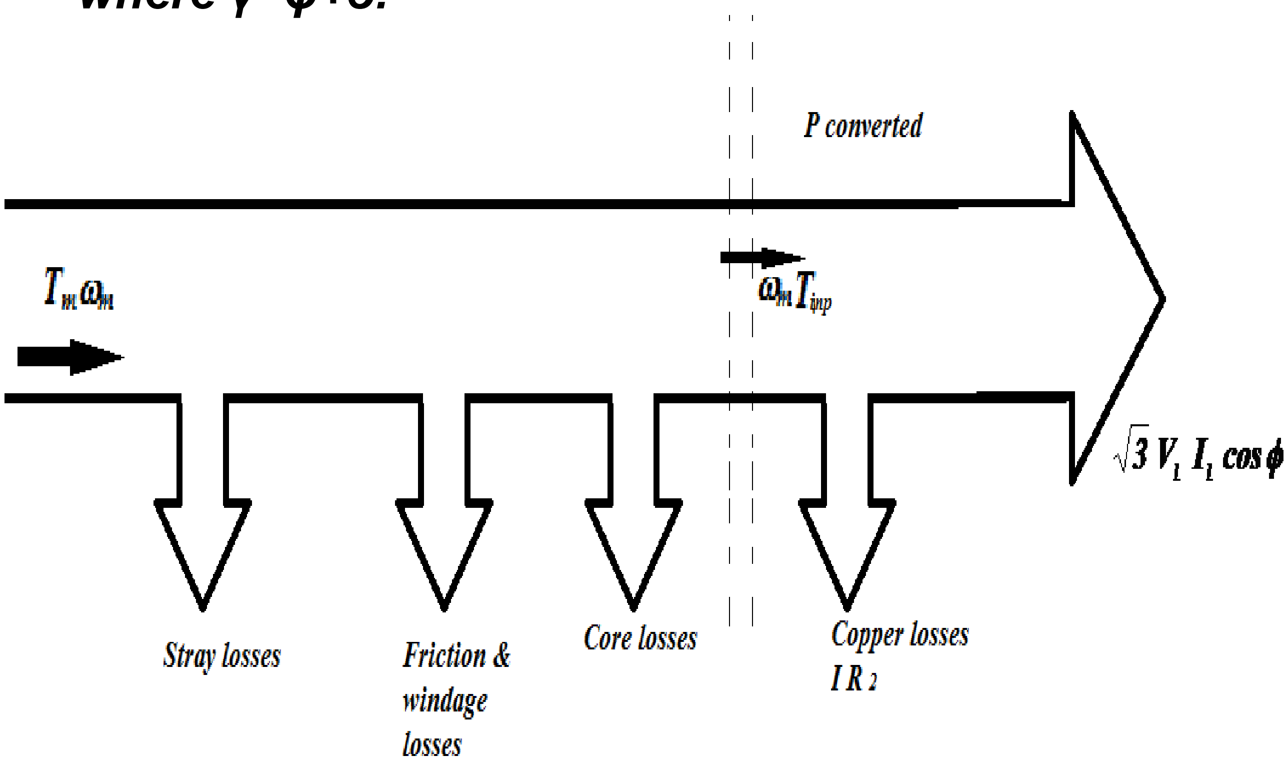
# ***Power Equation of Cylindrical S.M***

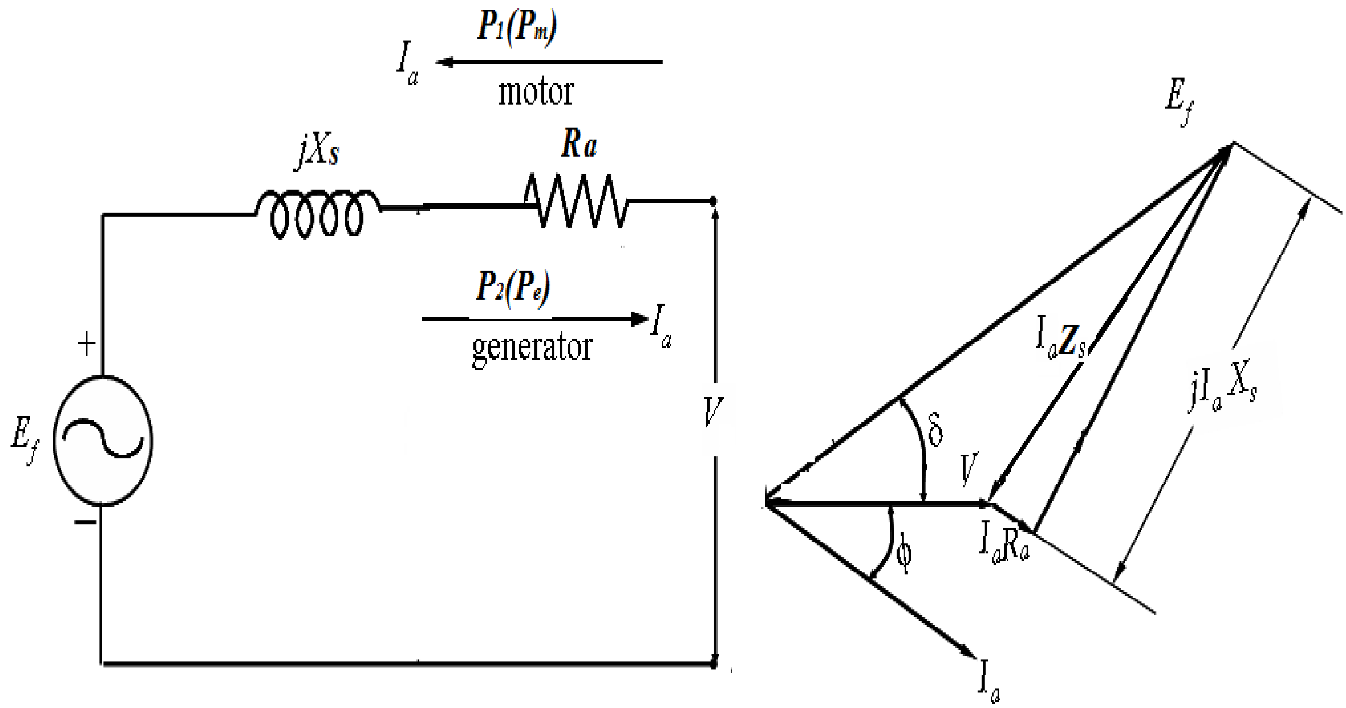
***The difference between input power and output power represents the losses of the machine. The input mechanical power is the shaft power in the generator ( $T_m\omega_m$) while the power converted from mechanical to electrical from internally ($3E_fI_a\cos\gamma$), where $\gamma=\varphi+\delta$.***




***The power flow diagram of synchronous generator***

## *18.1 The power angle characteristics:*

***The maximum power of synchronous machine can be delivered is determined by the maximum torque which can applied without loss of synchronism with the external system to which is connected.***




***The power ($P_2$) delivered through the impedance to the load:***

$$P_2 = VI_a \cos\phi$$

***The developed power,***

$$S = P + jQ$$

***From the Phasor diagram;***

$$\bar{V} = V + j0 = V\angle 0$$

$$I_a = \frac{E_f\angle\delta - V\angle 0}{Z_s\angle\phi_z} = \frac{E_f}{Z_s}\angle\delta - \phi_z - \frac{V}{Z_s}\angle -\phi_z$$

$$Z_s = R_a + jX_s = \sqrt{R_a^2 + X_s^2}\angle \tan^{-1}\frac{X_s}{R_a}$$

***The real part of current component in phase with V;***

$$I_a \cos\phi = \frac{E_f}{/Z_s/}\cos(\delta-\phi_z) - \frac{V}{/Z_s/}\cos(-\phi_z)$$

$$\cos(-\phi_z) = \cos(\phi_z) = \frac{R_a}{/Z_s/}$$

$$\therefore P_2 = \frac{VE_f}{/Z_s/}\cos(\delta-\phi_z) - \frac{V^2 R_a}{/Z_s/^2}$$

$$assume\ \ that\ \ \alpha_z = 90-\phi_z = \tan^{-1}\frac{R_a}{X_s}\ \ (usually\ is\ a\ small\ angle)$$

$$P_2 = \frac{VE_f}{/Z_s/}\sin(\delta+\alpha_z) - \frac{V^2 R_a}{/Z_s/^2}$$

$$for\ generator\ \mathrm{mod}e\ \ P_2 = P_e$$

***Similarly the power $P_1$ at source end $E_f$ of the impedance can be expressed:***

$$P_1 = P_m = \frac{-VE_f}{/Z_s/}\sin(\delta+\alpha_z) + \frac{V^2 R_a}{/Z_s/^2}$$

$$for\ motor\ \mathrm{mod}e\ (derive\ equation\ H.W)$$

***If resistance is neglected;***

$$P_1 = P_2 = P_e = P_m = \frac{E_f V}{X_s}\sin\delta$$

$$It\ is\ called\ \ power\ angle\ equation, The\ reactive\ \ power:$$

$$Q = V I_a \sin\phi = \frac{VE_f}{/Z_s/}\cos(\delta+\alpha_z) + \frac{V^2 X_s}{/Z_s/^2}$$

***$\delta$ is called power angle in synchronous machine, the maximum power occurs when $\delta = 90^0$.***

$$P_1 = P_2 = P_{max} = \frac{E_f\ V}{X_s}$$

$$P_{syn}\ (The\ synchronous\ power) = \frac{E_f\ V}{Z_s}\sin(\delta + \alpha_z)\quad for\ motoring$$

$$= \frac{-E_f\ V}{Z_s}\sin(\delta + \alpha_z)\ for\ generating$$

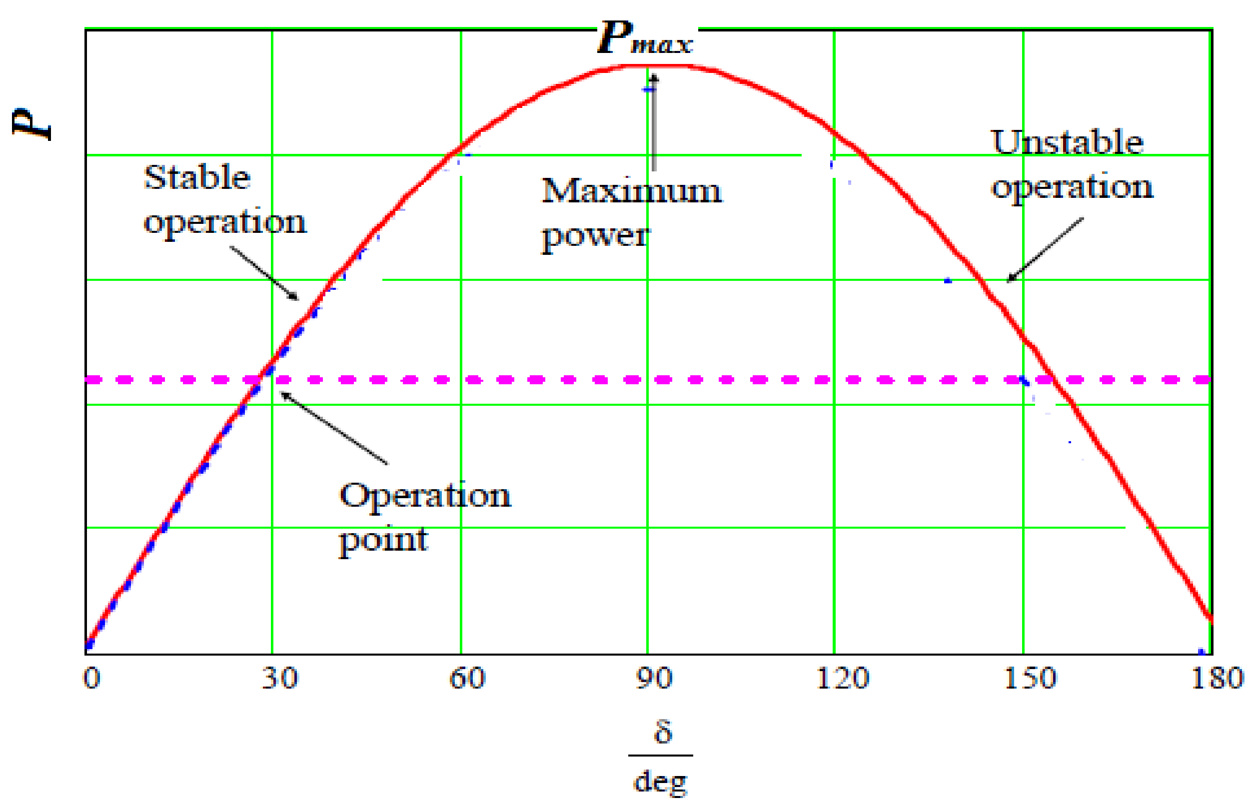
$$The\ synchronzed\ torque(T_{syn.}) = \frac{P_{syn}}{\omega_s}$$

***Above equations per phase for three phase every on multiplied by three;***

$$P = \frac{E_f\ V}{Z_s}\sin\delta$$

$$Note: If\ E_f = V\ then\ power\ angle\ is\ very\ small\ \sin\delta = \delta$$

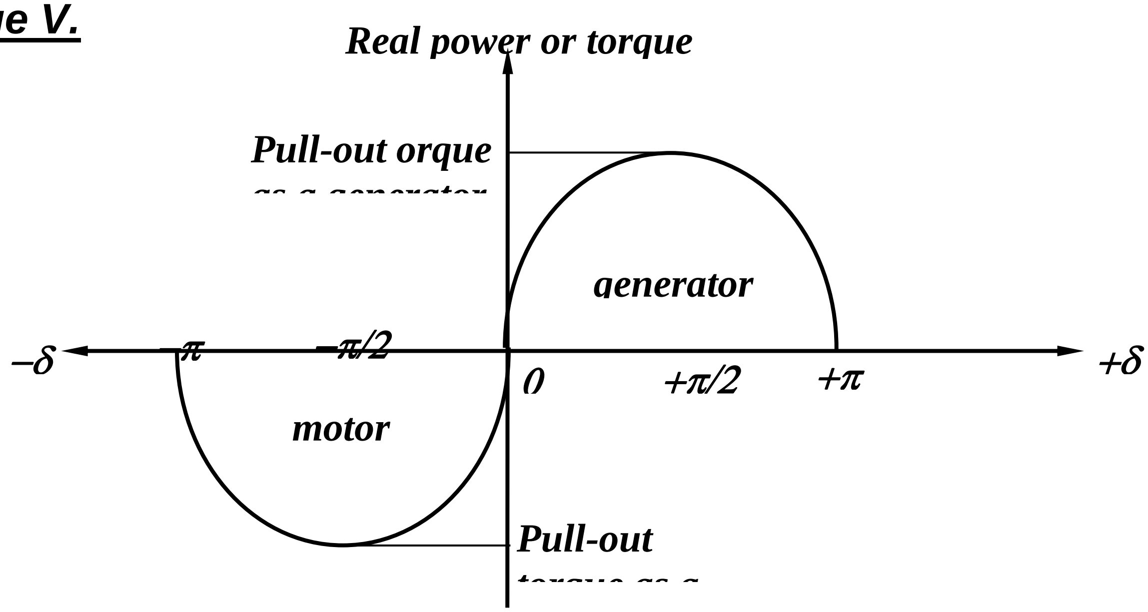
$$P = \frac{3\ V^2}{X_s}\delta$$



## *18.2 Load angle ($\delta$ power angle):*

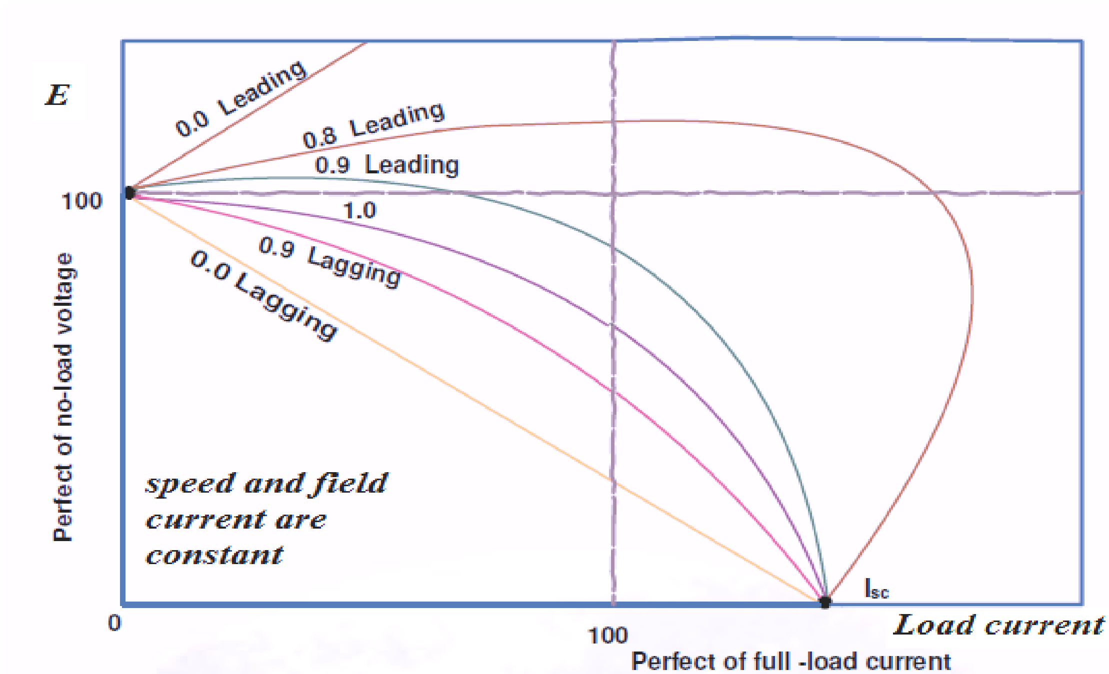
***The phase angle between $E_f$ & V is $\delta$ and its depend on the real power handled by the generator and its called <u>load angle</u>. The value of $\delta$ increases with increase in the <u>real power handled</u>.***

***<u>The load angle is positive for generator operation, the $E_f$ leading the direction with respect to the terminal voltage V.</u>***



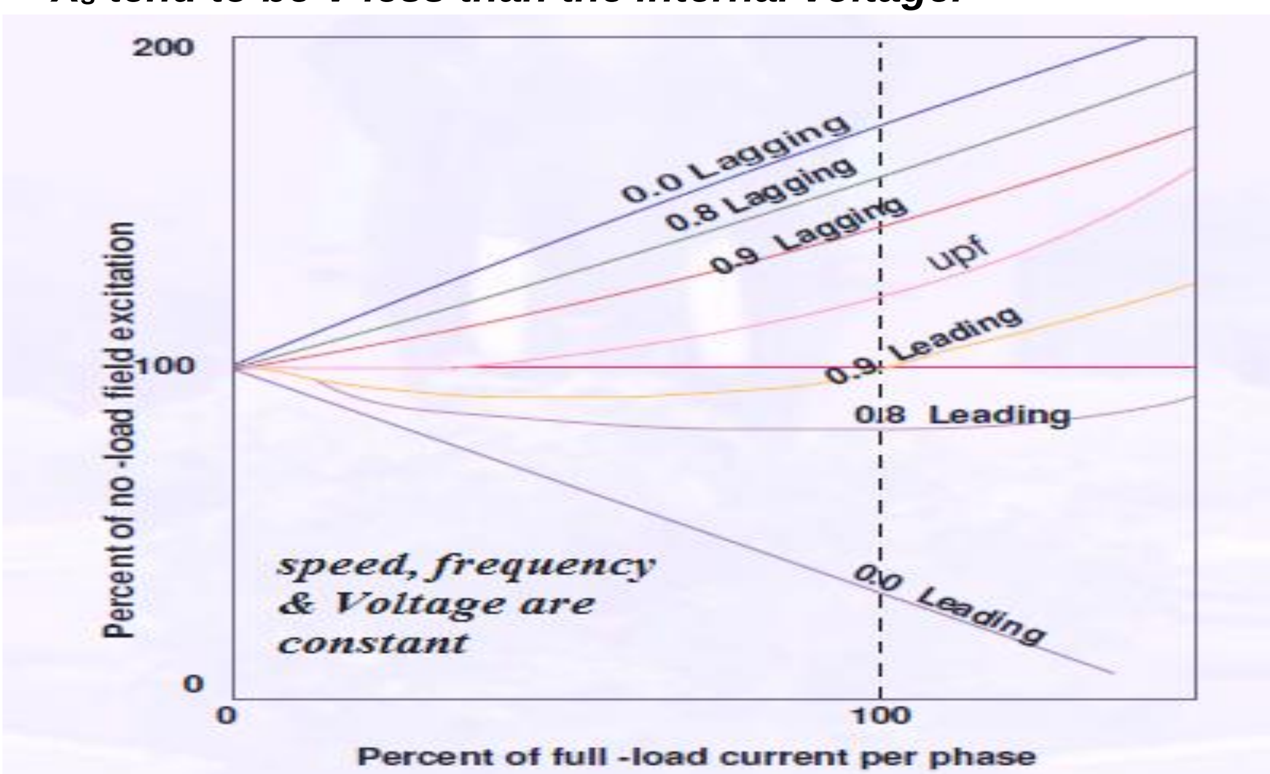

## *18.3 Load Characteristics(External Characteristics):*

***When alternator is driven by the prime mover at constant speed decided by the frequency of operation, at no load ( $V=E_f$) can be set up by passing the required field current to its excitation. With constant speed and excitation, when alternator is made to deliver power to its local load, the terminal voltage (V) is subjected to change due to the reasons drops in armature. The change in terminals voltage plotted with respect to current is called " Load Characteristics" of an alternator.***




# 18.4 Excitation Required for Constant Voltage ( Regulation Characteristics):

***To maintain and remain the terminal voltage constant, the excitation current may be increased or decreased according to type of load. The voltage drop due to $R_a$ & $X_s$ tend to be V less than the internal voltage.***



***Example 1:***
***Three phase alternator, 400V, Y-connected has impedance of ( 0.5+j5)Ω.. Calculate the load angle δ, $I_a$ and power factor, if alternator developing of 15KW at rated voltage and at excitation voltage $E_f$ of 500V.***

## *Solution:*

$$Z_s = 0.5 + j5 = 5.025\angle 84.23^0$$

$$P = \frac{3VE_f}{/Z_s/}\sin(\delta+\alpha_z) - \frac{3V^2R_a}{/Z_s/^2}$$

$$1500 = \frac{3*500/\sqrt{3}*400/\sqrt{3}}{5.025}\sin(\delta+\alpha_z) - \frac{38(400/\sqrt{3})^2*0.5}{5.025^2}$$

$$\delta+\alpha_z = 27.16^0 \Rightarrow \delta = 27.16^0 - \alpha_z = 27.16 - (190 - \tan^{-1}\frac{X_s}{R_a}) = 21.45^0 \ lead$$

$$E_f\angle\delta = V + I_aZ_s = 400/\sqrt{3} \quad + I_aZ_s \Rightarrow I_a = \frac{E_f\angle\delta - V\angle 0}{Z_s}$$

$$= \frac{500/\sqrt{3}\angle 21.45 - 400/\sqrt{3}}{5.025\angle 84.23}$$

$$= 22.3\angle -14.8^0 \ A \& \quad \cos\phi = \cos 14.8 = 0.97$$

***Example 2: 10MW alternator, Y-connected with 11KV line voltage and 0.85 lagging. $R_a$ & $X_s$ are 0.1 and 0.65 respectively. Calculate the line and its angle of emf generated.***
***Solution:***

$$E_f\angle\delta = V\angle 0 + I_aZ_s$$

$$P = \sqrt{3}\,V_l\ I_a\ \cos\phi$$

$$10\times 10^6 = \sqrt{3}\ \times 11000\times I_a \times 0.85 \Rightarrow I_a = 617.5A$$

$$E_f\angle\delta = 11000/\sqrt{3}\ \angle 0 + 617.5\angle -31.8\times(0.1 + j0.65)$$

$$= 6625.66\angle 2.717^0\ Volt$$

$$E_f\angle\delta(line to line) = \sqrt{3}\times 6625.66 = 11.476KV$$

***Example 3: A 400V, three phase synchronous motor , delta connection has resistance of 0.3Ω & $X_s$ of 1.2Ω per phase. The machine takes of 70A and operate at 0.5 power factor lagging, find the open circuit emf generated.***

***Solution:***

$$\phi = \cos^{-1}(0.5) = 60 lagging$$

$$E_f \angle \delta = V \angle 0 - I_a Z_s$$

$$= 400 - 70/\sqrt{3} \angle -60^0 \times (0.3 + j1.2)$$

$$= 352.27 \angle -2.24_0 V$$

$$E_f = 352.27$$

$$\delta = 2.24^0$$

***Example 5: Y- connected alternator having a resistance of 0.5Ω & $X_s$ is 10Ω per phase is excited to give on open circuit line voltage of 6050V. Find the terminal line voltage when the machine generated a line current of 100A at unity power factor.***

***Solution:***

$$E_f / \sqrt{3} \angle \delta = V \angle 0 - I_a Z_s$$

$$E_f / \sqrt{3} \cos\delta + jE_f / \sqrt{3} \sin\delta = V + I_a (R_a + jI_a X_s)$$

$$= V + 50 + j1000$$

$$By\ comparasion;$$

$$E_f / \sqrt{3} \cos\delta = V + 50 \qquad (1)$$

$$E_f / \sqrt{3} \sin\delta = 1000 \qquad (2)$$

$$6050 / \sqrt{3} \sin\delta = 1000 \Rightarrow \delta = \sin^{-1} \frac{\sqrt{3} \times 1000}{6050} = 16.6^0$$

$$sub(2)\ in\ (1):$$

$$E_f / \sqrt{3} \cos\delta = V + 50 \Rightarrow V = 3297.4 Volt$$