medical imaging techniques and this leads often to post-surgery recurrences. Therefore, mathematical models are called upon to help making predictions about the spread of bacteria and necrotic tissue and hence guide the resection. Such models rely on the essential biological facts: By insect bites *M. ulcerans* enters the sub-cutaneous tissue, produces mycolactone, and proliferates. The toxin spreads into the wound and beyond, penetrating the surrounding areas and diffusing further away, thereby necrotizing more tissue. Bacteria are (hapto)tactically attracted to the regions with high concentrations of necrotic tissue, where they produce more toxin and proliferate even further, thus ensuring the sustained spread of lesions.

According to the approach of random walks of particles on a lattice with the probabilities of transition between its sites depending on the local information about cells, tissue, and mycolactone concentration, the model of this disease is developed and this model is represented by reaction-diffusion-transport

equations. Additionally, the analysis and numerical simulations are made by the help of the obtained model.

This is a joint work with Christina Surulescu and Shimi Chettiparambil Mohanan from TU Kaiserslautern, Felix-Klein-Zentrum für Mathematik.

**Antonella Ritorto** (KU Eichstätt-Ingolstadt, Germany)

Connecting double-integrals and nonlocal supremals via power law approximation

The Cartesian separate convexity has been identified as the necessary and sufficient condition for weakly\* lower semicontinuity of nonlocal supremals, and plays a key role in structure-preservation during relaxation. It coincides with separate convexity in the scalar case and it is strictly stronger in higher dimensions. We show that the relaxation of a nonlocal supremal is the Gamma-limit of double-integrals for diverging integrability exponents. The result relies on a characterization of the asymptotic behavior of nonlocal inclusions, Young measure theory, and does not require any additional convexity assumption or Jensen's type inequality. Joint work with Carolin Kreisbeck (KU Eichstätt - Ingolstadt) and Elvira Zappale (Sapienza University of Rome).

**Hero Waisi Salih** (Salahaddin University, Iraq)

A direct method for the solution of a geometric inverse problem

In this paper, a geometric inverse problem where an unknown part of the boundary of the domain is constructed from the knowledge of partially overdetermined boundary conditions. We propose a method of solving this problem whose essence consists in solving a Cauchy problem for the Laplace equations followed by the resolution of independent nonlinear problems. For the Cauchy problem, the Fourier series is used to formulate a first kind Fredholm integral equation for the unknown function of the data. Then, a regularization method and the separable property by term of the kernel function. are used to obtain a closed-form regularized solution. The coordinates of the unknown part of the boundary are then obtained as roots of nonlinear equations using a Newton-like method.

**Leonie Schmeller** (Weierstrass institute, Germany)

Multi-phase dynamic systems at finite strain elasticity: Mathematical modelling, existence theory and numerical solutions

Phase-field models coupled to finite strain elasticity are the underlying model system to describe many phenomena in soft matter physics and biology, such as wetting of soft substrates or pattern formation during swelling or collapse of hydrogels. We show that the presented multi-phase dynamic system can be formulated as a gradient flow with a (conserved) Cahn-Hilliard type evolution for the phase fields.

We derive a weak formulation for a time discrete version of the introduced general class of models and develop numerical algorithms for different applications. We then study existence for the time discrete problem, obtained by an incremental minimisation scheme. This serves as the basis to deduce the existence of weak solutions for the time-continuous formulation of the problem.

**Caterina Sportelli**, (University of Bari, Italy)

Existence result for weighted quasilinear Schrödinger type equations via a variational approach

Variational methods have a long history which probably originated from the pioneering works of Euler (1744) and Lagrange, who introduced formalisms and techniques essentially still used today. Nowadays,



variational methods provide a solid basis for the existence theory of PDEs and other applied problems. In this talk, I will discuss the existence of solutions for the quasilinear elliptic Schrödinger-type equation

$$-\operatorname{div}(A(x, u)|\nabla u|^{p-2}\nabla u) + \frac{1}{p}A_t(x, u)|\nabla u|^p + V(x)|u|^{p-2}u = g(x, u) \text{ in } \mathbb{R}^N \quad (1)$$

with  $N \geq 3, p > 1$  and  $V : \mathbb{R}^N \rightarrow \mathbb{R}$  is a suitable measurable function. Here, we assume that a  $C^1$ -Carathéodory function  $A : \mathbb{R}^N \times \mathbb{R} \rightarrow \mathbb{R}$  exists such that  $A_t(x, t) = \frac{\partial A}{\partial t}(x, t)$  and a Carathéodory function  $g : \mathbb{R}^N \times \mathbb{R} \rightarrow \mathbb{R}$  can be found which satisfies a subcritical growth. Under suitable assumptions on  $A(x, t)$  and  $g(x, t)$  it is possible to recognize the variational structure of the problem (1). In fact, having  $G(x, t) = \int_0^t g(x, s) ds$ , its related functional is

$$\mathcal{J}(u) = \frac{1}{p} \int_{\mathbb{R}^N} A(x, u)|\nabla u|^p dx + \frac{1}{p} \int_{\mathbb{R}^N} V(x)|u|^p dx - \int_{\mathbb{R}^N} G(x, u) dx,$$

which is  $\mathcal{C}^1$  but does not comply with the classical Palais-Smale condition. I will discuss how to overcome the difficulties that arise in this case, by introducing a suitable Banach space and a suitable variant of the Palais-Smale condition. Finally, I will use a generalized version of the classical Ambrosetti-Rabinowitz Mountain Pass Theorem in order to prove the desired existence result. This is a joint work with Anna Maria Candela and Addolorata Salvatore.

**Elsy Wehbe** (University of Pau, France)

Existence, regularity of MHD equations with Navier BC in 2D

Magnetohydrodynamic (MHD) is the discipline studying the behaviour of conductive fluids of electricity when their movement is coupled to the electromagnetic field. Here we study in  $\Omega$ , a multi-connected two dimensional domain, the existence of solutions for a MHD coupling an equation of polymer aqueous solution with Maxwell equation of electromagnetic. These equations are presented, in the stationary case, as the following:

$$\begin{aligned} -\nu\Delta\mathbf{u} + (\mathbf{u} \cdot \nabla)(\mathbf{u} - \alpha\Delta\mathbf{u}) + \nabla\pi - (\mathbf{B} \cdot \nabla)\mathbf{B} + \frac{1}{2}\nabla(|\mathbf{B}|^2) &= \mathbf{f} \quad \text{in } \Omega, \\ -\Delta\mathbf{B} - (\mathbf{B} \cdot \nabla)\mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{B} + \nabla\theta &= 0 \quad \text{in } \Omega, \\ \operatorname{div} \mathbf{u} = 0, \operatorname{div} \mathbf{B} &= 0 \quad \text{in } \Omega, \end{aligned}$$

where  $\mathbf{u}$  and  $\mathbf{B}$  are the velocity field and the magnetic field,  $\pi$  is the pressure of the fluid,  $\theta$  is an unknown function related to the motion of heavy ions and  $\mathbf{f}$  is the external force acting on the fluid. We study the existence of solutions  $(\mathbf{u}, \mathbf{B}, \pi, \theta)$  in  $\mathbf{H}^2(\Omega) \times \mathbf{H}^2(\Omega) \times L^2(\Omega) \times H^1(\Omega)$  with the Navier-type boundary conditions :

$$\begin{aligned} \mathbf{u} \cdot \mathbf{n} = 0, \quad \operatorname{curl} \mathbf{u} &= 0, \quad \text{on } \partial\Omega, \\ \mathbf{B} \cdot \mathbf{n} = 0, \quad \operatorname{curl} \mathbf{B} &= 0, \quad \text{on } \partial\Omega. \end{aligned}$$

To solve our problem we need some estimations related to the Stokes associated problem. One of the difficulties is the geometry of the domain, supposed here non simply connected. On the other hand, it is shown an additional regularity in  $\mathbf{W}^{2,p}(\Omega)$  for the magnetic field.

**Mingmin Zhang** (Aix-Marseille University, France)

Reaction-diffusion fronts in funnel shaped domains

We consider bistable reaction-diffusion equations in funnel-shaped domains of  $\mathbb{R}^N$  made up of straight parts and conical parts with positive opening angles. We study the large time dynamics of entire solutions emanating from a planar front in the straight part of such a domain and moving into the conical part. We show a dichotomy between blocking and spreading, by proving especially some new Liouville type results on stable solutions of semilinear elliptic equations in the whole space  $\mathbb{R}^N$ . We also show that any spreading solution is a transition front having a global mean speed, which is the unique speed of planar fronts, and that it converges at large time in the conical part of the domain to a well-formed front whose position is approximated by expanding spheres. Moreover, we provide sufficient conditions on the size  $R$  of the straight part of the domain and on the opening angle  $\alpha$  of the conical part, under which the solution emanating from a planar front is blocked or spreads completely in the conical part. We finally show the openness of the set of parameters  $(R, \alpha)$  for which the propagation is complete. This is a joint work with Francois Hamel.

## List of Participants:

1. Arzu Ahmadova, Eastern Mediterranean University, North Cyprus, Turkey
2. Liliana Esquivel, Gran Sasso Science Institute, L'Aquila, Italy
3. María Ángeles García-Ferrero, BCAM, Spain
4. Antonella Ritorto, KU Eichstätt-Ingolstadt, Germany
5. Ritika Singh, IIT (BHU), India
6. Elsy Wehbe, University of Pau, France
7. Hind Al Baba, Lebanese University, Lebanon
8. Mara Sandoval, National Autonomous University of Mexico
9. Rajae Bentahar, National School of Applied Sciences, Morocco
10. Amira Meddah, Johannes Kepler University, Linz, Austria
11. Cintia Pacchiano, Aalto University, Finland
12. Elisabeta Peti, Tirana University, Albania
13. Leonie Schmeller, Weierstrass Institute, Germany
14. Antonella Nastasi, University of Palermo, Italy
15. Stefanie Schindler, Weierstrass Institute, Germany
16. Martina Conte, Universidad de Granada, Spain
17. Mingmin Zhang, Aix-Marseille University, France
18. Julie Yuan Merten, University of Groningen, Netherlands
19. Alexandra Holzinger, TU Wien, Austria
20. Rizwan Ali, J.C.Bose University of Science and Technology, India

21. Maria Cabrera Calvo, Laboratoire Jacques-Louis Lions, France
22. Anieza Maltsi, Weierstrass institute, Berlin
23. Alexia Papalazarou, Lund university, Sweden
24. Tania Biswas, IIT Delhi, India
25. Carina Geldhauser, Lund university, Sweden
26. Alice Marveggio, Institute of Science and Technology Austria
27. Stefania Lisai, Heriot-Watt University, UK
28. Pratima Hebbar, Duke University, USA
29. Neslihan Nesliye Pelen, Ondokuz Mayis University, Turkey
30. Sudad Musa Rashid, University of Sulaimani, Iraq
31. Rekha Khot, IIT Bombay, India
32. Sheetal Dharmatti, IISER TVM, India
33. Saliha Boukassa, University of Pau, France
34. Ruma Maity, IIT Bombay, India
35. Sabah Kaouri, Cadi Ayyad University, Morocco
36. Hero Waisi Salih, Salahaddin University, Iraq
37. Devika, IIT Madras, India
38. Caterina Sportelli, University of Bari, Italy
39. Fatima Zahra Lahbiri, Ibn Zohr University, Morocco
40. Elisabetta Brocchieri, Université d'Évry Val-d'Essonne, France
41. Chiara Gavioli, TU Wien, Austria
42. Thuyen Dang, University of Houston, USA
43. Elena Nikolova, Technical University of Sofia, Bulgaria
44. Lovy Jain, IIT Bombay, India
45. Melanie Koser, Humboldt University, Berlin
46. Abdellaoui Mohammed, Sidi Mohamed Ben Abdellah University, Morocco
47. Mahwash Aftab, Aligarh Muslim University, India
48. Nesibe Ayhan, Bilkent University, Turkey
49. Iulia Christian, University of Bonn, Germany
50. Andrea Aspri, University of Pavia, Italy
51. Abramo Agosti, University of Pavia, Italy
52. Matteo Fornoni, University of Pavia, Italy