# Chapter Four

# Torque-Slip Characteristics:

## Torque Equation:

Total torque developed electromagnetically can be obtained:

$$T_e = \frac{3P_{mech}}{\omega_m} = \frac{3P_g(1-S)}{\omega_s(1-S)} = \frac{3P_g}{\omega_s},$$

$$n = n_s(1-S) \;\&\; \omega_m = \omega_s(1-S)$$

# Chapter Four

## Torque-Slip Characteristics:

$$P_g / phase = (I_2^{/})^2 \; \frac{R_2^{/}}{S}$$

$$I_2^{/} \qquad = \frac{V / phase}{(R_1 + R_2^{/} / S) + j(X_1 + X_2^{/})}$$

$$T_e = \frac{3P_g}{\omega_s} = \frac{3(I_2^{/})^2 \times \frac{R_2^{/}}{S}}{\omega_s} = \frac{3[\frac{V}{(R_1 + R_2^{/} / S)^2 + j(X_1 + X_2^{/})^2}]^2 \times \frac{R_2^{/}}{S}}{\omega_s}$$

$$\therefore T_e \;\; = \frac{3 \times V^2 \times \frac{R_2^{/}}{S}}{\omega_s \; [(R_1 + \frac{R_2^{/}}{S})^2 + (X_1 + X_2^{/})^2]} \qquad (N.m)$$

$$Where \; V = stator \; per \; phase \; voltage$$

$$\omega_s = 2\pi n_s = 2\pi \frac{120 f}{60 P} \qquad (rad / \sec)$$

# *Chapter Four*

# *Torque-Slip Characteristics:*

$$Te = E_1 I_2 / \cos\phi_2)/\omega_s = ( 3*4.44*N*f*\phi_r*I_2/*\cos\phi_2)/\omega_s$$

***Increasing of rotor speed from standstill(0rpm) to rated speed (n):***

***Note: SE1 & I2/ decrease, but cos$\phi$2 improve (increase), so that initially there is an increase of torque with slip until the fall of the current is greater than the rise of power factor.***

***The electromagnetic torque is very nearly proportional to I2/cos$\phi$2, because the peak mutual- flux is approximately constant as in the transformer.***

# *Chapter Four*

# *Torque-Slip Characteristics:*

***The effect of variation in slip (S) alone are the most informative, this equation can be reformed as ;***

$$T_e = \frac{K_t R_2^{/}}{S} \frac{1}{(R_1 + R_2^{/} / S)^2 + X_{eq}{}^2}$$

$$= \frac{K_t R_2^{/}}{S R_1^2 + 2 R_1 R_2^{/} + \frac{(R_2^{l})^2}{S} + S X_{eq}{}^2}$$

$$K_t = \frac{3V^2}{\omega_s} \quad \& \quad X_{eq} = X_1 + X_2^{/}$$

# Chapter Four

## Torque-Slip Characteristics:

***The maximum torque or breakdown torque can be obtained by differentiation ;***

$$\frac{dT_e}{ds} = \frac{0 - K_t R_2^{/}[R_1^2 - (\frac{R_2^{/}}{S})^2 + X_{eq}^{\ 2})]}{[denominator]^2},$$

$$At\ maximum\ point\ \frac{dT_e}{ds} = 0$$


$$\frac{dT_e}{ds} = \frac{0 - K_t R_2^{/}[R_1^2 - (\frac{R_2^{/}}{S})^2 + X_{eq}^{\ 2})]}{[denominator]^2},$$

$$At\ maximum\ point\ \frac{dT_e}{ds} = 0$$

# Chapter Four

# Torque-Slip Characteristics:

$$\frac{R_2'}{S} = \mp\sqrt{(R_1)^2 + (X_{eq})^2}$$

*The maximum slip at which maximum torque occurs*

*is called* $S_{max}$

$$S_{max} = \mp\frac{R_2'}{\sqrt{(R_1)^2 + (X_{eq})^2}}$$

The corresponding power factor( if we assume that rotor and stator reactance are equal and stator resistance is neglected).

$\phi_2 = \tan^{-1}\dfrac{X_2'}{\frac{R_2'}{S}} \approx 45^\circ$, Substituting $S_{max}$ in original Torque equation:

$$T_{e\max} = \frac{K_t[\mp\sqrt{(R_1)^2 + (X_{eq})^2}]}{[R_1^2 \mp (\sqrt{(R_1)^2 + (X_{eq})^2})^2 + X_{eq}^2]}$$

$$= \frac{K_t[\mp\sqrt{(R_1)^2 + (X_{eq})^2}]}{[R_1^2 + X_{eq}^2 \mp R_1^2 \mp X_{eq}^2 \mp 2R_1\sqrt{(R_1)^2 + (X_{eq})^2}]}$$

$$= \frac{K_t}{2[R_1 \mp \sqrt{(R_1)^2 + (X_{eq})^2}]}$$

$$\therefore T_{\max} = \frac{3V^2}{2\omega_s[R_1 \mp \sqrt{(R_1)^2 + (X_{eq})^2}]} \qquad (N.m)$$

# Chapter Four

# Torque-Slip Characteristics:

## The Recommended IEEE and Thivenen Equivalent Circuit:

$$\frac{R_2^{/}}{S} = \pm\sqrt{(R_{th})^2 + (X_{th} + X^{/}{}_2)^2}$$

$$The \max imum\, slip\, at\, which \max imum\, torque\, occurs$$

$$is\, called\;\; S_{\max}$$

$$S_{\max} = \pm \frac{R_2^{/}}{\sqrt{(R_{th})^2 + (X_{th} + X_2^{/})^2}}$$

$$T_{\max} = \frac{3V_{th}{}^2}{2\omega_s[R_{th} + \sqrt{(R_{th})^2 + (X_{th} + X_2^{/})^2}\,]} \qquad (N.m)$$

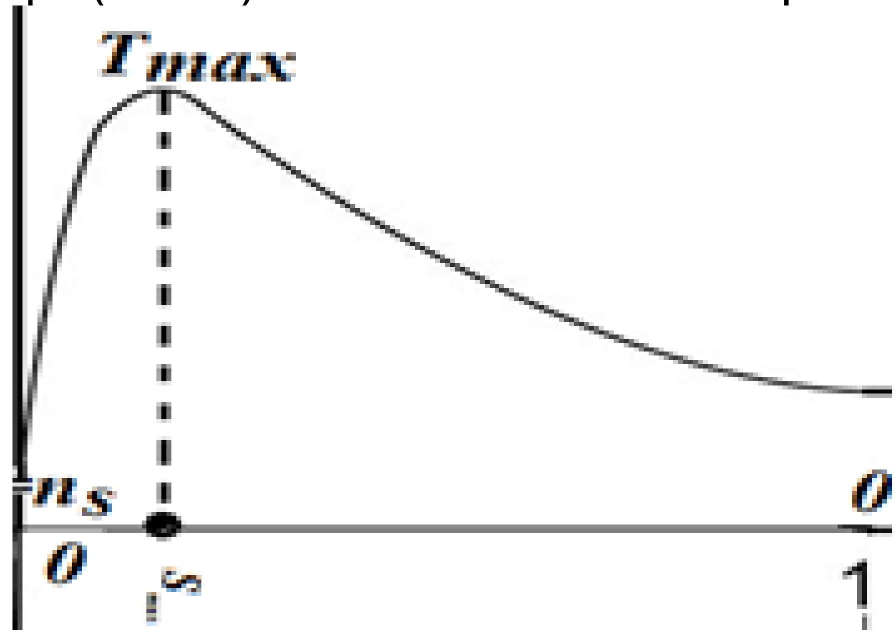
Note:

1- From the above equation the maximum torque is independent on the $R_2^{/}$ value, but the value of $S_{max}$ at which the maximum torque occur is directly proportional to $R_2^{/}$.

2- The starting Torque is produced when S=1,Then

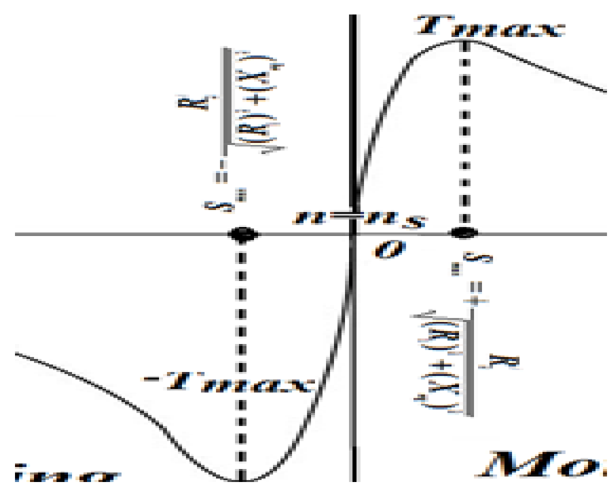
$$T_s = \frac{3 \times V^2 \times R_2^{/}}{\omega_s \, [(R_1 + R_2^{/})^2 + (X_1 + X_2^{/})^2]} \quad (N.m)$$

The starting torque developed perhaps (20-30)% of the maximum torque for simple squirrel cage IM.



-For negative value of slip the maximum torque is greater than for positive value but occur at the same value of slip. The negative value of slip indicates of operating of IM at super synchronous speed ($n>n_S$), in this condition the machine operates as a induction generator.

-From general torque equation $T_e$, when (S=0), the motor must be run as the same speed of rotating field and there is no relative motion speed. It results rotor emf, rotor current and then $T_e=0$.



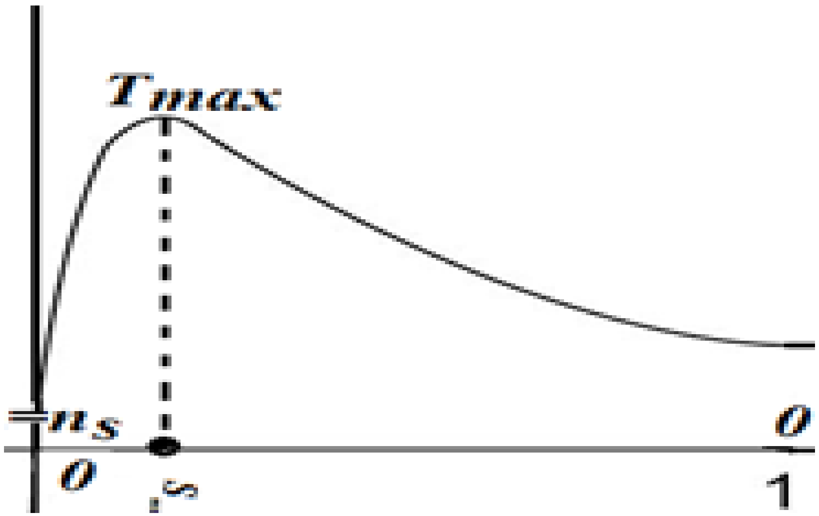
-When slip is very small, all denominator term are approximately equal to$(R_2^{/}/S)$ and then,

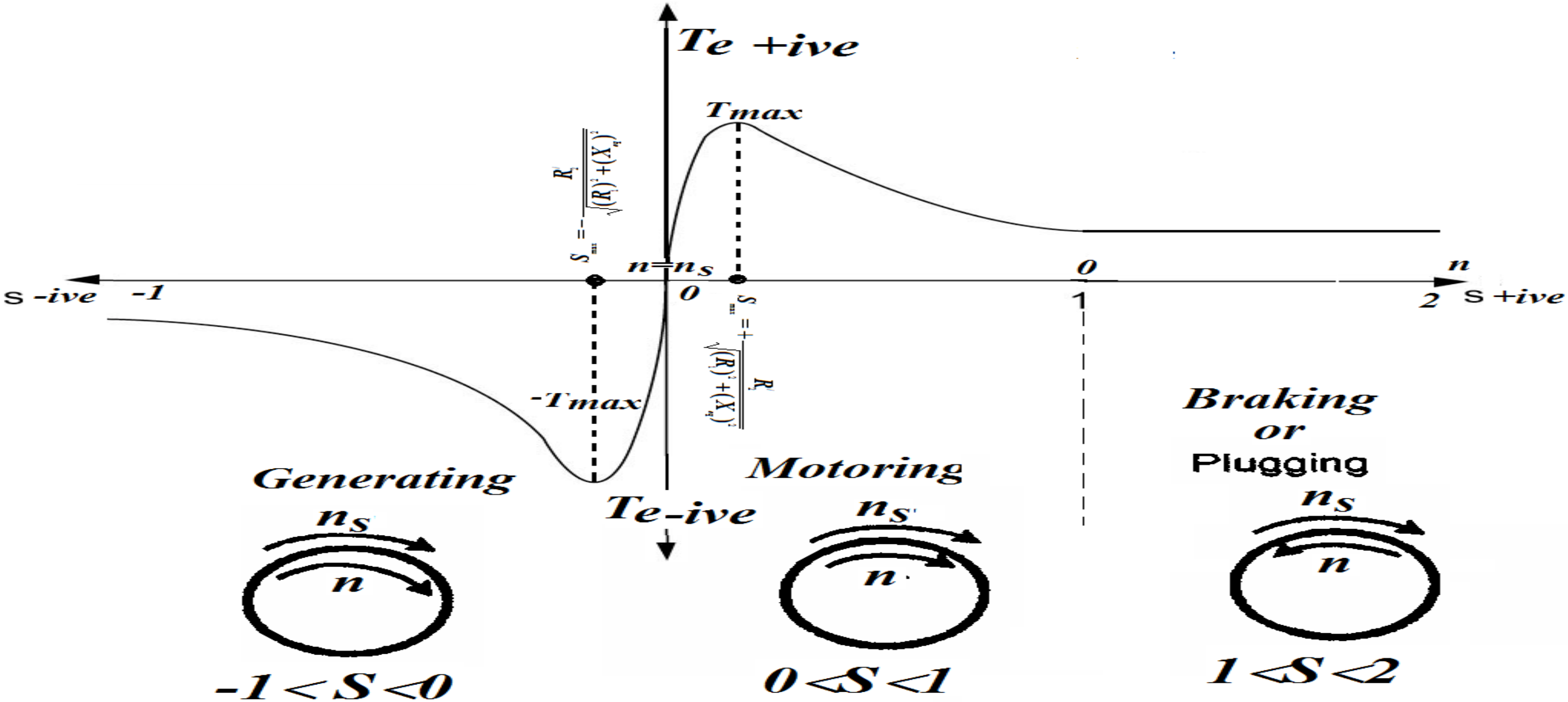
$$T_e \approx \frac{3V^2\ S}{\omega_s R_2^{/}} \quad ,$$

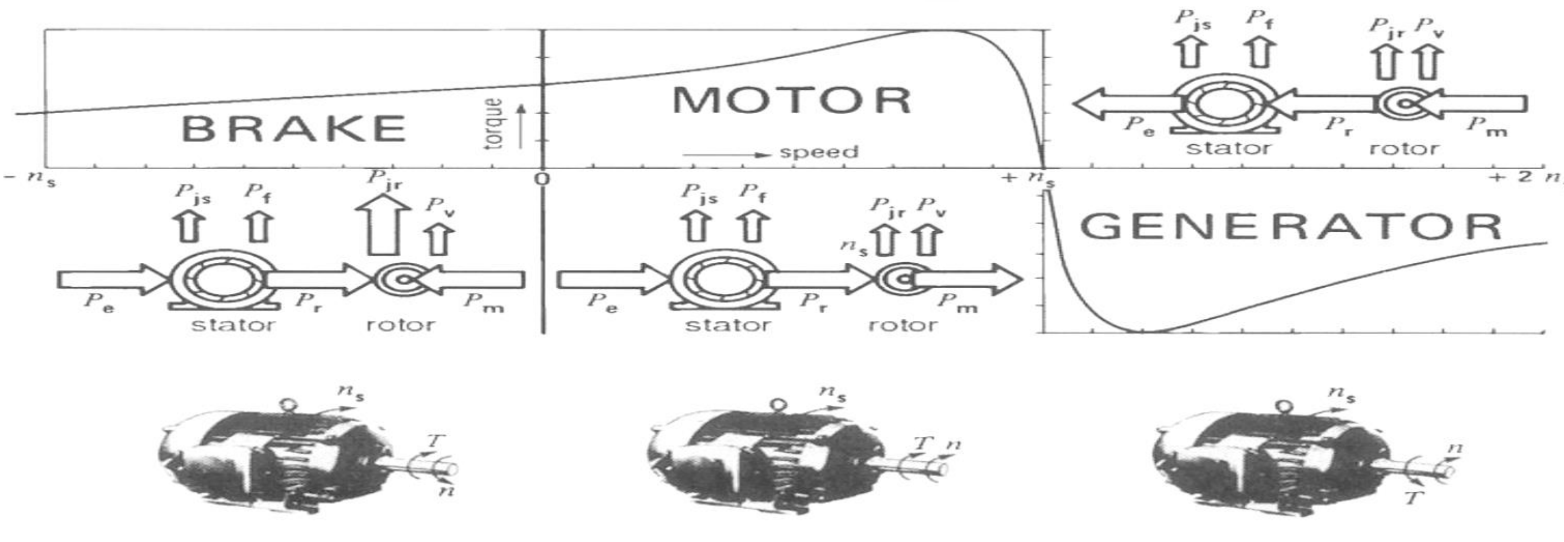
so that torque approximately proportional to S. Near S=0, the curve is a asymptomatic to a straight line and departs from this to reach the maximum value.

-At higher value of slip the curve falls to approach the hyperbolic asymptote which is inversely proportional with slip.

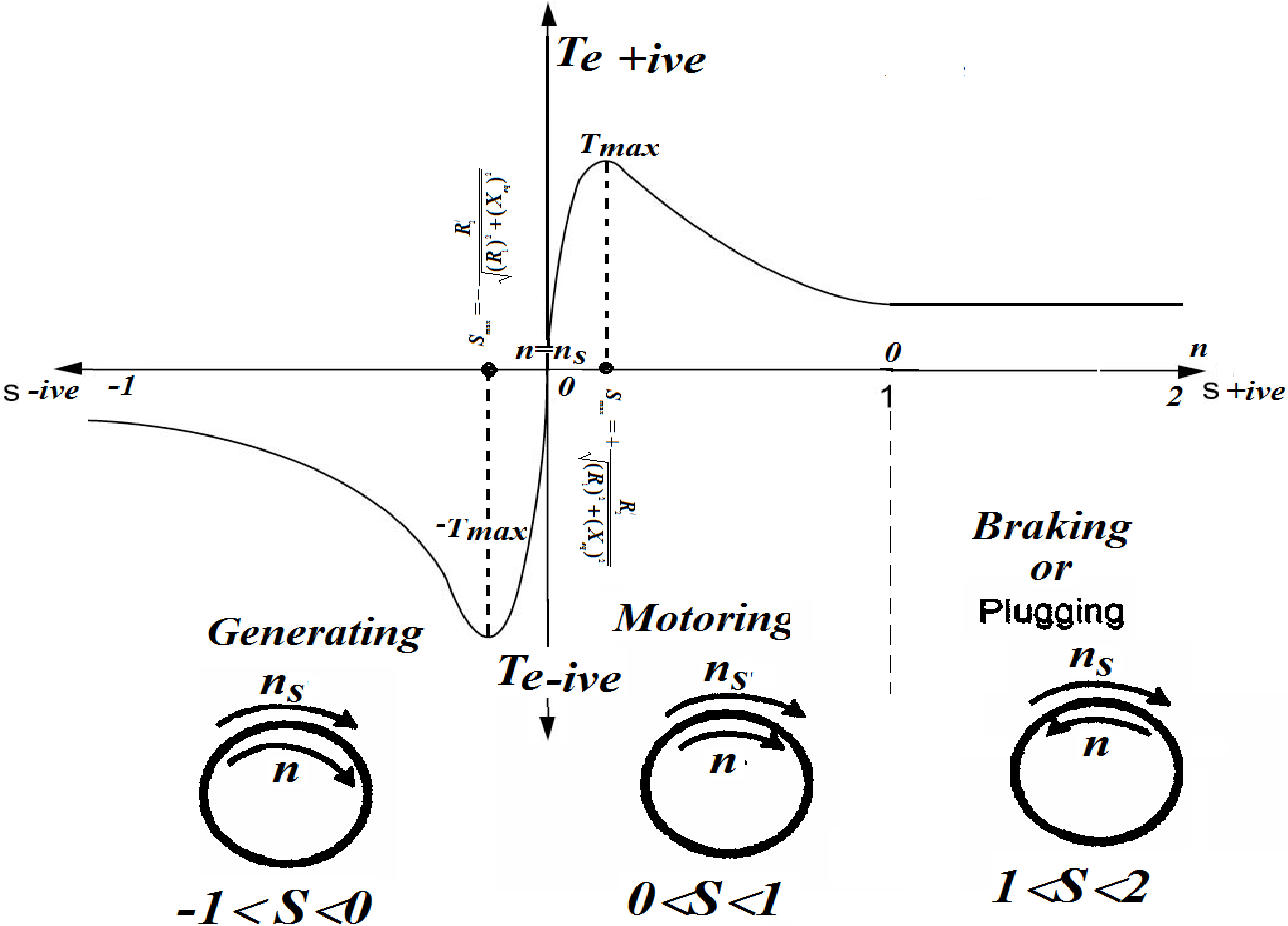
$$T_e \approx \frac{3 \times V^2 \times \dfrac{R_2^{/}}{S}}{\omega_s\ [(R_1)^2 + (X_1 + X_2^{/})^2]}$$



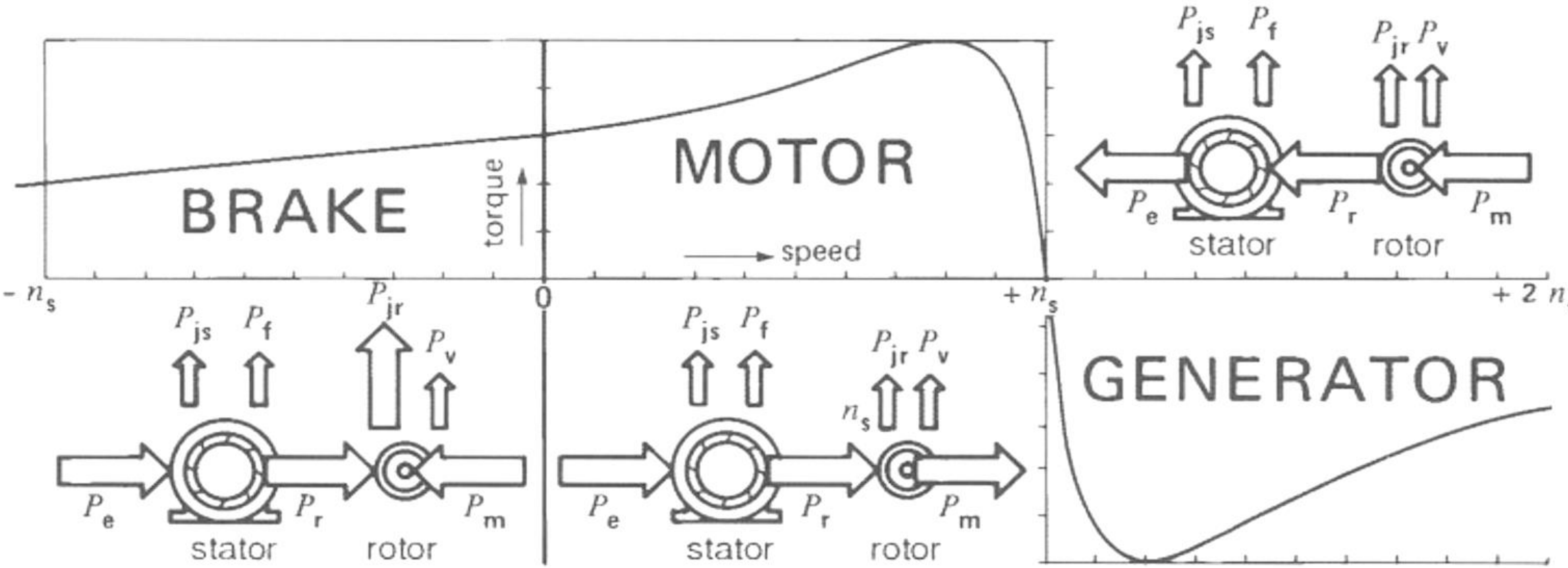


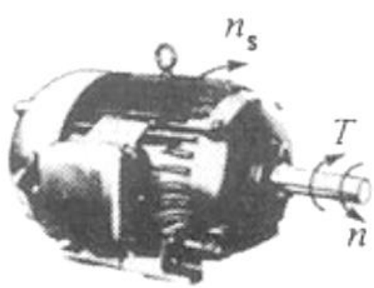




T = torque developed by the machine
n = speed of rotation
ns = synchronous speed of the revolving field

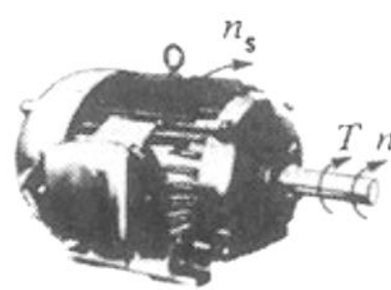

Te +ive
Tmax
$S_{max} = -\frac{R_2'}{\sqrt{(R_1)^2 + (X_{eq})^2}}$
n=ns
0
n
S -ive
-1
0
1
2
S +ive
$S_{max} = +\frac{R_2'}{\sqrt{(R_1)^2 + (X_{eq})^2}}$
-Tmax
Braking
or
Plugging
Generating
Motoring
Te-ive
ns
n
ns
n
ns
n
-1 < S < 0
0 < S < 1
1 < S < 2

# Chapter Four
# Torque-Slip Characteristics:

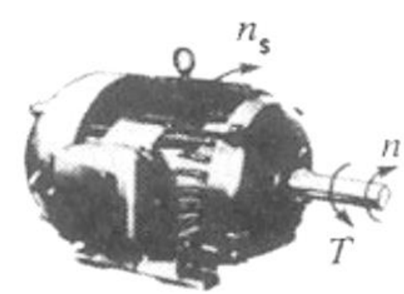



$T$ = torque developed by the machine




$n$ = speed of rotation




$n_s$ = synchronous speed of the revolving field

# *Chapter Four*

## *Slip Ring IM ( Wound Rotor IM)(WRIM):*

*The rotor winding is accessible for external connection through the terminals D, E&F. Usually three phase resistance are added to the rotor winding which causes the rotor circuit resistance $R_2$ to become,*

$$R_{2new} = R_2 + R_{ex}$$

*Where $R_{ex}$=external resistance & $R_2$ the original rotor resistance.*
*The external resistance gives two advantages;*

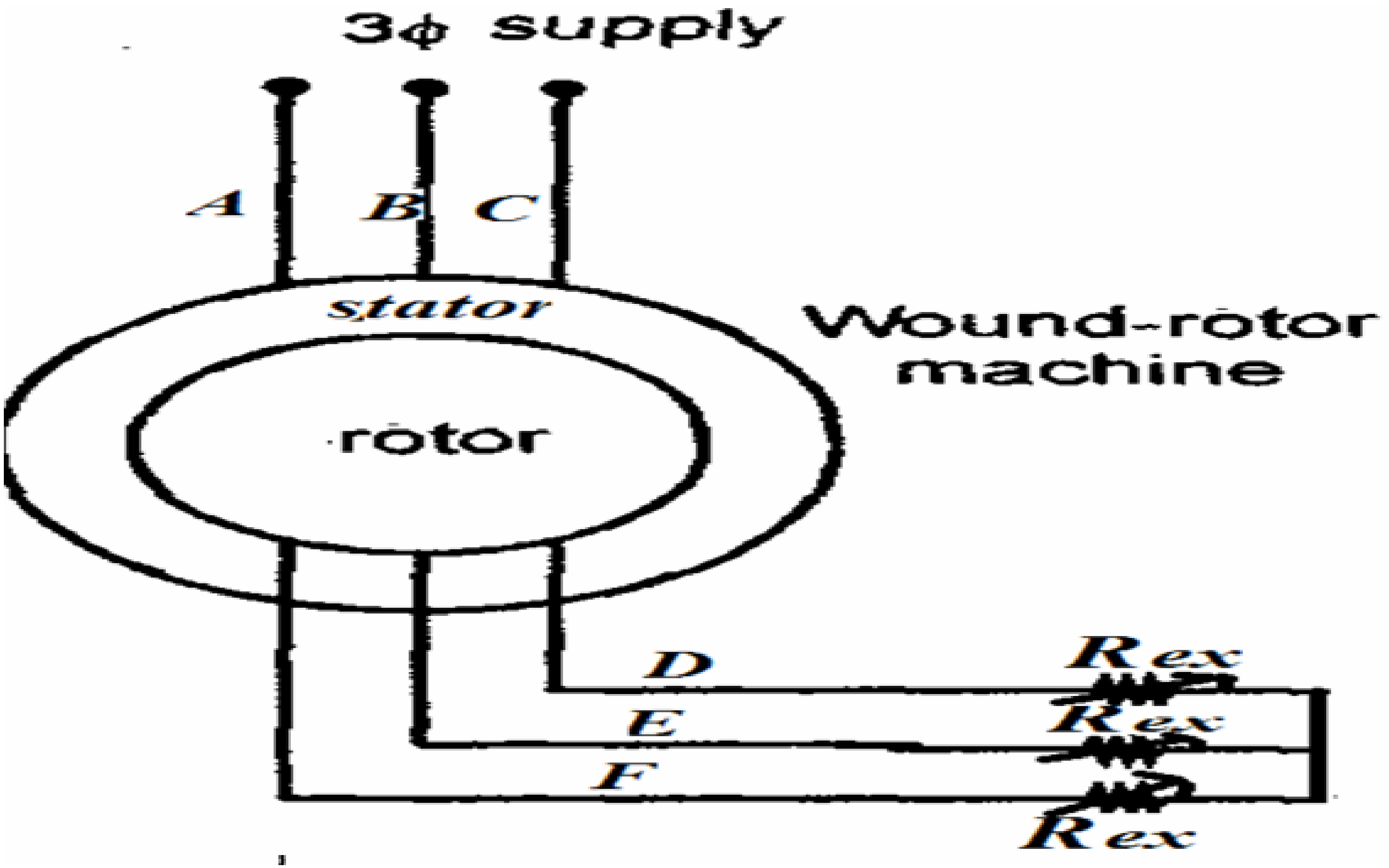
*1.Reduce the rotor current, hence the current drawn from the supply.*

*2.Improving the starting torque developed by improving the rotor current power factor in high proportion to decrease in rotor current.*

*3-During stable operation region, having an external resistance will lead to more slip or large drop in speed and poor efficiency.*

# *Chapter Four*

## *Slip Ring IM ( Wound Rotor IM)(WRIM):*



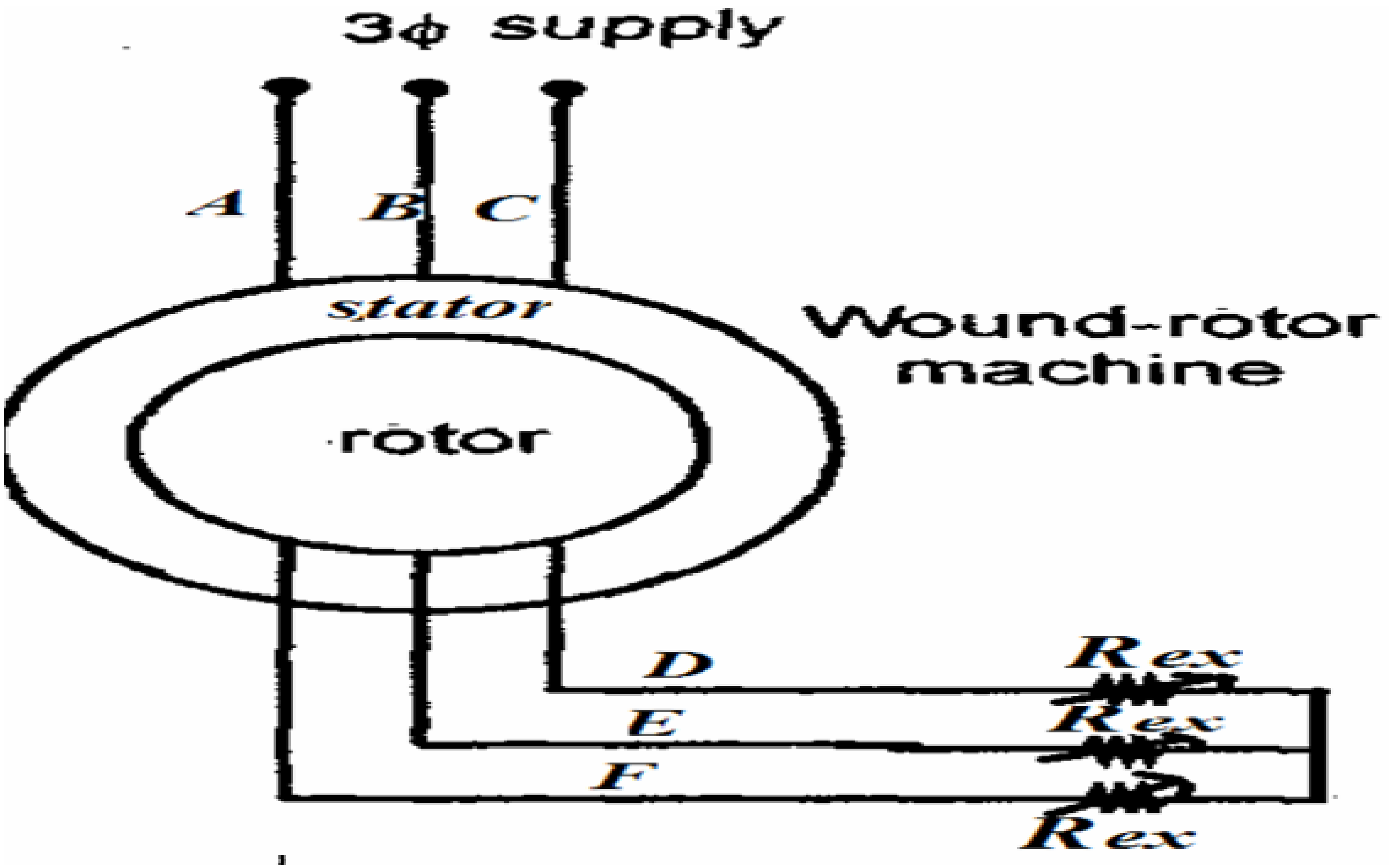
# *Chapter Four*

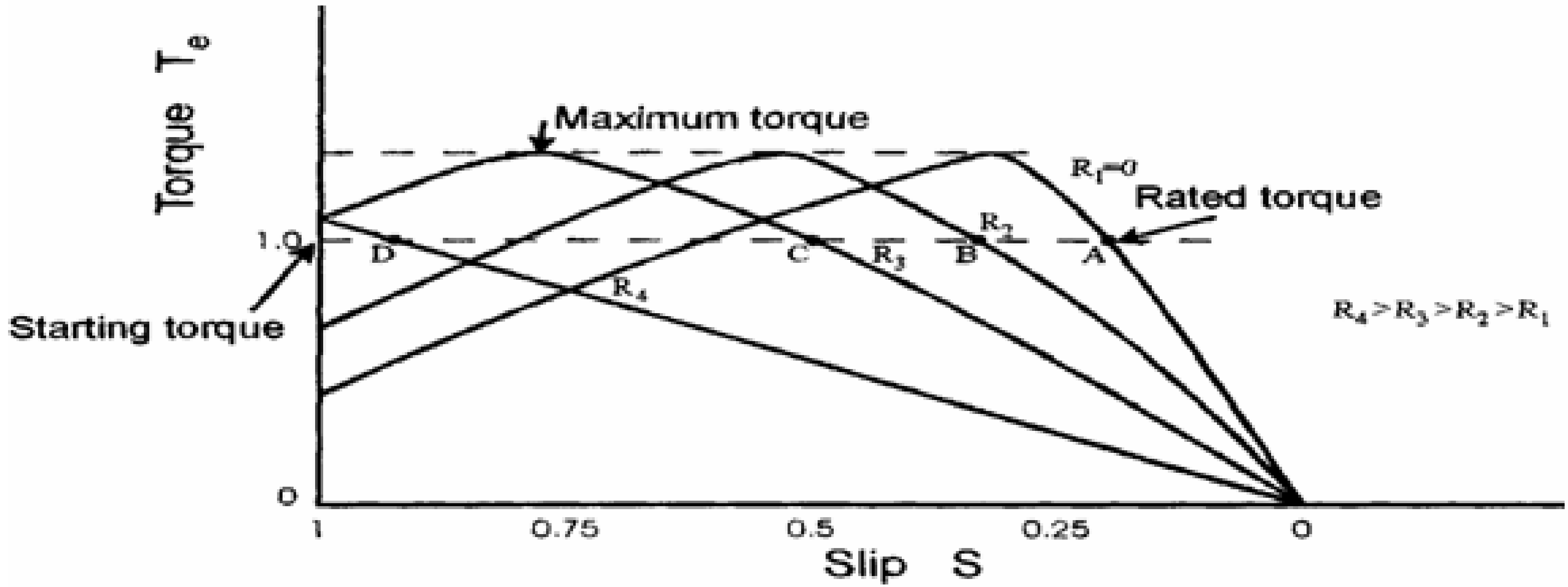
## *Slip  Ring IM ( Wound Rotor IM)(WRIM):*

$$T_S = \frac{3 \times V^2 \times \frac{R'_{2new}}{S}}{\omega_s \left[ \left( R_1 + \frac{R'_{2new}}{S} \right)^2 + \left( X_1 + X'_2 \right)^2 \right]} \qquad (N.m)$$

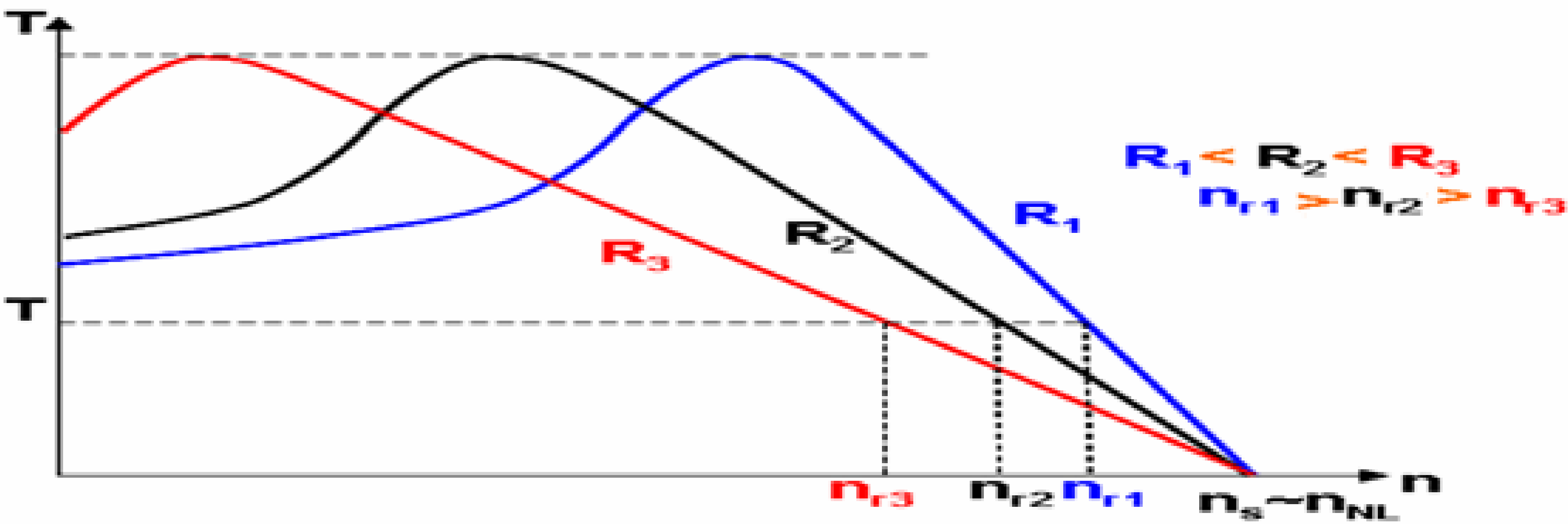
# *Chapter Four*

## *Slip Ring IM ( Wound Rotor IM)(WRIM):*






Torque-slip curves of motor with variable rotor resistance



***Example: A 6-pole, 50Hz, three phase IM has a rotor resistance of 0.09Ω/phase. If its pull out speed is 850rpm, find the value of the resistance to be added to the rotor circuit to develop maximum torque at starting.***

***Solution:***

$$n_s = \frac{120f}{p} = 1000rpm, \qquad S_{\max} = \frac{n_s - n}{n_s} = \frac{1000-850}{1000} = 0.15$$

$$S_{\max} = \frac{R_2}{X_2}, \; 0.15 = \frac{0.09}{X_2}, \; X_2 = 0.6\Omega$$

$$n = 0 \qquad at\,starting (S = 1)$$

$$1 = \frac{R_{new}}{0.6}, R_{new} = 0.6\Omega$$

$$R_{new} = R_2 + R_{ex}, \Rightarrow 0.6 = 0.09 + R_{ex}, \; R_{ex} = 0.51\Omega$$

# Chapter Four

***Example: A 8-pole, 50Hz, WRIM has rotor resistance and standstill reactance per phase of 0.03Ω & 0.15Ω respectively.***

***i. Find the speed at which $T_{max}$ occur.***

***ii. Find the value of Rex /phase to be added to the rotor circuit in order to develop starting torque three fourth of maximum torque at starting.***

***Solution:*** $n_s = 750\,rpm,$

$$i - S_{\max} = \frac{R_2}{X_2} = \frac{0.03}{0.15} = 0.2$$

$$Rotor\,speed\;\;at\;S_{\max} = n_s\;(1 - S_{\max})$$

$$= 750(1 - 0.2) = 600\,rpm$$

$$ii - \frac{T_s}{T_{\max}} = 0.75\;\;(neglecting\;the\;stator\;impedance)$$

$$\frac{\dfrac{3V^2 R'_{2new}}{\omega_s (R'_{2new})^2 + (X'_2)^2}}{\dfrac{3V^2}{2\omega_s X'_2}} = \frac{\dfrac{R_{2new}}{(R_{2new})^2 + (X_2)^2}}{\dfrac{1}{2X_2}} = 0.75$$

$$(\text{since}\,X_2 = 0.15\Omega)$$

$$R_{2new}^2 - 0.4R_{2new} + 0.0225 = 0, \Rightarrow R_{2new} = 0.332\Omega\;\&0.0667\Omega$$

$$\therefore R_{ex} = 0.3\Omega\;\;or\;\;\;0.037\Omega$$

# Chapter Four

***Example: A 8-pole, 400V, 50Hz, WRIM, delta connected, has stator to rotor turn ratio is 2:1. The stator impedance per phase is (0.13+j0.6)Ω and rotor circuit standstill impedance per phase is (0.035+j0.15)Ω. Find;***

***1- The Smax & Tmax( for motoring and Generating)***

***2-The Ts***

***3-The percentage variation in maximum torque and the speed at this torque, if resistance of rotor circuit is increased by 40%.***

## *Solution:*

$$\omega_s = 2\pi n_s = 78.53 \, rad/\sec, \; Z_1 = 0.13 + j0.6\Omega,$$

$$Z_2 = 0.035 + j0.15\Omega, \Rightarrow Z_2^{/} = 2^2(0.035 + j0.15) = 0.14 + j0.6 \;\; \Omega$$

$$S_{max} = \mp \frac{R_2^{/}}{\sqrt{(R_1)^2 + (X_{eq})^2}} = \mp \frac{0.14}{\sqrt{0.13^2 + (0.6 + 0.6)^2}} = \mp 0.115$$

$$T_{\max} = \frac{3V^2}{2\omega_s[R_1 \mp \sqrt{(R_1)^2 + (X_{eq})^2}]}$$

$$= \frac{3(400)^2}{2\times 78.53[0.13 \mp \sqrt{(0.13)^2 + (1.2)^2}]} = +2285 N.m (motoring)$$

$$= -2837.6 N.m (generating)$$

$$T_S = \frac{3\times V^2 \times R_2'}{\omega_s [(R_1 + R_2')^2 + (X_1 + X_2')^2]} = 566 N.m$$

$$The\ percenatge\ variation\ in\ \max.torque = \frac{T_{\max 1} - T_{\max 2}}{T_{\max 1}} \times 100$$

$$= 0\% (because\ the\ value$$

$$\max imum\ torque\ not\ change\ by\ changing\ the\ rotor\ resistance$$

$$(T_{\max 1} = T_{\max 2}).$$

$$R'_{2new} = 1.4 R'_{2old} = 1.4 \times 0.14 = 0.21\Omega$$

$$S_{\max 1} = 0.115, \qquad \therefore n_{r1} = 750(1-0.115) = 663.75 rpm$$

$$S_{\max 2} = \frac{0.21}{\sqrt{0.13^2 + 1.2^2}} = 0.174$$

$$n_{r2} = 750(1-0.174) = 619.5 rpm$$

$$Variation\ in\ rotor\ speed\% = \frac{663.75 - 619.5}{663.75} \times 100 = 6.66\%$$

***Example:* A 460V, 25hp, 60Hz, 4-pol3, Y-connected WRIM has the following impedance in ohm per phase referred to stator;**

***$R1= 0.641\Omega$ $R2^{/}= 0.332\Omega$ $X1=1.106\Omega$ $X2^{/}=0.464\ \Omega$ $Xm=26.3\Omega$; use recommended IEEE model for calculation.***

***What is the maximum torque? At what speed and slip does it occur?***

***What is the starting torque of this motor?***

***When the rotor resistance is doubled, what is the speed at which maximum torque now occur? What is the new starting torque of the motor?***

***Solution:***

$$V_{th} = V_1 \frac{X_m}{\sqrt{R_1{}^2 + (X_1 + X_m)^2}}$$

$$= 266 \quad \frac{26.3}{\sqrt{0.641^2 + (1.106 + 26.3)^2}} = 255.2V$$

$$R_{th} \approx R_1 \left(\frac{X_m}{X_1 + X_m}\right)^2 = 0.641 \left(\frac{26.3}{1.106 + 26.3}\right)^2 = 0.59\Omega$$

$$\& \ X_{th} \approx X_1 = 1.106\Omega$$

$$S_{\max} = \mathrm{m} \frac{R_2^{/}}{\sqrt{(R_{th})^2 + (X_{th} + X_2{}^{/})^2}} = 0.198$$

*The corrspondsrotor speed,*

$$n = n_s(1 - S_{\max}) = 1800(1 - 0.198) = 1444 rpm$$

$$T_{\max} = \frac{3V_{th}{}^2}{2\omega_s[R_{th} + \sqrt{(R_{th})^2 + (X_{th} + X_2{}^{/})^2}]} \qquad (N.m)$$

$$= 229 N.m$$

**b-**

$\therefore$ *The electromagnetic torque at starting,*

$$T_s = 3\frac{V_{th}^{2} \; R_2^{/}}{\omega_s\left((R_{th}+R_2^{/})^2 + (X_{th}+X_2)^2\right)}$$

$$= 104N.m$$

***c- If the rotor resistance is doubled, $S_{max}$=0.396***

***n=1800(1-0.396)= 1087rpm***

***$T_{max}$=229N.m is still as before.***

***$T_S$=170N.m***

***Quiz No.1***

**A three phase 440V, 6-pole, 50Hz, delta connection WRIM has rotor resistance of 0.3Ω and leakage reactance of 1Ω per phase referred to stator side. It runs at a full load slip of 3%. What resistance must be inserted in the rotor circuit to obtain a rotor speed of 800rpm? Neglect stator impedance. The stator to rotor turns ratio is(2.2).**