Chapter Three
Equivalent Circuit of IM

Losses in I.M are three types ;

i-Iron losses (magnetic losses)

ii-Copper losses (Electrical losses)

iii-Mechanical losses (f + w (friction + windage))losses, or rotational losses.

Chapter Three
Equivalent Circuit of IM

Iron losses:

The presence of rotating magnetic field in the stator core account for major portion of the magnetic losses. As the stator core is made of silicon content steel-iron stampings ,also which is called core losses, iron losses consist of

1-Hystereas losses Ph

2-Eddy current losses Pe

Chapter Three
Equivalent Circuit of IM

Hysteresis losses:

Every part of stator core is subjected to alternate changes in the magnetic field due to its rotating nature. The frequency of flux reversal (f) is similar to the supply frequency.

The flux reversals take place through the B-H loop or the hysteresis loop causing hysteresis which is given by,

$$P_h = K_h \times B_m^{x} \times f \qquad \text{in watt}$$

Where Bm =Maximum flux density in Tesla in stator core,

f= stator supply frequency,

x= Variable depends on the quality of the material which is stator core stamping are made of.

Kh= Hysteresis constant also depends of materials of stator core.

If the silicon is added to stator core in the manufacture of core the value of Kh is decrease, and then the hysteresis losses is decreases.

Chapter Three
Equivalent Circuit of IM
Eddy Current Losses:

The stator core steel may be account as a good medium for magnetic field. Therefore, according to Farady's law the flux reversal in the stator core causes an emf which is called eddy emf induced in stator core. The value of this emf according to this law proportional to flux density and the frequency of flux reversal f. This emf small in value, drives a current called eddy current loss.

To decrease the eddy current losses, the core resistance must be as large as possible. This is achieved by building the stator core with thin laminations, stacked and riveted at right angles to the path of eddy current. The laminations are insulated from each other by a thin coat of varnish.

$$P_e = K_e \, B_m^2 \, f^2 \qquad \text{in watt}$$

Where Ke= eddy current constant, proportional to the squire of thickness of core lamination and is inversely proportional to the resistively of the core material.

Chapter Three

Equivalent Circuit of IM

Iron Losses

Piron = Ph + Pe

The frequency of supply f and flux density Bm is constant, hence Pi may be considered as constant under all conditions. Since the frequency of rotor is very small, the iron losses rotor can be neglected at lower slip (high speed).

Chapter Three

Equivalent Circuit of IM

Copper Losses

It is occur due to ohmic resistance of stator and rotor resistance of three phase windings.

Copper loss= stator cu loss + rotor cu loss

$$=3I_1^2R_1 + 3I_2^2R_2$$

Since the I_1&I_2 vary with load, therefore the cu loss vary with square proportion of the current. Hence cu losses or electrical loss can otherwise be called as variable losses.

Chapter Three
Equivalent Circuit of IM

Mechanical Losses (Pm):

Friction Loss (Pf):

Which is occur between solid parts of the machines (friction of bearing). In WRIM the slip ring friction loss added to bearing loss, therefore the WRIM has more friction loss than squirrel cage type.

$$P_f = K_h \quad n \qquad\qquad (Watt)$$

Where n= rotor speed and Kh is friction loss constant.

Windage Loss(Pw):

This loss occur between a rotating solid part of the machine and the air around the surface of the rotor. Addition windage losses may be added if fan blades are fitted on one side of the rotor for ventilation purposes.

$$Pw = K_w \quad n^2 \qquad\qquad (Watt)$$

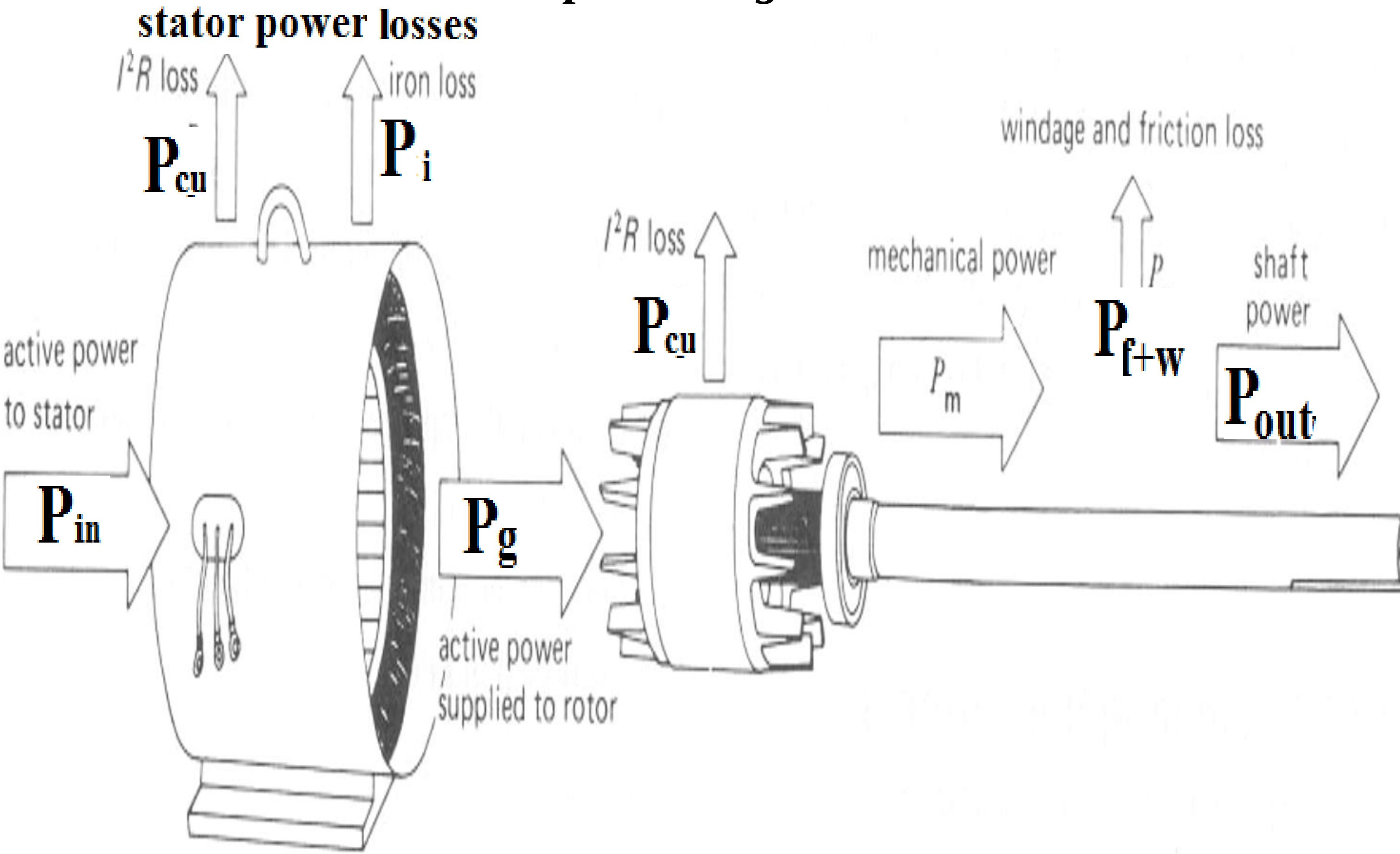
Where Kw = windage loss constant.

The total mechanical losses $P_m = P_f + P_w = P_{f+w}$

The change in speed from no-load to full load speed is very small, therefore , the mechanical losses are considered to be constant losses as similar to iron losses.

Chapter Three
Equivalent Circuit of IM

The various power stages shown below.



Chapter Three
Equivalent Circuit of IM

$P_{inp} = \sqrt{3}\ V_L\ I_L \cos\phi$

$= \text{rotor input} + \text{stator cu losses}$

$P_g = \text{rotor input} = \text{air gap power}$

$= E_1 I_2' \cos\phi_2$

The air gap power P_g has two parts:

1- $(I_2')^2\ R_2'$ $\Rightarrow$ It is converted to electrical power and its dissipated as a heat losses.

2- $(1\text{-}S)\ (I_2')^2\ R_2' / S$ $\Rightarrow$ It is converted to mechanical energy

$P_g\ /\ phase = (I_2')^2\ R_2' / S$

$= (I_2')^2\ R_2' + (1\text{-}S)\ (I_2')^2\ R_2' / S$

$= \text{Rotor cu loss} (P_{cur}) + \text{mechanical loss} (p_{mec})$

Chapter Three
Equivalent Circuit of IM

$$P_g \; / phase \qquad = S\,P_g \; + P_g \; (1\text{-}S)$$

The rotor output is converted into mechanical energy. some is lost due to windage and friction losses in the rotor and the rest appears as the useful or shaft torque T_{sh} or T_{out}

$$P_g = T_{airgap} \cdot \omega_s$$

$$= 2\pi n_s \; . \; T_e$$

Te it is called air gap torque or gross torque.

$$T_e = \frac{P_g}{\omega_s}$$

$$\frac{Rotor\,cu\,loss}{Rotor\,input} = \frac{SP_g}{P_g} = S = \frac{n_s - n}{n_s}$$

$$Rotor\,cu\,loss = Sp_g$$

$$Rotor\,gross\,output = P_g(1-S)$$

$$\frac{Rotor\,output(P_{mech})}{Rotor\,Input(P_g)} = \frac{P_g(1-S)}{P_g} = 1 - S = 1 - \frac{n_s - n}{n_s}$$

$$\frac{P_{mech}}{P_g} = \frac{n}{n_s}$$

$$Rotor\ efficiency = \frac{P_{mech}}{P_g} = \frac{n}{n_s}$$

$$T_{out} = \frac{P_{mech}}{\omega_m} = \frac{P_{mech}}{2\pi n_r}$$

Example:

The power input to the rotor of 415V, 50Hz, 4-pole , three phase IM is 55kW, the frequency of rotor is 2Hz, find, the slip, the rotor speed (n), the rotor cu losses and mechanical power developed.

Solution:

$n_s = 120f/p = 1500rpm$

$f_r = Sf \quad \Rightarrow S = f_r/f = 2/50 = 0.04$

$$0.04 = \frac{1500 - n}{1500}, \quad \therefore n = 1440 rpm$$

Rotor cu loss $P_{cur} = SP_g = 0.04 * 55kW \qquad = 2.2kW$

$P_{mech} = P_g(1-S) \qquad = 55*10^3(1-0.04) \qquad = 52.8kW$

Chapter Three
Equivalent Circuit of IM

Example: A 415V, 4-pole ,, three phase IM develops 15.25kW with mechanical losses when running at 1426rpm. The supply frequency 50.5Hz with power factor of 0.89. find; the speed, rotor copper losses, input power to stator if stator losses are 1.4kW, input line current and rotor frequency .

Solution:

$$n_s = \frac{120 \times 50.5}{4} = 1515 rpm, \qquad S = \frac{1515 - 1462}{1515} = 0.035$$

$$P_{mek} = P_s(1-S) \Rightarrow P_s = \frac{p_{mek}}{1-S} = \frac{15.25 \times 10^3}{1-0.035} = 15803 Watt$$

$$CUr = SP_s = 0.035 \times 15803 = 553 Watt$$

$$P_{inp} = P_s + stator \quad loss = 15803 + 1400 = 17.203 kWatt$$

$$P_{inp} = \sqrt{3} \ V_L \ I_L \ \cos\phi \Rightarrow 17203.1 = \sqrt{3} \quad 415 \quad I_L \quad 0.89 \Rightarrow$$

$$I_L = 26.891A, \ \& f_r \qquad = 0.035 \times 50.4 = 1.76 Hz$$

Example:

A 4-pole, 415V, 50Hz, three phase IM deliver a shaft useful torque of 25.04N.m with a speed of 1411rpm when the supply frequency is 49Hz. The stator losses equal to 320Watt and input power factor is 0.86. If the mechanical torque lost in (f+w) is 0.5N.m. find; the input current and efficiency.

Solution:

$$n_s = \frac{120 \times 49}{4} = 1470 rpm, \quad S = \frac{1470 - 1411}{1470} = 0.04$$

$$T_{out} = 25.04 N.m \Rightarrow P_{out} = \omega_m T_{out} = 147.76 \times 25.04 = 3700 Watt$$

$$T_{mech} = 25.04 + 0.5 = 25.54 N.m$$

$$P_{mech} = \omega_m T_{mech} = 147.76 \times 25.54 = 3773.79 Watt$$

$$P_{mech} = P_s (1 - S) \Rightarrow 3773.79 = p_s (1 - 0.04)$$

$$P_s = 3931.588 Watt$$

$$P_{input} = P_s + sator \quad losses = 3931.588 + 320 = 4251.6 Watt$$

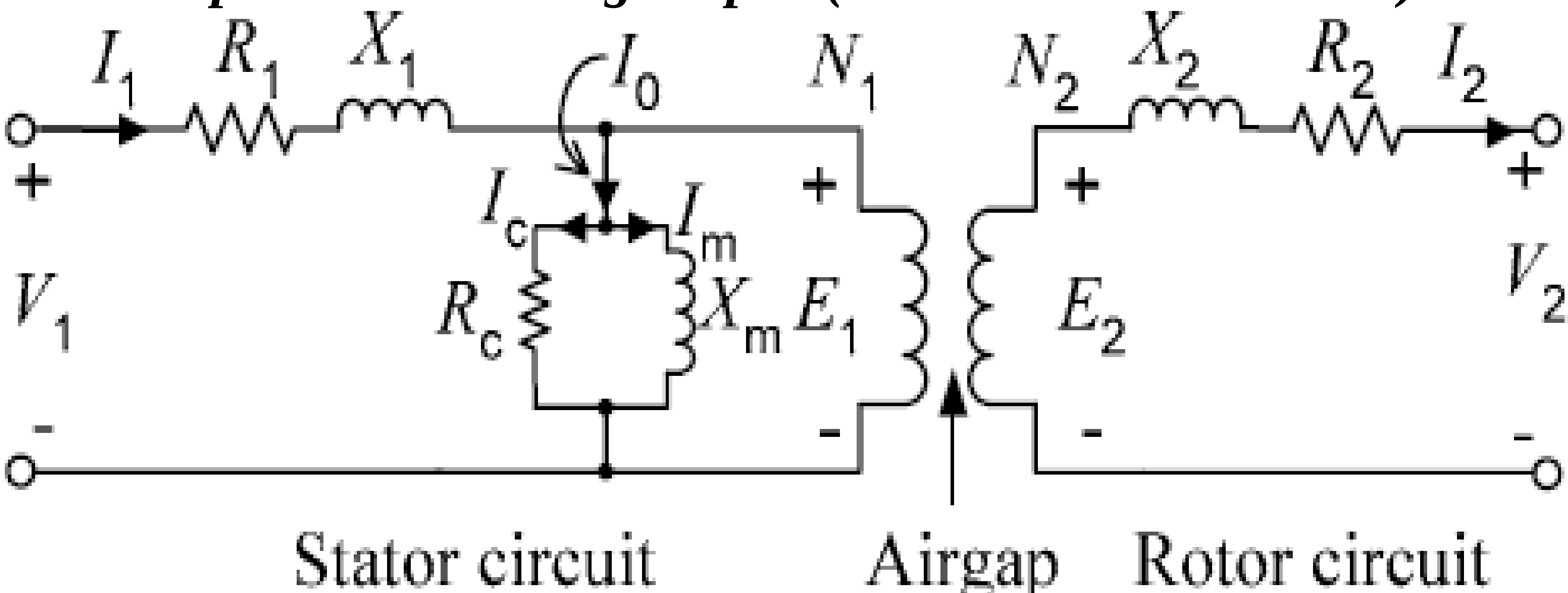
$$P_{input} = P_{inp} = \sqrt{3}\ V_L\ I_L\ \cos\phi \qquad I_L = 6.88A$$

$$\eta\% = \frac{3700}{4251.6} \times 100 = 87\%$$

Chapter Three
Equivalent Circuit of IM
Conventional equivalent circuit:

Note: 1-*Never use three-phase equivalent circuit. Always use per-phase equivalent circuit.*
2-The equivalent circuit always bases on the Y connection regardless of the actual connection of the motor.

Step1: Rotor winding is open (The rotor will not rotate)



Chapter Three
Equivalent Circuit of IM
Conventional equivalent circuit:

V_1 – stator voltage, per phase ($V_1 = V_{LL}/\sqrt{3}$)

R_1, R_2 – stator and rotor winding resistance

$X_1 = 2\pi f_1 L_1$ – stator leakage reactance

$X_2 = 2\pi f_1 L_2$ – rotor leakage reactance

R_c – resistance representing core loss, per phase

X_m – magnetizing reactance, per phase

N_1, N_2 – effective number of turns of stator and rotor windings.

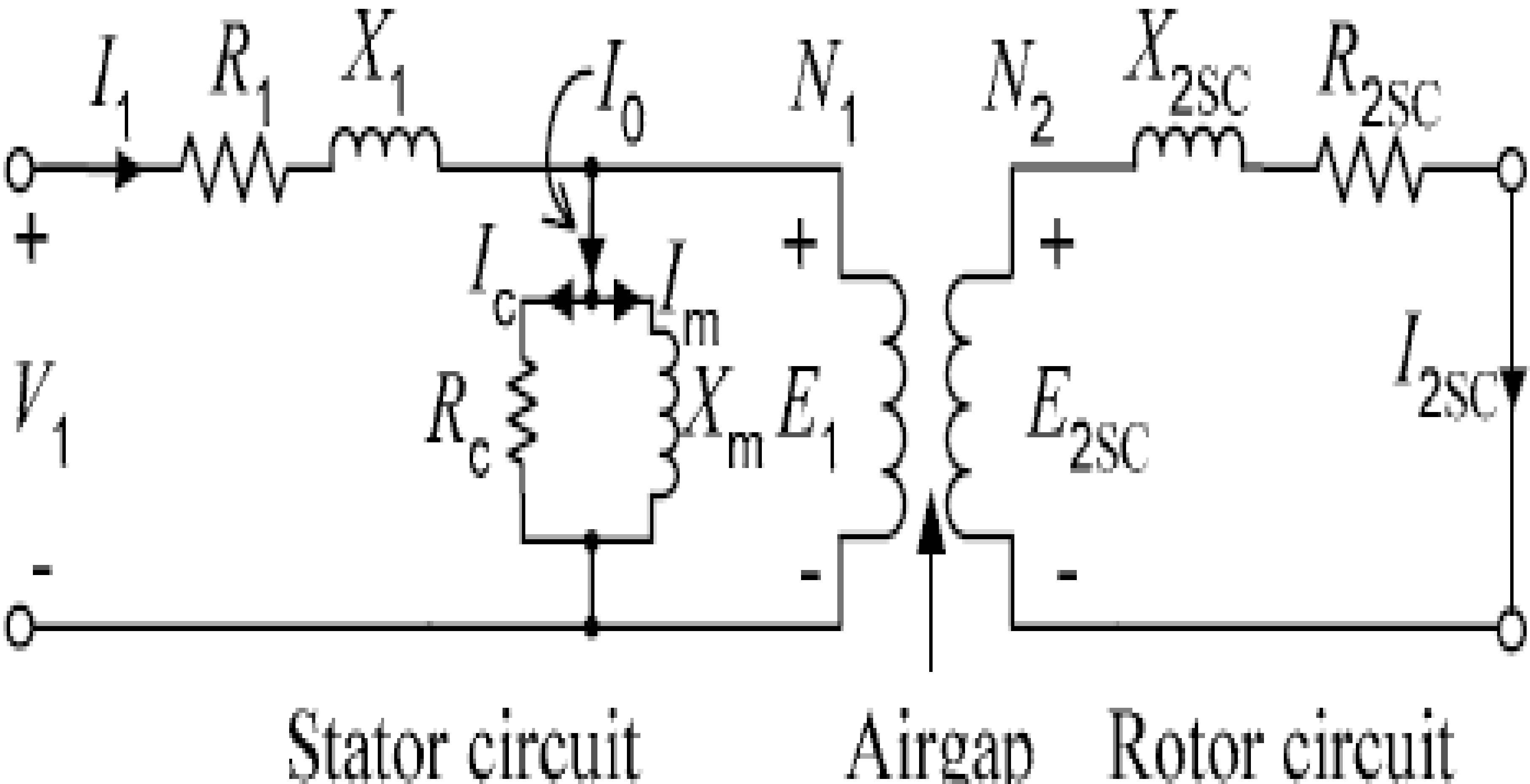
$E_1 = 4.44 f_1 N_1 \Phi$, where Φ is flux per pole

$E_2 = 4.44 f_1 N_2 \Phi$

Chapter Three
Equivalent Circuit of IM

Conventional equivalent circuit:

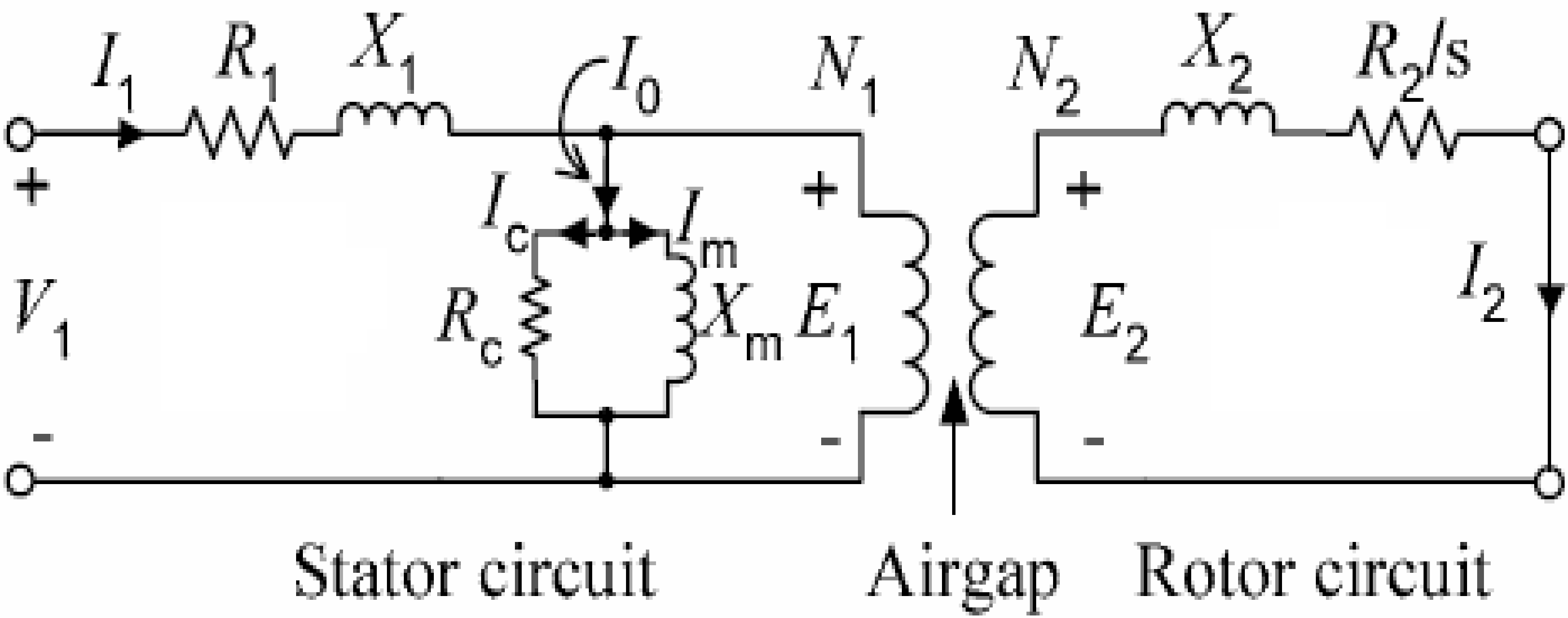
Step2: Rotor winding is shorted (Under normal operating conditions, the rotor winding is shorted. The slip is s)



Chapter Three
Equivalent Circuit of IM
Conventional equivalent circuit:

•*Note: the frequency of E_2 is f_r=Sf because rotor is rotating.Step3 : Eliminate f_2*

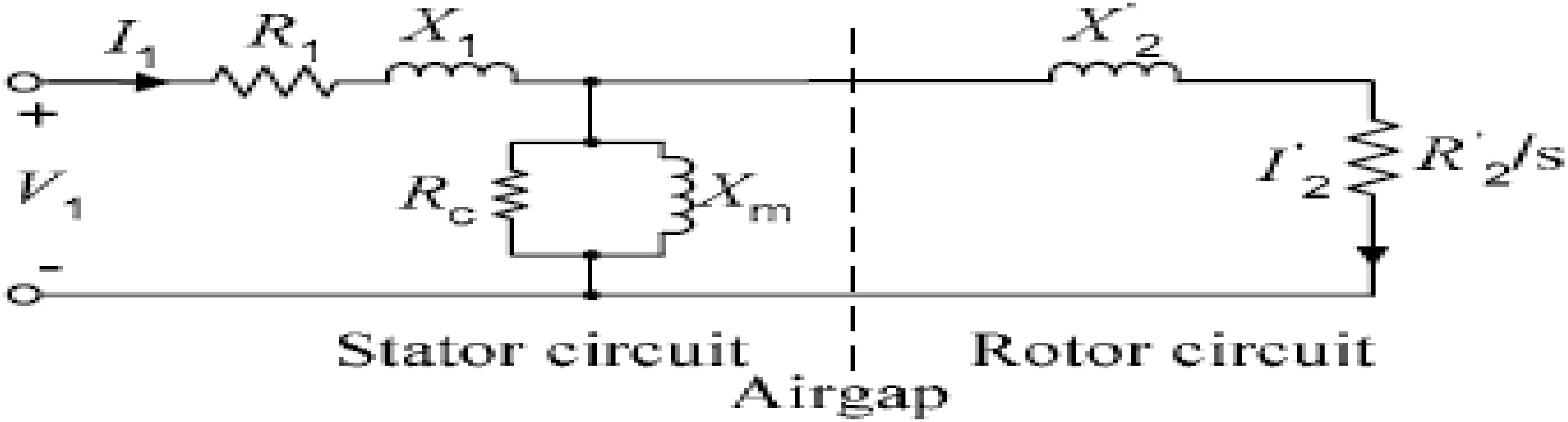
$$I_{2sc} = \frac{E_{2sc}}{R_{2sc} + jX_{2sc}} = \frac{sE_2}{R_2 + jsX_2} = \frac{E_2}{\frac{R_2}{s} + jX_2} = I_2$$



Chapter Three
Equivalent Circuit of IM

The Exact equivalent circuit:

Step 4 :Referred to the stator side



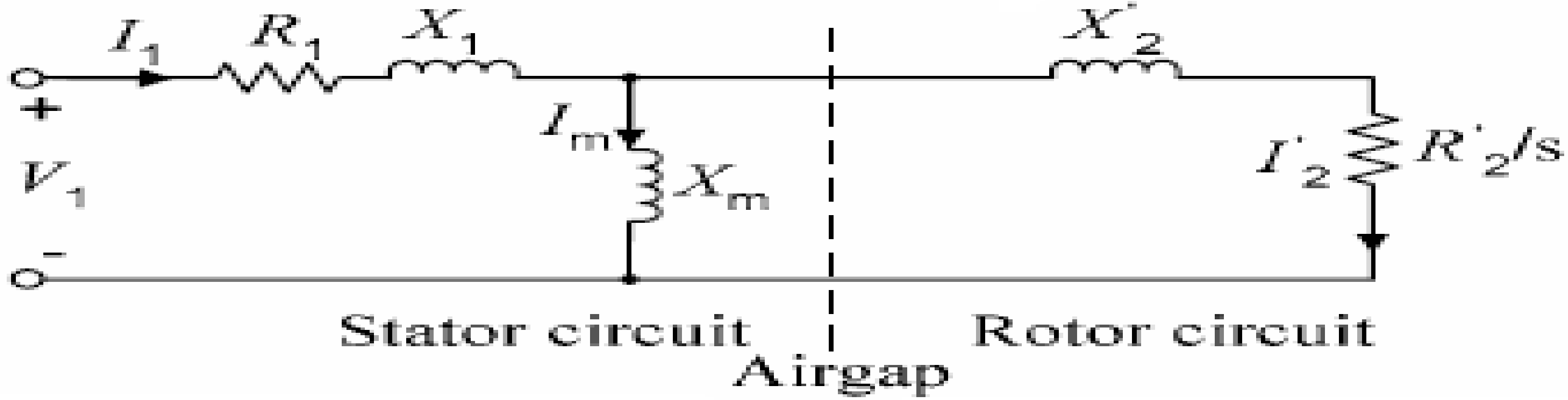
The Exact equivalent circuit:

$$X_2' = a^2 X_2,$$

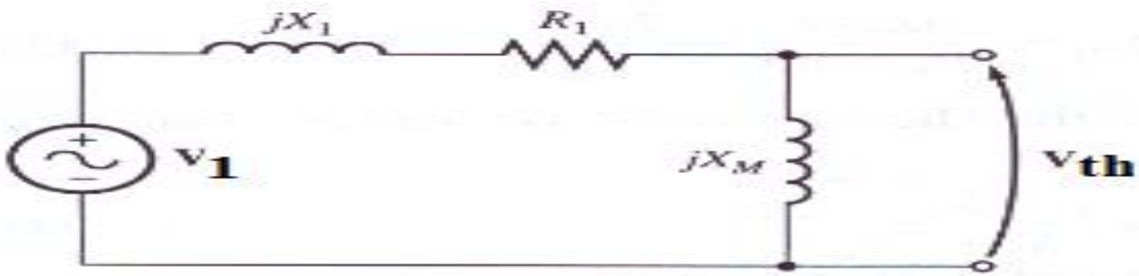
$$R_2' = a^2 R_2,$$

$$I_2' = \frac{1}{a} I_2,$$

$$\text{where } a = \frac{N_1}{N_2}$$

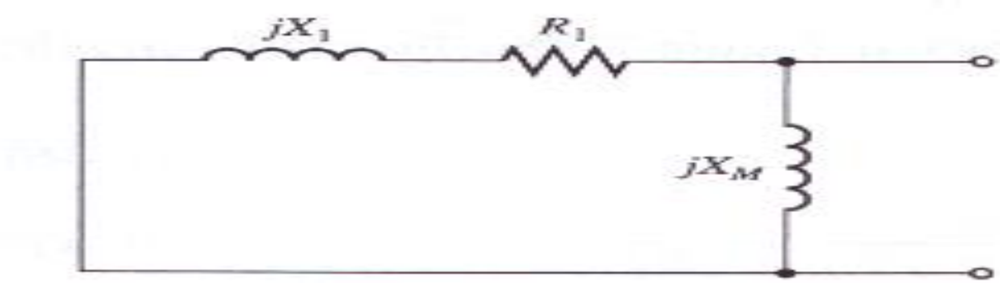


IEEE recommended equivalent circuit

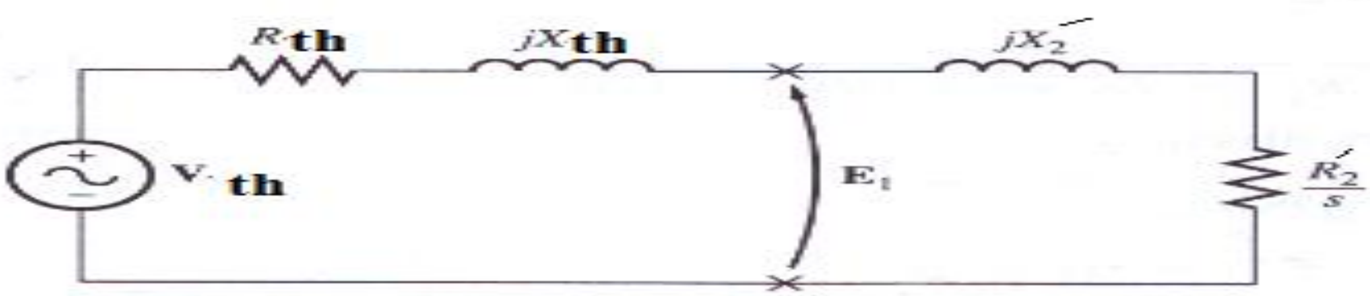


$$V_{th} = \frac{jX_M}{R_1 + jX_1 + jX_M} V_1$$

$$V_{th} = \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}} V_1$$



$$Z_{th} = \frac{jX_M (R_1 + jX_1)}{R_1 + j(X_1 + X_M)}$$



$$V_{th} = V_1 \frac{jX_m}{R_1 + jX_1 + jX_m}$$, *The magnitude of the Thevenin voltage* V_{th} *is*

IEEE recommended equivalent circuit

$$V_{th} = V_1 \frac{X_m}{\sqrt{R_1^{\,2} + (X_1 + X_m)^2}} \approx V_1 \frac{X_m}{X_1 + X_m}$$

(because $X_m >> (X_1 \ \& \ R_1)$**).**

The Thevenin impedance $Z_{th} = \frac{Z_1 Z_m}{Z_1 + Z_m}$

$$Z_{th} = R_{th} + j\ X_{th} = \frac{jX_m(R_1 + jX_1)}{R_1 + jX_1 + jX_m}$$

The Thevenin resistance and reactance are approximately

because $X_m >> X_1 \ \& \ X_m + X_1 >> R_1$

$$R_{th} \approx R_1 \left(\frac{X_m}{X_1 + X_m}\right)^2 \ \& \ X_{th} \approx X_1$$

Chapter Three

Equivalent Circuit of IM

Conventional equivalent circuit:

IEEE recommended equivalent circuit

$$I_2' = \frac{V_{th}}{Z_{th} + Z_2} = \frac{V_{th}}{R_{th} + R_2'/s + jX_{th} + jX_2}$$

$$I_2' = \frac{V_{th}}{\sqrt{(R_{th} + R_2'/s)^2 + (X_{th} + X_2)^2}}$$

$$P_g = 3(I_2')^2 \frac{R_2'}{s} =$$

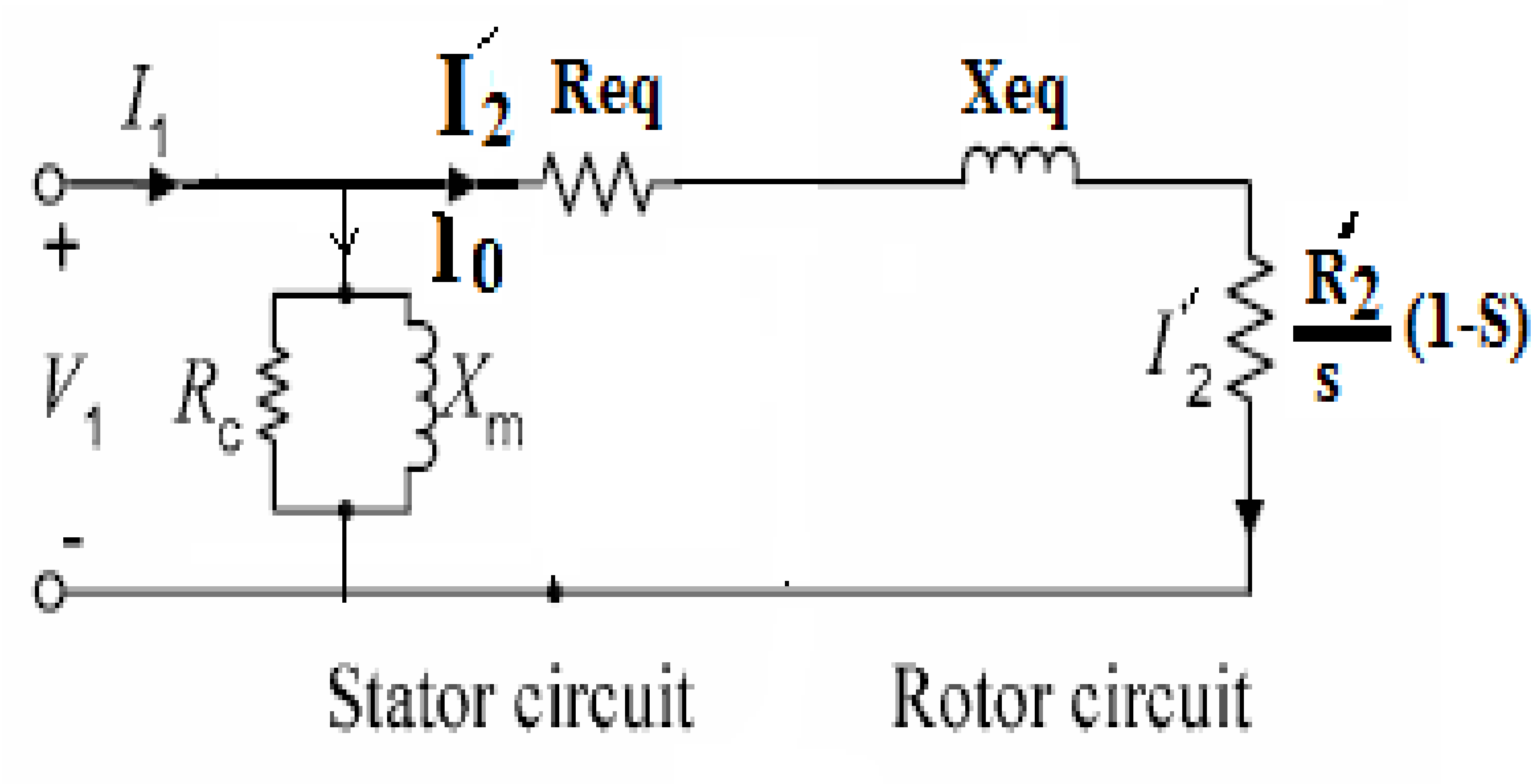
$$3\frac{V_{th}^{\;2}}{(R_{th} + R_2'/s)^2 + (X_{th} + X_2)^2}\frac{R_2'}{S}$$

$$\therefore The\ electromagnetic\ torque,$$

$$T_e = 3\frac{V_{th}^{\;2}}{\omega_s (R_{th} + R_2'/s)^2 + (X_{th} + X_2)^2}\frac{R_2'}{s}$$

Chapter Three
Equivalent Circuit of IM

Approximate equivalent circuit:



Chapter Three
Equivalent Circuit of IM

The Phaser Diagram of Three Phase IM:

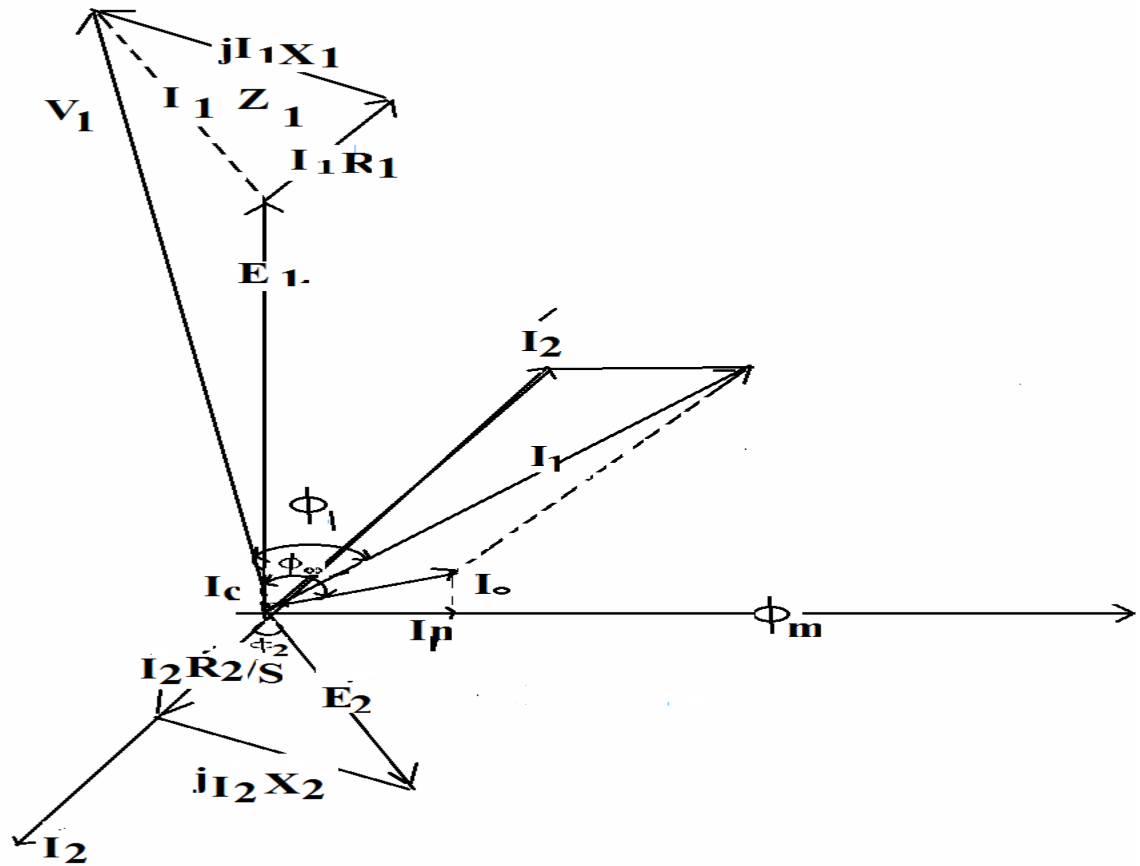
If the rotating magnetic field presented in the air gap considered as a reference, then;

$\phi m \perp E1$ Lag. & ϕm is in phase with Im

$\phi 1$ is the angle between I1 & V1

$\phi 0$ is the angle between I0 & V1

$\phi 2$ is the angle between I2 & E2



Chapter Three
Equivalent Circuit of IM
Example

Example:

A 415V, delta connected three phase IM has an equivalent circuit of stator impedance (1+j2) Ω and an equivalent rotor impedance of (1.2+j1.8) Ω. The magnetizing branch impedance of j50Ω and 500Ω per phase.

1- Use the exact equivalent circuit, find input power, input current, power factor and efficiency at a slip of 0.04. If the mechanical loss equal to 220Watt.

2- Use the approximate equivalent circuit, find input power, input current, power factor and efficiency at a slip of 0.04. If the mechanical loss equal to 220Watt.

3- Use the recommended IEEE standard equivalent circuit, find input power, input current, power factor and efficiency at a slip of 0.04. If the mechanical loss equal to 220Watt.

Chapter Three
Equivalent Circuit of IM
Example

Solution:

1- $Z_2^{/} = R_2^{/} + jX_2^{/} + R_{mech}^{/} = 30 + j1.8 = 30\angle 3.4^0 \Omega$

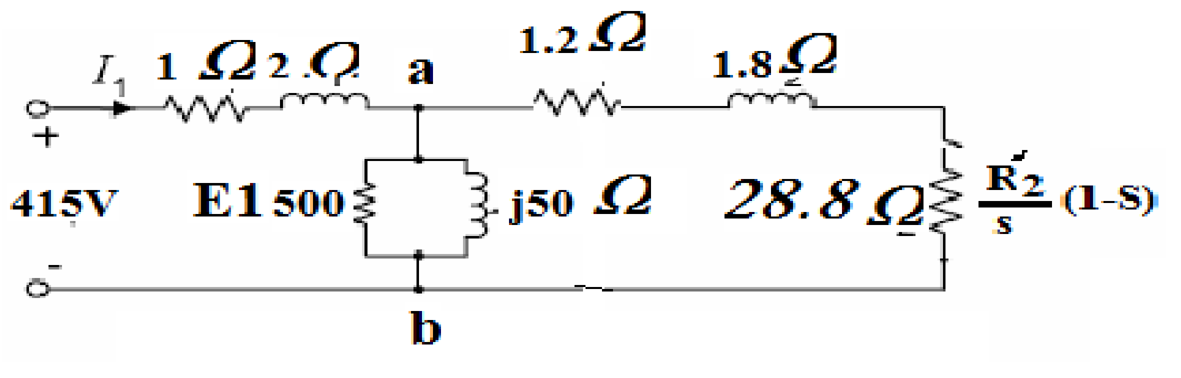
$$z_M = \frac{500 \times j50}{500 + j50} = 49.75\angle 84.29^0 \Omega$$

$$Z_{ab} = Z_2^{/} // Z_m = 24\angle 31.9^0 \Omega$$

$$Z_{inp} = Z_{ab} + Z_1 = 25.95\angle 34.45_0 \Omega$$

$$I_1 = \frac{V_1}{Z_{inp}} = \frac{415\angle 0}{25.95\angle 34.45} = 16\angle -34.45^0 A$$

$$\cos\varphi = \cos 34.45 = 0.82 lagging$$



Chapter Three
Equivalent Circuit of IM
Example

$$P_{inp} = \sqrt{3}\, V_L I_L \cos\varphi = \sqrt{3}\ \ 415\ \sqrt{3}\ \ 16\ \ 0.82$$
$$= 16.426 kWatt$$

$$I_2^{/} = I_1 \ \frac{Z_m}{Z_m + Z_2^{/}} = 12.84\angle -5.9^{\circ} A$$

$$P_{mech} = 3\,(I_2^{/})^2\ R_{mech} = 3\,(12.84)^2 \ \ 28.8 = 1411.244 kWatt$$

$$P_{out} = P_{mech} - P_{(f+w)} = 14.244 - 0.22 = 14.024 kWatt$$

$$\eta\% = \frac{P_{0ut}}{P_{inp}} \times 100 = \frac{14.024}{16426} \times 100 = 85.38\%$$

$$1 - Stator\,cu\,loss = 3(I_2^{/})^2\ R_1 \ = 3(16)^2\ 1 = 768 Watt$$

$$2 - Stator\,Core\,loss :$$

$$First\,method\ : I_o = \frac{Z_2^{/}}{Z_2^{/} + Z_m} \times I_1 = 7.74\angle -86.78^{\circ} A$$

$$P_{core\ loss} = 3\ \left|I_o\right|^2\ R_{series\ resis\tan ce\ of\ magnetizing\ branch} = 3(7.74)^2\ 4.95 = 890 Watt$$

$$Second\ method\ : by\ calculating\ \ E_1 = V_1\angle 0 - I_1 Z_1$$

$$E_1 = 415 - 16\angle -34.45^{\circ} \times 2.23\angle 63.43^{\circ} = 384.24\angle -2.58^{\circ} V$$

$$I_c = \frac{E_1}{R_c} = \frac{384.2}{500} = 0.768 A$$

$$P_{core\ loss} = 3(I_c)^2\ R_c = 886 Watt$$

$$cu_{rotor} = 3(I_2^{/})^2\ R_2^{/} = 3(12.84)^2\ \ 1.2 = 593.5 Watt$$

$$mechanical\ \ loss(f + w) = 220 Watt(\,from\,quistion)$$

$$Total\,loss = 768 + 890 + 593.5 + 220 = 2471.5 Watt$$

$$Also\,total\,loss = P_{inp} - P_{out} = 16426 - 14024 = 2402 Watt$$

$$! - The\ small\ difference\ is\,due\ to\,round\ off\ \ error.$$

2- The Approximate Equivalent Circuit:

$$Z_{inout} = (Z_1 + Z_2^{\prime}) // Z_m = 31.23\angle 7^{\circ} // 500 // j50$$

$$= 24.16\angle 35.29^{\circ}\,\Omega$$

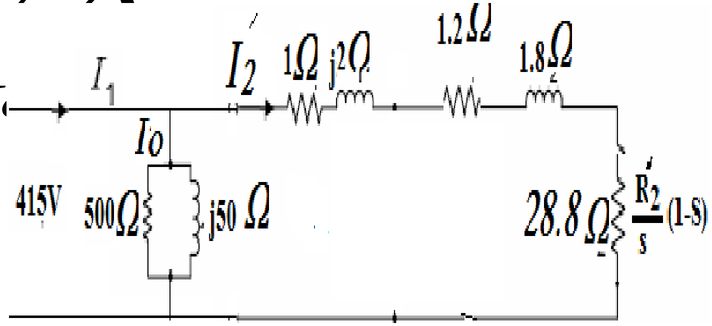
$$I_1 = \frac{415\angle 0^{\circ}}{24.16\angle 35.29^{\circ}} = 17.17\angle -35.32^{\circ}\,A$$

$$\cos\phi = \cos 35.32 = 0.816$$

$$P_{input} = \sqrt{3}\ 415\ \sqrt{3}\ 17.17\quad 0.816 = 17442\,Watt$$

$$I_2^{\prime} = I_1\ \frac{Z_m}{Z_m + Z_1 + Z_2^{\prime}} = 13.28\angle -7^{\circ}\,A$$

$$P_{mechanical} = 3\ (I_2^{\prime})^2\ R_{mech} = 15.237\,kW$$



$$P_{out} = 15237 - 220 = 15017\,Watt$$

$$\eta\% = \frac{15017}{17442} \times 100 = 86\%$$

$$Stator\ copper\ loss = 3(I_2^{\prime})^2\,R_1 = 529\,Watt$$

$$I_c = \frac{V_1}{R_m} = \frac{415}{500} = 0.83A$$

$$P_{core\ loss} = 3\ I_c^{\ 2}\ R_c = 1033.35\,Watt = 3\frac{V_1^{\ 2}}{R_c}$$

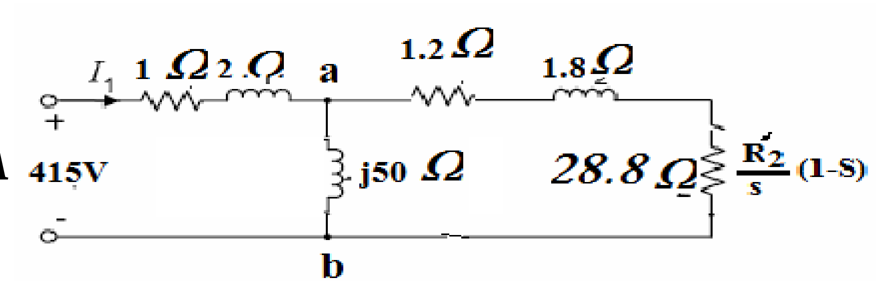
$$cu_{rotor} = 3(I_2^{\prime})^2\ R_2^{\prime} = 634.9\,Watt$$

$$Total\ \ loss = 529 + 1033.35 + 634.9 + 220 = 2417\,Watt$$

3- Neglecting Rc

$$Z_{ab} = Z_2^{/} \,//\, jX_m = (30 + j1.8) \,//\, j50 = 25.06\angle 33.5^0 \Omega$$

$$Z_{inp} = Z_{ab} + Z_1 = 27\angle 35.86^{\circ} \Omega$$



$$I_1 = \frac{V_1}{Z_{inp}} = \frac{415\angle 0}{27\angle 35.86} = 15.37\angle -33.86^0 A$$

$$\cos\varphi = \cos 33.86 = 0.83 lagging$$

$$P_{inp} = \sqrt{3}\, V_L I_L \cos\varphi = \sqrt{3}\ \ 415\ \ \sqrt{3}\ \ 15.3\ \ 0.83$$

$$= 15.882 kWatt$$

$$I_2^{/} = I_1 \ \frac{jX_m}{jX_m + Z_2^{/}} = 12.83\angle -3.04^0 A$$

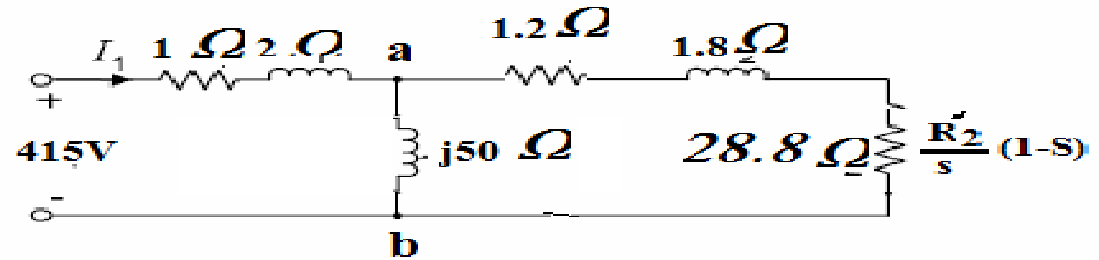
$$P_{mech} = 3\,(I_2^{/})^2\, R_{mech} = 3\,(12.83)^2\ \ 28.8 = 14.22 kWatt$$

$$P_{out} = P_{mech} - P_{(f+w)} = 14.222 - 0.22 = 14.0 kWatt$$

$$\eta\% = \frac{P_{0ut}}{P_{inp}} \times 100 = \frac{14}{15.882} \times 100 = 88.1\%$$

$$Stator\ cu\ loss = 3(I_2^{/})^2\, R_1 \ = 3(15.37)^2\ \ 1 = 708.7 Watt$$

4- Recommended IEEE



$$V_{th} \approx V_1 \frac{X_m}{X_1 + X_m} \approx 415 \times \frac{50}{2+50} = 399V$$

$$R_{th} \approx R_1 \left(\frac{X_m}{X_1 + X_m}\right)^2 \ \& \ X_{th} \approx X_1$$

$$R_{th} \approx 1 \times \left(\frac{50}{50+2}\right)^2 = 0.924\Omega$$

$$X_{th} = j2\Omega$$

$$I_2' = \frac{V_{th}}{\sqrt{(R_{th} + R_2'/s)^2 + (X_{th} + X_2)^2}}$$

$$= 12.8A$$

$$P_g = 3(I_2')^2 \frac{R_2'}{s} =$$

$$= 3 \frac{V_{th}{}^2}{(R_{th} + R_2'/s)^2 + (X_{th} + X_2)^2} \frac{R_2'}{S}$$

$$= 14.76 kWatt$$

$$P_m = P_g - Rotor\,cu\,loss$$

$$= 14.76kW - 3 \times 12.8^2 \times 1.2 = 14170Watt$$

$$P_{out} = 14170 - 220 = 13.95 kWatt$$

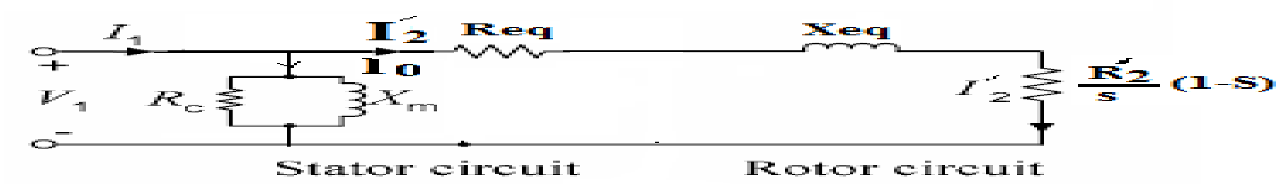
$$\eta\% = \frac{13950}{15882} \times 100 = 87.83\%$$

Comparison between methods

	Exact	Approximate	IEEE	Exact without Rm
Efficiency %	85.38	86	87.83	88.1
Stator current (A)	16	17.17	15.37	15.37
Power Factor	0.82	0.816	0.83	0.83

Example: A 6-pole, 415V, 50Hz, three phase IM, delta connected has the following data: $Z_1=(2.2+j5.8)\,\Omega=Z_2'$, no-load line current $I_o=3.54A$, iron loss =280Watt, mechanical loss=60Watt. Determine the input current, power factor, out put torque and efficiency if slip is 0.03 using approximate equivalent circuit.

Solution:



$$I_2' = \frac{15\angle 0}{2.2 + j5.8 + 2.2/0.03 + j5.8} = 5.431\angle -8.73^\circ A$$

$$I_o / phase = \frac{3.54}{\sqrt{3}} = 2.044\ A$$

$$P_{input\ no-load} = \sqrt{3} V_o I_o \cos\phi_o \Rightarrow 280 = \sqrt{3} \times 415 \times 3.54 \times \cos\phi$$

$$\therefore \cos\phi_o = 0.11\ lag\ .$$

$$I_o = 2.044\angle \cos^{-1} 0.11 = 2.044\angle -83.68^\circ A$$

$$I_1 = I_o + I_2' = 6.28\angle -27^\circ A,\ \cos\phi = \cos(-27) = 0.89\ la$$

$$P_{input} = \sqrt{3}\ \ 415\ \ \sqrt{3}\ \ 6.28\ \ 0.89 = 6958.6\,Watt$$

$$P_{out} = P_2 - (cu_2 + P_{f\&w}) = 3\times(I_2')^2\frac{R_2'}{S}(1-S) - P_{f\&w}$$

$$= 6234.4\,Watt$$

$$T_{out} = \frac{6234.4}{2\pi\times 970/60} = 61.4\,N.m$$

$$\eta\% = \frac{6234.4}{6958.6}\times 100 = 89.6\%$$