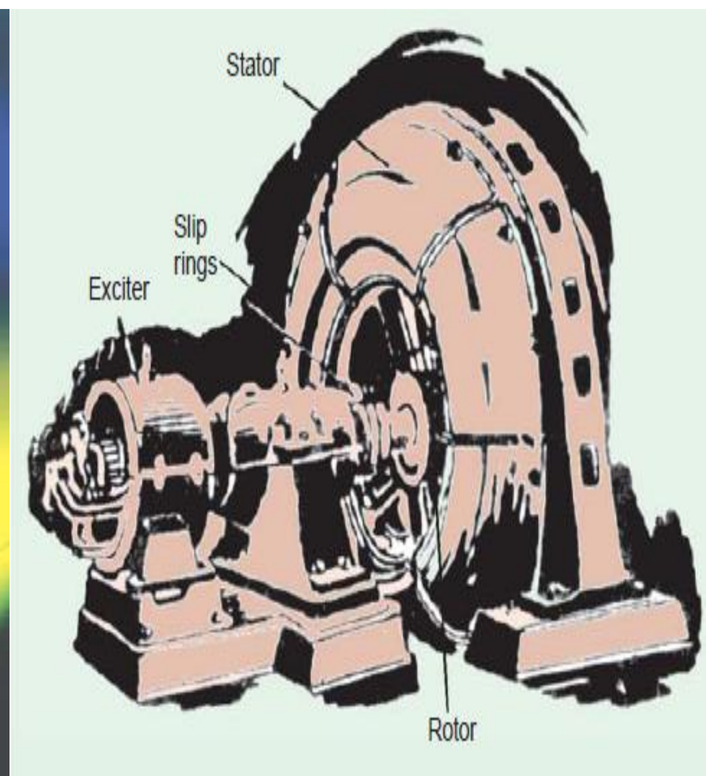
The Synchronous Motor

Introduction:

Synchronous motors are synchronous machines used to convert electrical power to mechanical power. The speed of the motor is always in synchronism with frequency of the ac voltage applied to it.

The synchronous motors posses characteristics of;

1. ***It is not self starting.***
2. ***The speed of operation is always in synchronism with supply frequency.***
3. ***May be operated at different power factor.***

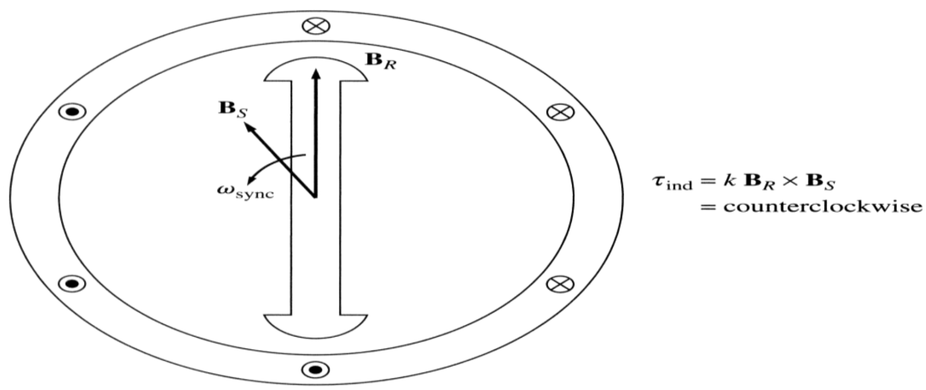



20.1 Motor Operation:

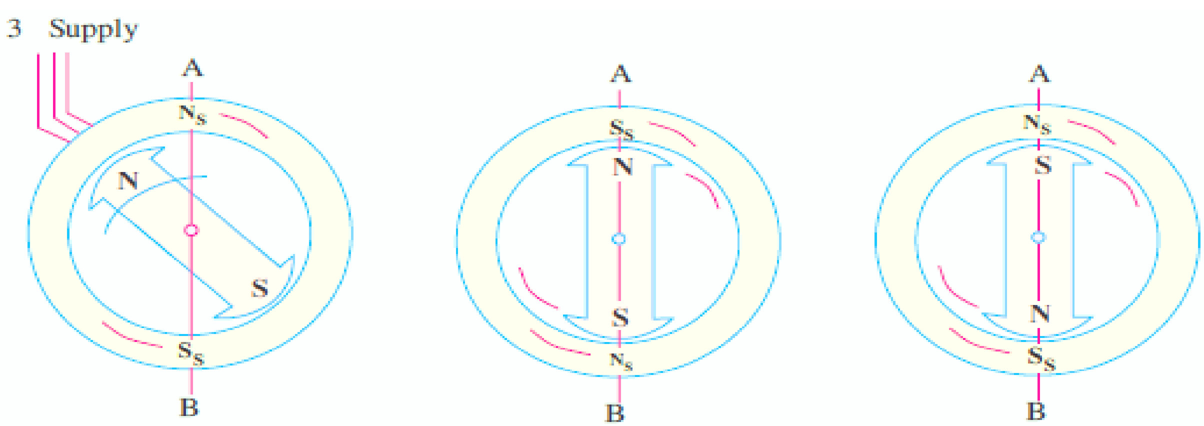
The field current IF of the motor produces a steady-state rotor magnetic field BR. A 3-phase set of voltages applied to the stator produces a 3-phase current flow in the windings. A 3-phase set of currents in an armature winding produces a uniform rotating magnetic field Bs. Two magnetic fields are present in the machine, and the rotor field tends to align with the stator magnetic field. Since the stator magnetic field is rotating, the rotor magnetic field will try to catch up pulling the rotor.

The larger the angle between two magnetic fields (up to a certain maximum), the greater the torque on the rotor of the machine.

When a three phase Windings are supplied by three phase supply, a rotating magnetic field at synchronous speed is produced.



Consider a 2-pole stator With a magnetic field rotate In clock wise direction with rotor position as shown. Suppose the stator poles are situated at point A and B. The two similar poles N of the rotor and Ns for the stator as well as S and Ss will repel other, which result that the rotor tends to rotate in anticlockwise direction.

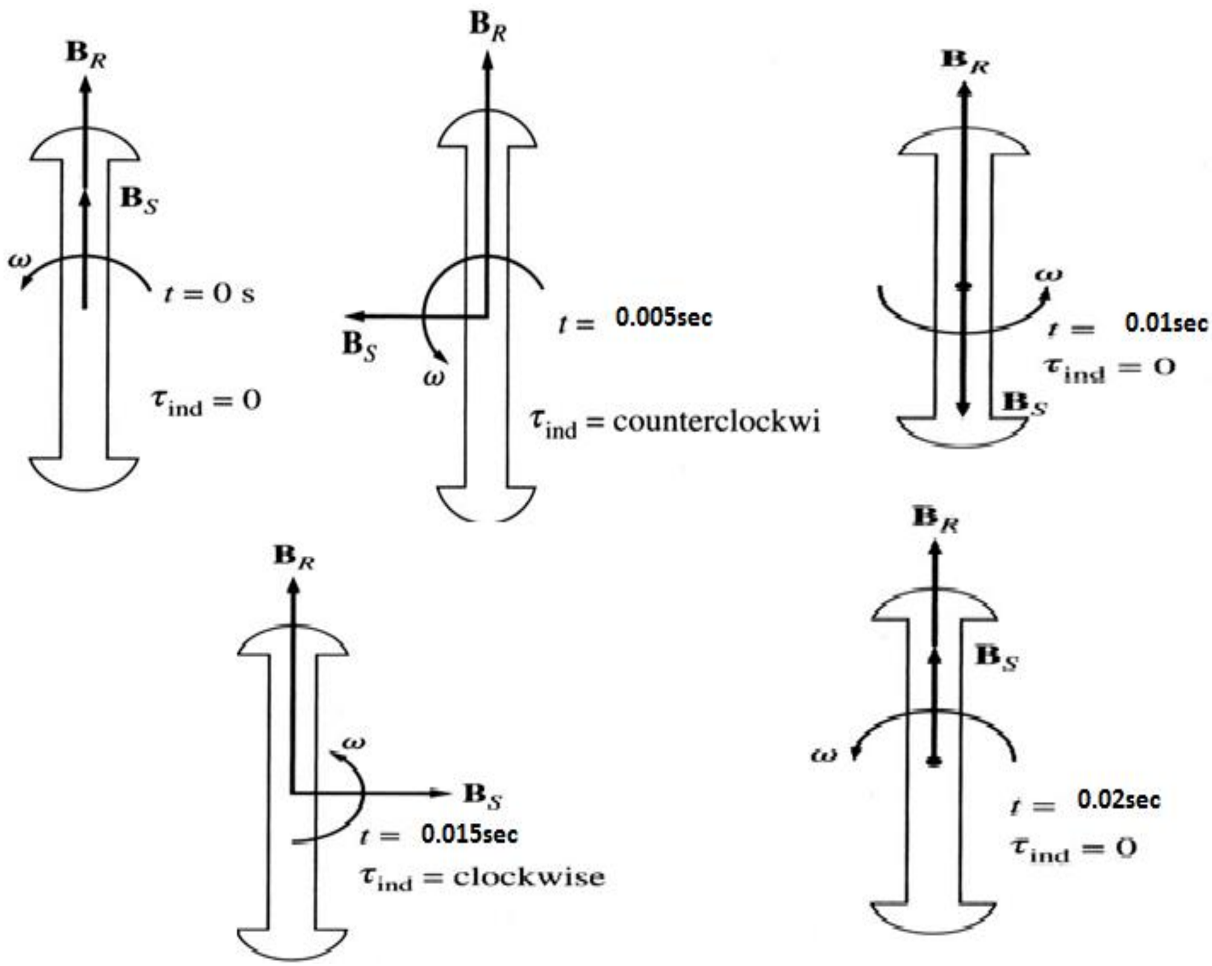


20.2 Starting synchronous motors

Consider a 60 Hz synchronous motor. When the power is applied to the stator windings, the rotor (and, therefore its magnetic field BR) is stationary. The stator magnetic field BS starts sweeping around the motor at synchronous speed.

Note that the induced torque on the shaft

$$\tau_{ind} = kB_R \times B_S$$



During one electrical cycle, the torque was counter-clockwise and then clockwise, and the average torque is zero. The motor will vibrate heavily and finally overheats!

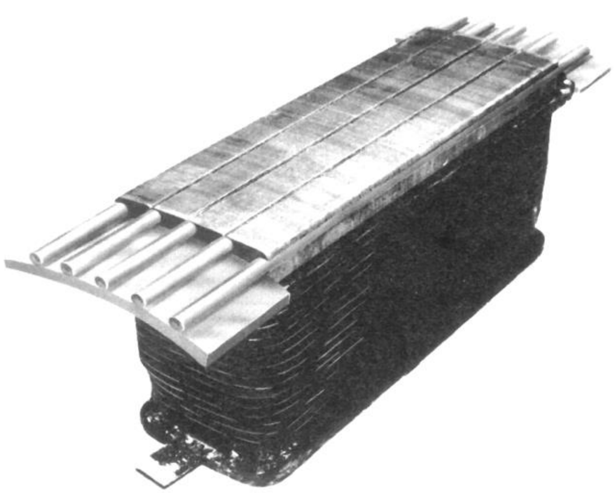
Three basic approaches can be used to safely start a synchronous motor:

1- Reduce the speed of the stator magnetic field to a low enough value that the rotor can accelerate and two magnetic fields lock in during one half-cycle of field rotation. This can be achieved by reducing the frequency of the applied electric power (which used to be difficult but can be done now).

2-Use an external prime mover to accelerate the synchronous motor up to synchronous speed, go through the paralleling procedure, and bring the machine on the line as a generator. Next, turning off the prime mover will make the synchronous machine a motor.

3-Use damper windings or amortisseur windings – the most popular.

Amortisseur (damper) windings are special bars laid into notches carved in the rotor face and then shorted out on each end by a large shorting ring.

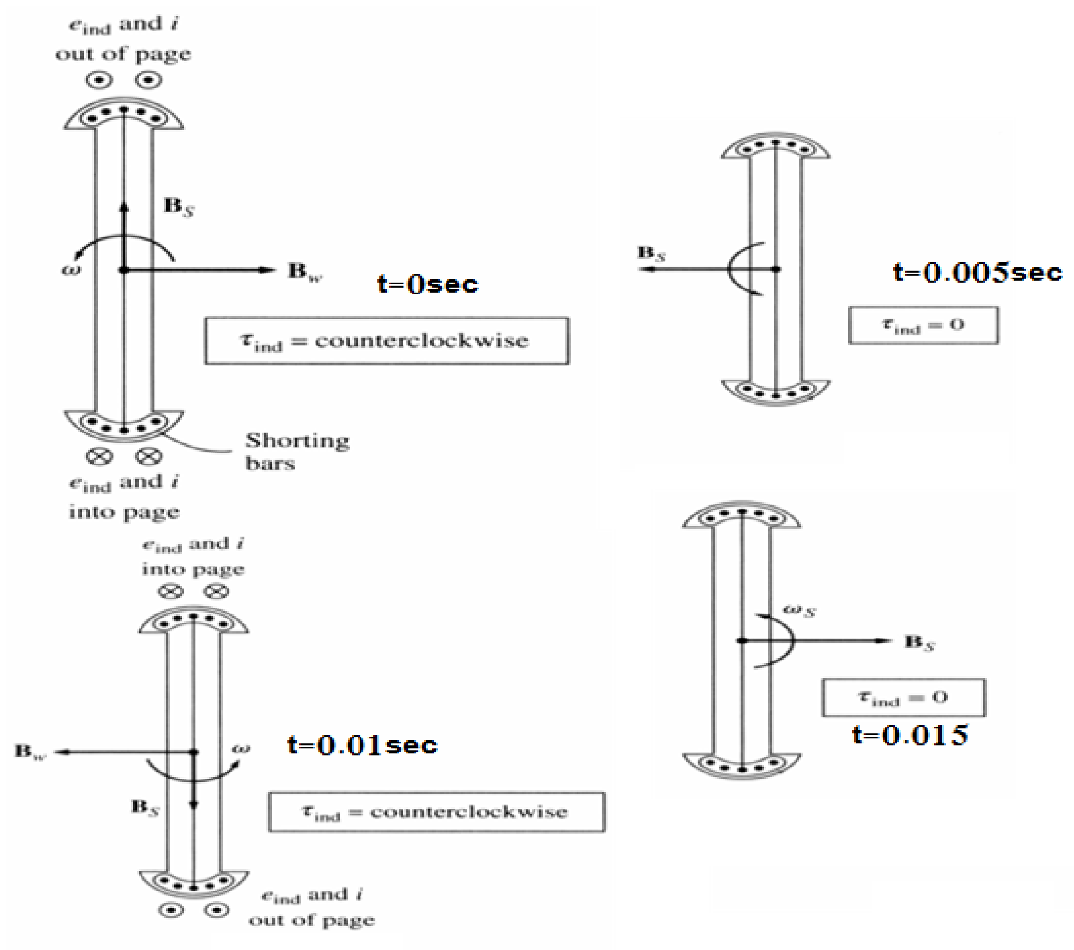


A diagram of a salient 2-pole rotor with an amortisseur winding, with the shorting bars on the ends of the two rotor pole faces connected by wires.

The bars at the top of the rotor are moving to the right relative to the magnetic field: a voltage, with direction out of page, will be induced. Similarly, the induced voltage is into the page in the bottom bars. These voltages produce a current flow out of the top bars and into the bottom bars generating a winding magnetic field B_W to the right. Two magnetic fields will create a torque

$$\tau_{ind} = kB_W \times B_S$$

The resulting induced torque will be counter-clockwise.



We observe that the torque is either counter-clockwise or zero, but it is always unidirectional. Since the net torque is nonzero, the motor will speed up.

However, the rotor will never reach the synchronous speed! If a rotor was running at the synchronous speed, the speed of stator magnetic field BS would be the same as the speed of the rotor and, therefore, no relative motion between the rotor and the stator magnetic field. If there is no relative motion, no voltage is induced and, therefore, the torque will be zero.

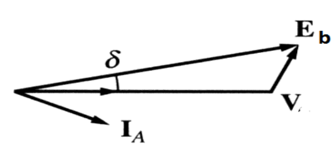
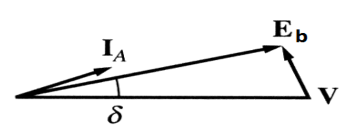
Instead, when the rotor's speed is close to synchronous, the regular field current can be turned on and the motor will operate normally. In real machines, field circuit are shorted during starting. Therefore, if a machine has damper winding:

1. ***Disconnect the field windings from their DC power source and short them out;***
2. ***Apply a 3-phase voltage to the stator and let the rotor to accelerate up to near-synchronous speed. The motor should have no load on its shaft to enable motor speed to approach the synchronous speed as closely as possible;***
3. ***Connect the DC field circuit to its power source: the motor will lock at synchronous speed and loads may be added to the shaft.***

20.3 Relationship between synchronous generators and motors:

Synchronous generator and synchronous motor are physically the same machines!
A synchronous machine can supply real power to (generator) or consume real power (motor) from a power system. It can also either consume or supply reactive power to the system.

1. ***The distinguishing characteristic of a synchronous generator (supplying P) is that E_A lies ahead of V while for a motor E_b lies behind V.***
2. ***The distinguishing characteristic of a machine supplying reactive power Q is that $E_b\cos\delta > V$ (regardless whether it is a motor or generator). The machine consuming reactive power Q has $E_b\cos\delta < V$.***

<table>
<tr><td></td><td>Supply reactive power Q $E_b \cos\delta > V$</td><td>Consume reactive power Q $E_b \cos\delta < V$</td></tr>
<tr><td>Supply power P
Generator
E_b leads V</td><td>
</td><td>
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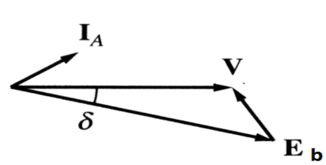
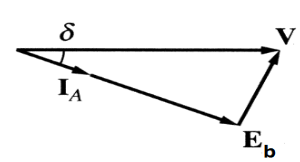
<tr><td>Consume power P
Motor
E_b lags V</td><td>
</td><td>
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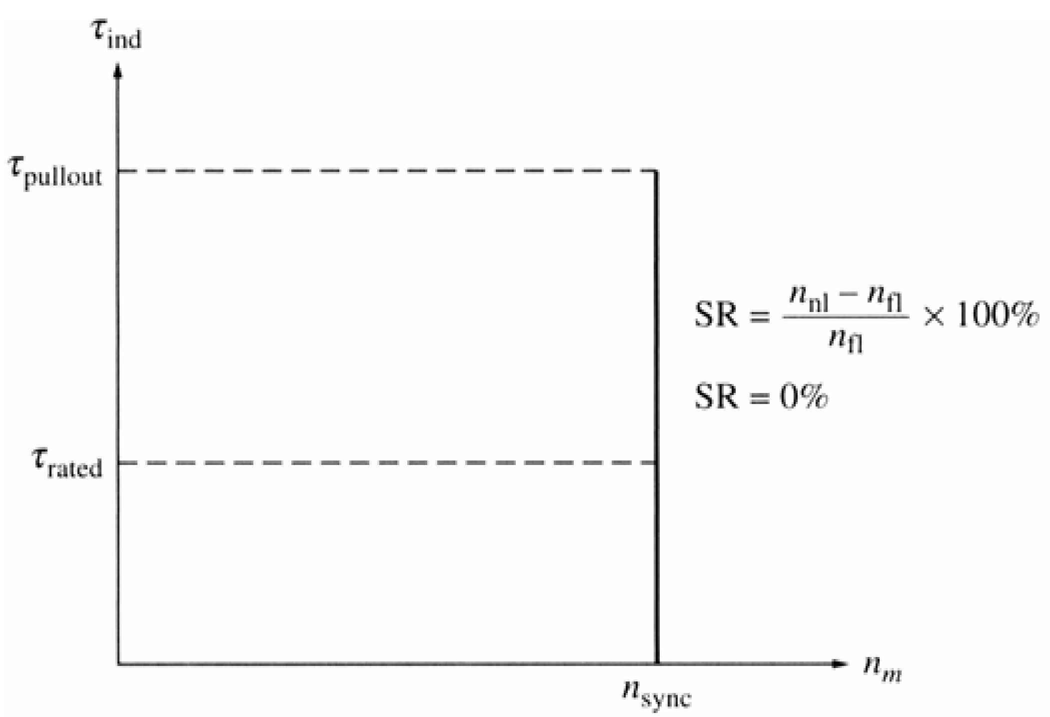
20.4 Torque-speed curve

Usually, synchronous motors are connected to large power systems (infinite bus); therefore, their terminal voltage and system frequency are constant regardless the motor load. Since the motor speed is locked to the electrical frequency, the speed should be constant regardless the load.

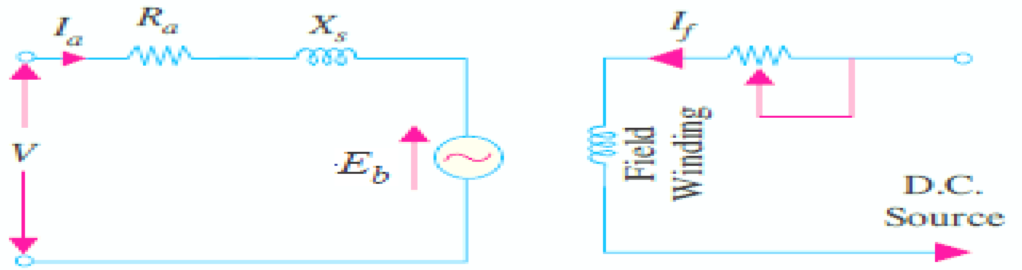
The steady-state speed of the motor is constant from no-load to the maximum torque that motor can supply (pullout torque). Therefore, the speed regulation of synchronous motor is 0%.

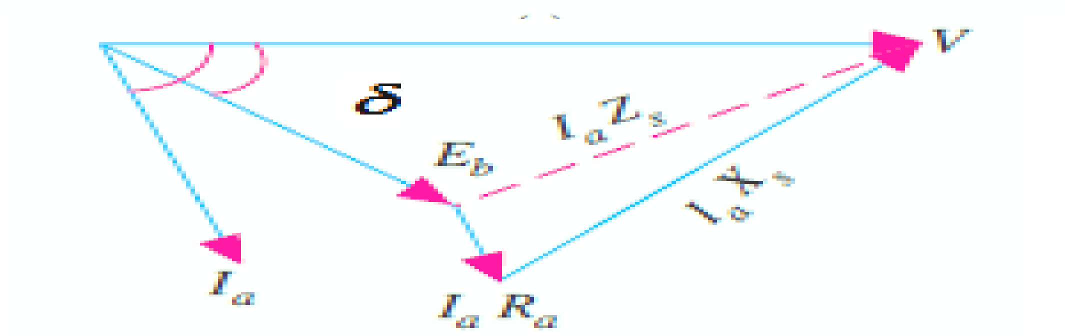
The induced torque is



$$\tau_{ind} = \frac{3V \; E_b}{\omega_m X_s} \sin \delta$$

20.5 Equivalent Circuit of Synchronous Motor:





$$V = E_b \angle -\delta + jI_a Z_s$$

Power input/phase in stator

$P = VI_a \cos \phi$

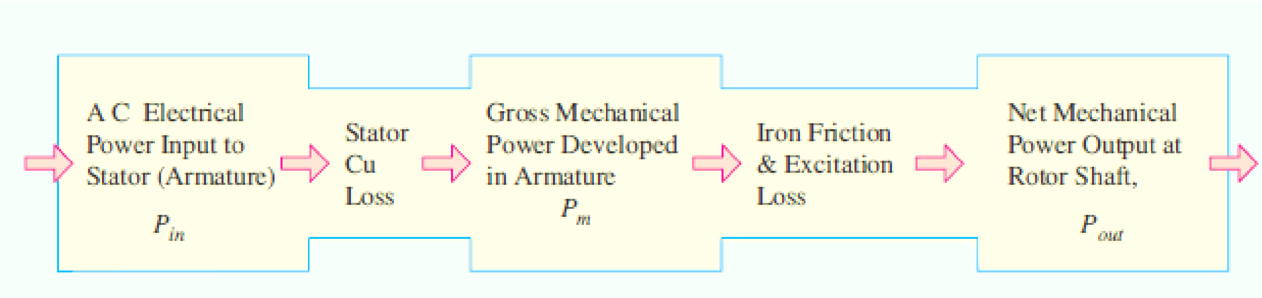
Armature (*i.e.*, stator) Cu loss $= I_a^2 R_a$

Mechanical power in armature $P_m = E_b I_a \cos(\delta - \phi)$

Iron, excitation & friction losses

Output power P_{out}

Different power stages in a synchronous motor are as under :



20.5 The (V-Curve) Relation between the excitation and armature current:

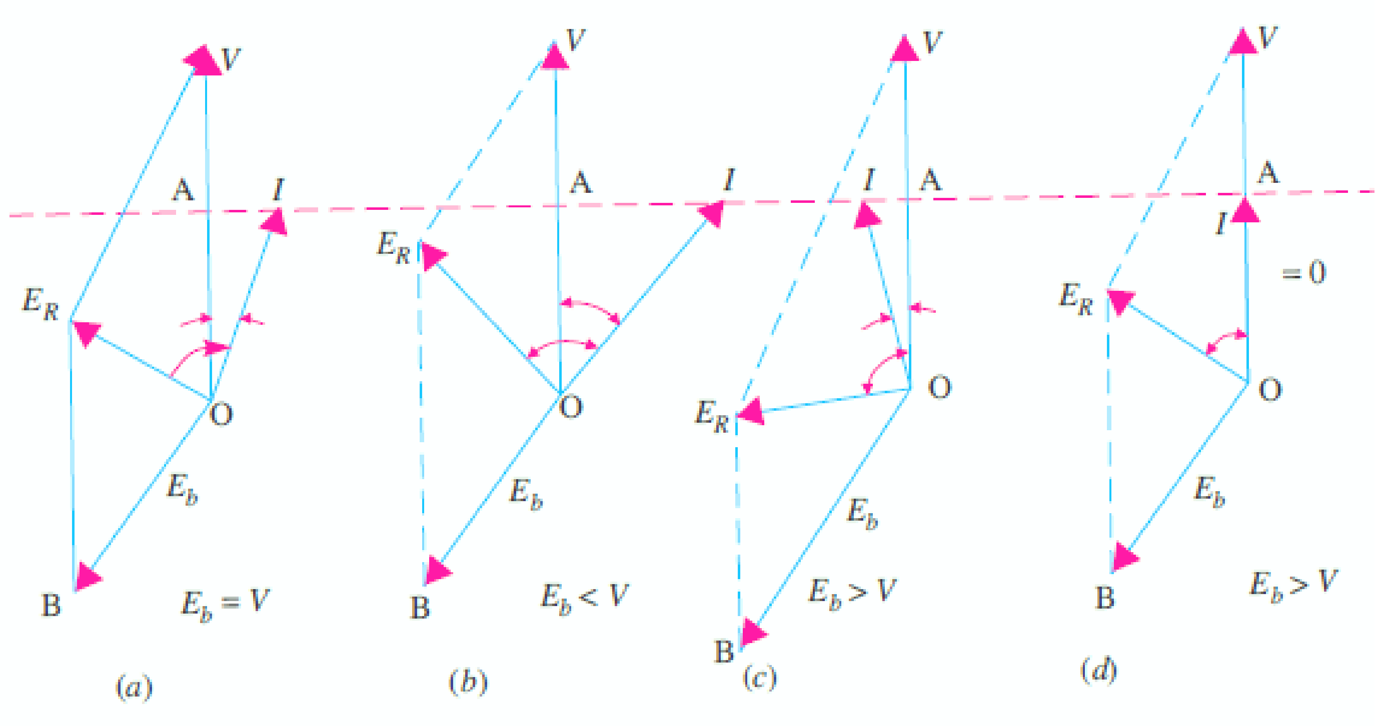


Figure a: ***E_b =V, 100% excitation, the armature current lags behind V by a small angle φ. The angle θ is between E_R and I_a. Where tan θ= X_S/R_a.***

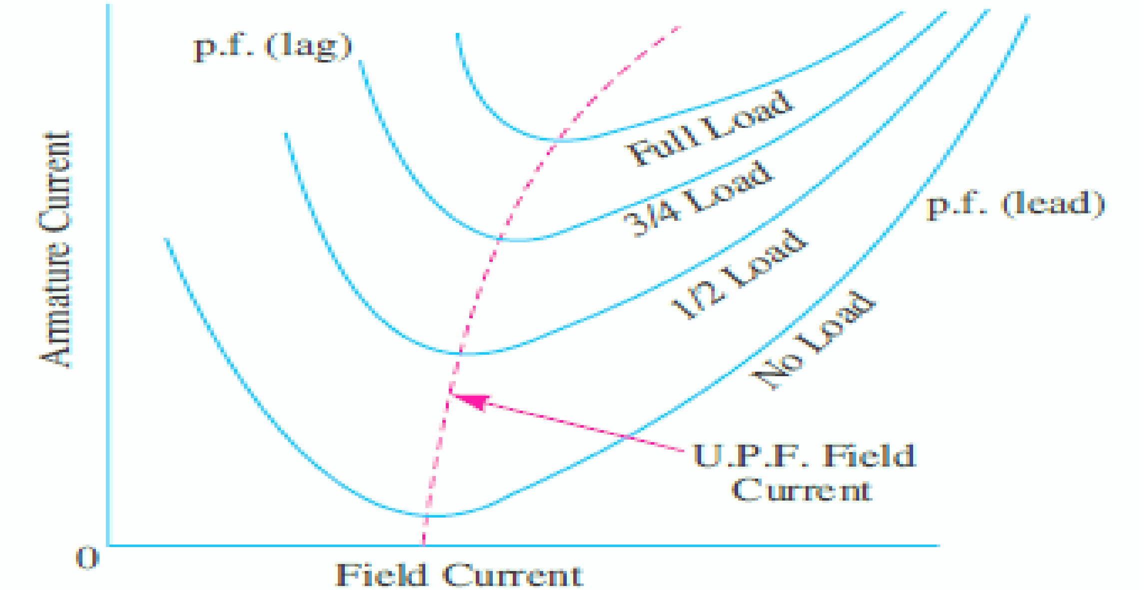
Figure b: ***The excitation is less than 100%. E_b<V(under-excited motor). The armature current increased but its power factor is decreased. Because V is constant $I_a$cosφ remains constant as before but reactive component of armature current ($I_a$sinφ) is increased. Hence the excitation is decreased, I_a increase p.f will decrease so that power consumed $I_a$cosφ =OA will remain constant.***

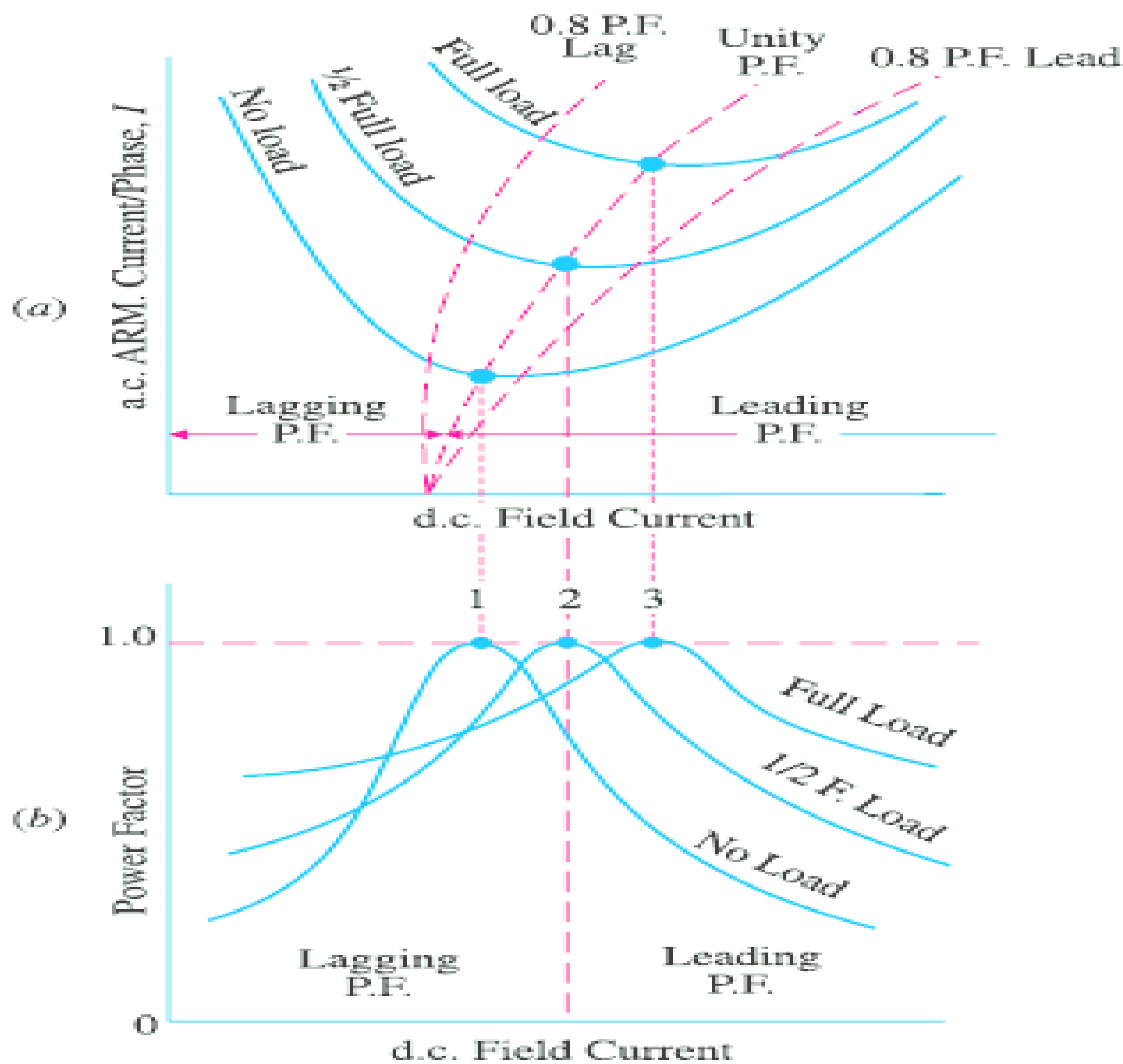
Figure c: ***E_b>V(over-excited motor) its over excited synchronous motor. The E_R is pulled anticlockwise direction and so I_a. Then the motor draws a leading power factor current.***

Figure d: (Critical- excited) At this time the current drawn by the motor is minimum.

The important conclusion points:

- ***i- The magnitude of armature current varies with the excitation. The armature current has large value both for low and high value of excitation current.***
- ***ii- The armature current is lagging for low value of excitation current and is leading for high value of excitation current. In between them, it has minimum value corresponding to a certain excitation.***
- ***iii- The variation of armature current I_a with excitation(field) current are known as V-curves because of their shape.***
- ***iv- Since the I_a varies over a wide range and so causes the p.f. also to vary accordingly. The curve of p.f. likes inverted V curve, It would be noted that minimum armature current corresponding to unity power factor.***
- ***v- The V- curve of synchronous motor shows how armature current varies with its field current when motor input is kept constant. There is a family of such curves each corresponding to definite power intake.***


p.f. (lag)
Full Load
3/4 Load
1/2 Load
No Load
p.f. (lead)
U.P.F. Field Current
Armature Current
0
Field Current


0.8 P.F. Lag
Unity P.F.
0.8 P.F. Lead
No load
½ Full load
Full load
(a)
a.c. ARM. Current/Phase, I
Lagging P.F.
Leading P.F.
d.c. Field Current
1
2
3
1.0
Full Load
1/2 F. Load
No Load
(b)
Power Factor
Lagging P.F.
Leading P.F.
0
d.c. Field Current

20.6 The Synchronous motor Application:

The Synchronous motors are used;

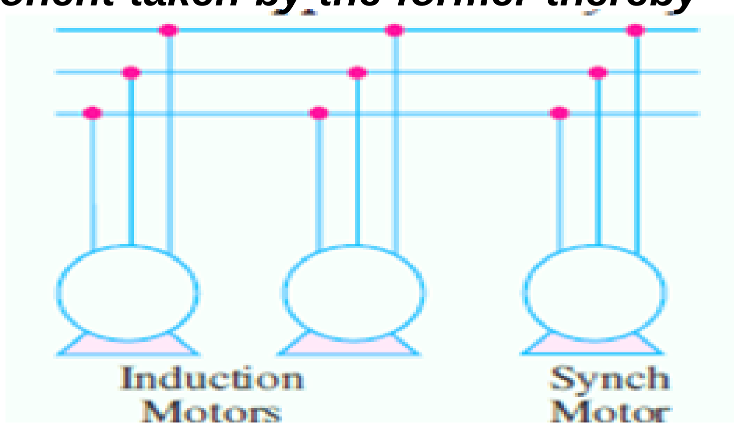
1. ***Power factor Correction***
2. ***Constant speed drives***
3. ***The voltage regulation***

20.6.1 The power factor correction:

In order to improve the power factor the synchronous motor operates at overexcited region with leading power factor of those power systems which employ a large number of induction motors and other devices having lagging p.f. such welders and fluorescents.

The synchronous motor can be operated overexcited to supply reactive power Q for a power system. In fact, at some time in past synchronous motor runs without a load, simply for power factor correction.

Using the synchronous motor in conjunction with induction motors and transformers, the lagging reactive power required by the latter is supplied locally by the leading reactive component taken by the former thereby relieving the line And generators of much of reactive components.



Where the synchronous motor is used for this purpose is called a synchronous condenser, because it draws like a capacitor leading current from the line. Most synchronous capacitors are rated between 20MVAR and 200MVAR and many are hydrogen cooled.

20.6.2 The constant speed applications:

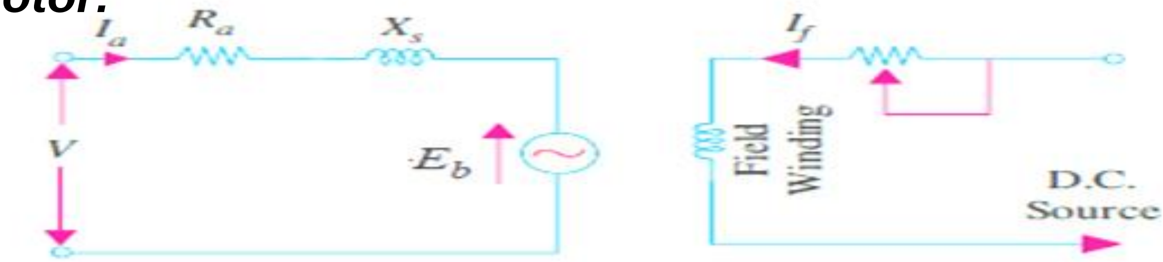
Because of its high efficiency and high speed, the synchronous motor are well suited for loads where constant speed is required such as centrifugal pumps, belt-driven reciprocating compressors, blowers, line shafts, rubber and paper mills etc.

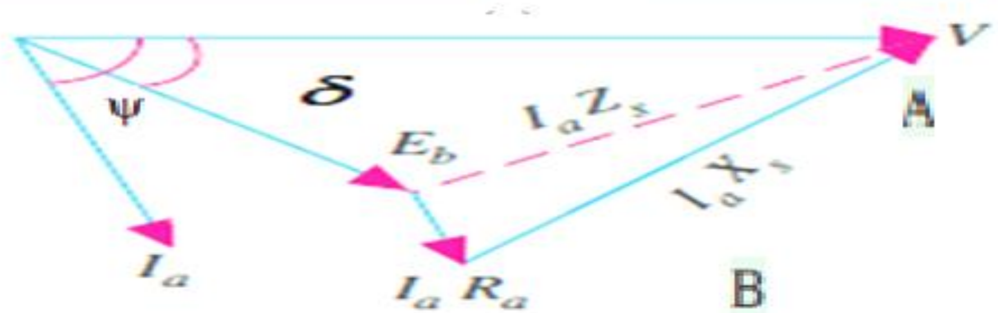
20.6.3 The Voltage Regulation:

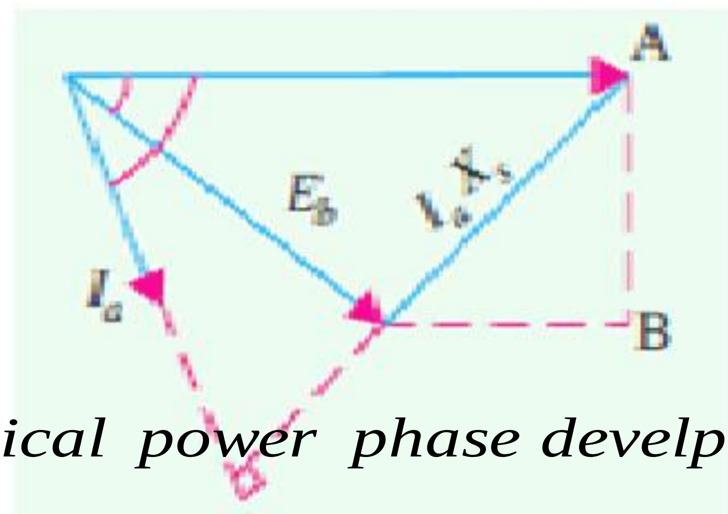
The voltage of the end of a long transmission line varies greatly especially when large inductive loads are present. When an inductive load is disconnected suddenly voltage tend to rise above its normal value because of the line capacitance. By installing a synchronous motor with a field regular, this voltage rise can be controlled.

When line voltage decreases due to inductive load, motor excitation is increased, thereby raising its p.f. which components for the line voltage drop. If, on the other hand, line voltage rises due to line capacitive effect, motor excitation is decreased, thereby making it p.f. lagging helps to maintain the line voltage at its normal value.

20.7 The power equation of the synchronous motor:







The mechnical power phase develped in the rotor

$$P_m = E_b I_a \cos\Psi$$

$$`I_a Z_s \cos\Psi = V\cos(\theta-\delta) - E_b \cos\theta$$

$$I\cos\Psi = \frac{V}{Z_s}\cos(\theta-\delta) - \frac{E_b}{Z_s}\cos\theta$$

$$P_m = E_b\left[\frac{V}{Z_s}\cos(\theta-\delta) - \frac{E_b}{Z_s}\cos\theta\right]$$

$$= \frac{E_b V}{Z_s}\cos(\theta-\delta) - \frac{E_b^2}{Z_s}\cos\theta$$

$$Where\ \theta = \tan^{-1}\left(\frac{X_s}{R_a}\right)$$

For maximum power developed can be found this equation with respect to load angle:

$$\frac{dP_m}{d\delta} = -\frac{E_b V}{Z_s}\sin(\theta-\delta) = 0$$

$$or\ \sin(\theta-\delta) = 0,\ \therefore \theta = \delta$$

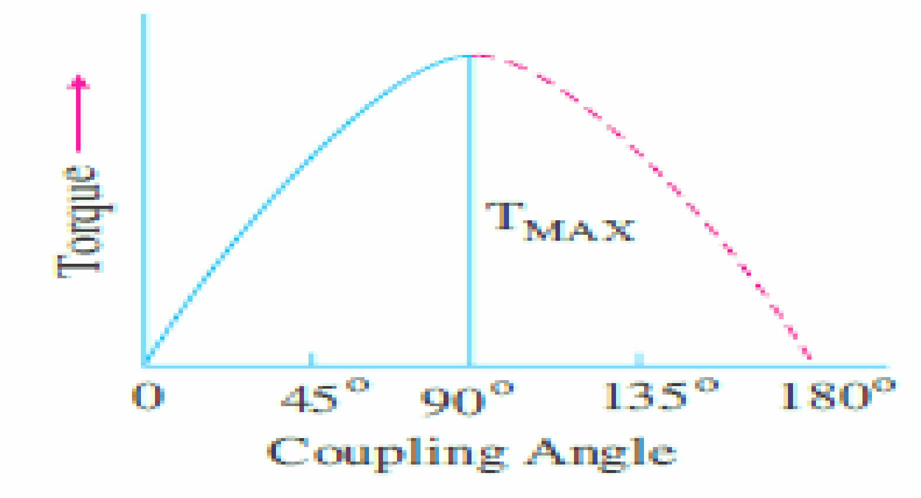
$$neglecting\ Ra,$$

$$P_m = \frac{E_b V}{X}\cos(90-\delta) = \frac{E_b V}{X_s}\sin\delta$$

$$The\,value\ of\ \max imum\ power$$

$$P_m(\max) = \frac{E_b V}{Z_s} - \frac{E_b^2}{Z_s}\cos\delta$$

$$= \frac{E_b V}{Z_s} - \frac{E_b^2}{Z_s}\cos\theta$$



The maximum power and its torque depends on V and E_b. For all value of V and E_b this limiting value of δ is the same but maximum torque will be proportional to the maximum power developed as shown in above fig.

If Ra is neglected then,

$$Z_s \approx X_s \quad and \quad \theta = 90, \cos\theta = 0$$

$$P_m = \frac{E_b V}{X_s} \sin\delta$$

This is corrsponding to pull out torque

$$P_1 = P_{inp} = \frac{-VE_f}{/Z_s/} \sin(\delta + \alpha_z) + \frac{V^2 R_a}{/Z_s/^2}$$

for motor mode (derive equation H.W)

$$\cos(-\phi_z) = \cos(\phi_z) = \frac{R_a}{/Z_s/}$$

$$assume\ that\ \alpha_z = 90 - \phi_z = \tan^{-1}\frac{R_a}{X_s}\ (usually\ is\ a\ small\ angle)$$

Example 1:

The input to an 11000-V, 3-phase, star-connected synchronous motor is 60 A. The effective resistance and synchronous reactance per phase are respectively 1 ohm and 30 ohm. Find (i) the power supplied to the motor (ii) mechanical power developed and (iii) induced emf for a power factor of 0.8 leading.

Solution. (*i*) Motor power input = $\sqrt{3} \times 11000 \times 60 \times 0.8 = 915$ kW

(*ii*) stator Cu loss/phase = $60^2 \times 1 = 3600$ W; Cu loss for three phases = $3 \times 3600 = 10.8$ kW

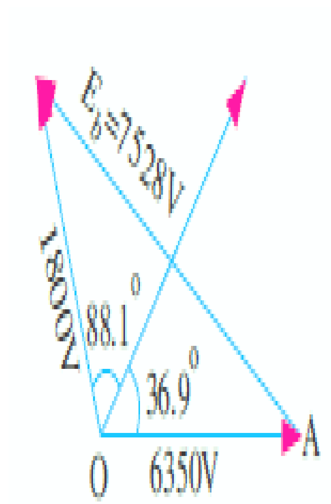
$$P_m = P_2 - \text{rotor Cu loss} = 915 - 10.8 = 904.2 \text{ kW}$$

$$V_p = 11000/\sqrt{3} = 6350 \text{ V}; \quad \phi = \cos^{-1} 0.8 = 36.9°;$$

$$\theta = \tan^{-1}(30/1) = 88.1°$$

$$Z_S \cong 30\ \Omega; \text{ stator impedance drop / phase} = I_a Z_S$$

$$= 60 \times 30 = 1800 \text{ V}$$



$$V = E_b + I_a Z_S$$

$$6350\angle 0 = E_b + 60\angle 36.9 \times 30\angle 88.1$$

$$\therefore E_b = 7528V$$

Example 2:

A 500-V, 1-phase synchronous motor gives a net output mechanical power of 7.46 kW and operates at 0.9 p.f. lagging. Its effective resistance is 0.8 Ω. If the iron and friction losses are 500 W and excitation losses are 800 W, estimate the armature current. Calculate the commercial efficiency.

Solution. Motor input $= VI_a \cos\phi$; Armature Cu loss $= I_a R_a^2$

Power developed in *armature* is $P_m = VI_a \cos\phi - I_a^2 R_a$

$$\therefore \quad I_a^2 R_a - VI_a \cos\phi + P_m = 0 \quad \text{or} \quad I_a = \frac{V\cos\phi \pm \sqrt{V^2\cos^2\phi - 4R_a P_m}}{2R_a}$$

Now, $P_{out} = 7.46\text{ kW} = 7{,}460\text{ W}$

$$P_m = P_{out} + \text{iron and friction losses} + \text{excitation losses}$$

$$= 7460 + 500 + 800 = 8760\text{ W}$$

$$I_a = \frac{500\times 0.9 \pm \sqrt{(500\times 0.9)^2 - 4\times 0.8\times 3760}}{2\times 0.8}$$

$$= \frac{450 \pm \sqrt{202{,}500 - 28{,}030}}{1.6} = \frac{450 \pm 417.7}{1.6} = \frac{32.3}{1.6} = 20.2\text{ A}$$

Motor input $= 500\times 20.2\times 0.9 = 9090\text{ W}$

$$\eta_c = \text{net output / input} = 7460 / 9090 = 0.8206 \quad \text{or} \quad 82.06\%.$$

Example 3: A three phase 6600V, 50Hz, star connected synchronous motor takes 50A current. The resistance and synchronous reactance per phase are 1ohm and 20ohm respectively. Find the power supplied to the motor and its induced emf for a power factor of 0.8lagging and leading.

Solution:

$1-\ p.f.=0.8lag.$

$P_{inp}=\sqrt{3}\times 6600\times 50\times 0.8=457.248kW$

$$phase\ voltage=\frac{6600}{\sqrt{3}}=3810V$$

$$\phi=36.87\ \deg ree\ \ \theta=\tan^{-1}\frac{20}{1}=87.7\deg ree$$

$Z_s=\sqrt{20^2+1^2}=20\Omega$

$V=E_b\angle-\delta+I_aZ_S$

$3810\angle 0=E_b\angle-\delta+50\angle-36.87\times 20\angle 87.7$

$\therefore E_b\angle-\delta=3810\angle 0-50\angle-36.87\times 20\angle 87.7$

$=3270\angle-13.7^0 Volt$

$Line\ Voltage\ of\ E_b=5660V$

$2-\ p.f=0.8leading$

$The\ power\ remain\ the\ same=457.248kW$

$E_b\angle-\delta=3810\angle 0-50\angle 36.87\times 20\angle 87.7$

$=4450\angle-10.7^0 Volt$

$Line\ Voltage\ of\ E_b=7700V$

Example 4: A 6-pole synchronous motor has an armature impedance of 10Ω and resistance of 0.5Ω, when it operates on 2kVphase voltage, 25Hz supply mains. Its field excitation is such that the emf phase induced is 1600V. Calculate the maximum total torque.

Solution:

$$\cos\theta = \frac{0.5}{10} = 0.05$$

$$P_m(\max) = \frac{E_b V}{Z_s} - \frac{E_b^2}{Z_s}\cos\delta$$

$$= \frac{E_b V}{Z_s} - \frac{E_b^2}{Z_s}\cos\theta$$

$$= \frac{2000\times1600}{10} - \frac{1600^2}{10}\times0.05$$

$$= 307.2kW$$

$$The\ \max imum\ torque = \frac{307.2kW}{52.33}$$

$$= 5868N.m$$

Example 5: A three phase 3.3kV, Y-connected synchronous motor has an effective resistance and synchronous reactance of 2Ω and 18Ω per phase respectively. If the open circuit generated emf is 3800V between lines, calculate 1- The maximum mechanical power that the motor can be develop, and 2- The current and p.f. at maximum mechanical power.

Solution:

$$\theta = \tan^{-1}\frac{18}{2} = 83.7^0,\ V/Phase = 3300/\sqrt{3} = 1905V,$$

$$E_b = 3800/\sqrt{3} = 2195V$$

$$At\ \max.torque\ \theta = \delta$$

$$Z_s = \sqrt{2^2 + 18^2} = 18.11\angle 83.66\Omega$$

$$1 - P_{\max} = \frac{3\times 2195\times 1905}{18.11} - \frac{2195^2\times 2}{18.11^2}$$

$$= 604.56kW$$

$$2 -$$

$$V = E_b\angle -\delta + I_a Z_S$$

$$1905\angle 0 = 2195\angle -83.66 + I_a \times 18.11\angle 83.66$$

$$\therefore I_a = 151\angle 31A,\ p.f. = \cos 31 = 0.85$$

$$The\ power\ input\ at\ \max imum =$$

$$= \sqrt{3}\times 3300\times 151\times 0.85 = 743.26kW$$

Example 6: A three phase, 8-pole, 50Hz, 6600V, Y-connected synchronous motor has a synchronous impedance of (0.66+j6.6)ohm per phase. When excited to give emf of 4500V/phase it takes an input of 2.5MW. Calculate the electromagnetic torque, input current, power factor and load angle.

Solution:

$$P_{inp} = \frac{-VE_f}{/Z_s/}\sin(\delta+\alpha_z) + \frac{V^2R_a}{/Z_s/^2}$$

$$Z_s = 0.66 + j6.6 = 6.63\angle 84.28^0\,\Omega$$

$$2500000 = \frac{3\times -6600/\sqrt{3}\times 4500}{6.63}\sin(\delta+\alpha_z) + 3\times\frac{(6600/\sqrt{3})^2\times 0.66}{6.63^2}$$

$$\delta+\alpha_z = -13.84^0 \Rightarrow \delta = -13.84 - \tan^{-1}\frac{0.66}{6.6}$$

$$= -19.5^0$$

The mechanical output of the motor;

$$P_m = \frac{3E_bV}{Z_s}\cos(\theta-\delta) - \frac{3E_b^2}{Z_s}\cos\theta$$

$$= \frac{3\times 4500\times 6600/\sqrt{3}}{6.63}\cos(84.28-19.5) - \frac{3\times 4500^2}{6.63^2}\cos(84.28)$$

$$= 2.387MW$$

$$T_{mechanical} = \frac{P_m}{\omega_s} = \frac{2.387\times 10^6}{2\pi\frac{120f}{60P}} = 30.4\times 10^3\,N.m$$

$$E_f\angle\delta = V - I_aZ_s \Rightarrow I_a = \frac{V - E_f\angle\delta}{Z_s}$$

$$= \frac{3810.5-(4242-j1501)}{6.633\angle 89.29}$$

$$= 235.5\angle 21.8A$$

$$\cos\phi = \cos(21.81) = 0.93 lagging$$

$$Cheaking\ input\ power = \sqrt{3}\times 6600\times 235.5\times\cos(21.8)$$

$$= 2500KW$$

20.8 Hunting of Synchronous Machine:

Whenever a synchronous machine is subjected to a sudden change in load, the rotor axis to oscillation before it settle at the new position corresponding to the new load angle. This process of settling up of oscillations is called the hunting of the load angle in synchronous machine, the process of searching for the new load angle position by synchronous machine is called the hunting of synchronous machines.

$$Natural\ Undamped\ frequency(without\ damper\ winding);$$

$$f_{UD}=\frac{1}{2\pi}\sqrt{\frac{stiffness}{Inertia}}=\frac{1}{2\pi}\sqrt{\frac{\frac{dTe}{d\delta}}{J}}(mech.rad/sec)\ or\ Hz$$

$$also\ The\ stored\ energy\ constant=\frac{stored\ energy\ at\ rated\ speed}{Rated\ Volt\ Ampere}$$

$$=\frac{\frac{1}{2}J\omega_s^{\,2}}{MVA}\qquad in(sec)$$

Example 7: A 6-pole, Y-connected synchronous motor a current of 690A at unity pf when connected to 5000V, 50Hz, infinite bus bar. The synchronous reactance is 1.05Ω per phase, moment of inertia is 7720kg.m². Calculate the load angle the natural period oscillation of the system for a small displacement.

Solution:

$$V = E_b\angle -\delta + jI_a X_s$$

$$5000/\sqrt{3} = E_b\angle -\alpha + 690\angle 0 \times 1.05\angle 90$$

$$E_b = 2976.27 Volt$$

$$\alpha = 14.1^0$$

$$P_m = \frac{3E_bV}{X_s}\sin\delta \quad = \frac{3\times 5000/\sqrt{3}\times 2976.27}{1.05}\sin 14.1$$

$$= 5980227.5 Watt$$

$$T_d = \frac{5980227.5}{2\pi\frac{120*50}{6\times 60}} = 57107 N.m$$

$$\frac{dT_e}{d\alpha} = \frac{3E_bV}{\omega_s X_s}\cos\alpha = \frac{23808283.63}{2\pi\frac{120*50}{6\times 60}} = 227352.3 N.m/elec.\deg ree$$

$$\frac{dT_e}{d\alpha} = 227352.3 N.m/elec.\deg re \times \frac{6}{2} = 682057 N.m/mech.\deg$$

$$f_{UD} = \frac{1}{2\pi}\sqrt{\frac{682057}{7720}} = 1.49 Hz$$

Example 8: A 10MVA, 4-pole, 6600V, 50Hz, 3-phase Y-connected synchronous generator has X_s of 1 Ohm. It operates on constant V &f bus bar, if natural frequency of oscillation period has 1.5sec operated at unity power factor. Find the moment of inertia and the stored energy constant.

Solution:

$$S = \sqrt{3} V I_a \Rightarrow 10 \times 10^6 = \sqrt{3} \times 6600 \times I_a \Rightarrow I_a = 874.8\,A$$

$$E_f \angle \delta = 6600 / \sqrt{3} + j874.8 \times 1$$

$$= 3910 \angle 13 Volt$$

$$P_e = \frac{3VE_f}{X_s} \sin\delta = \frac{3 \times 6600 / \sqrt{3} \times 3910 \sin 13}{1}$$

$$\frac{dT_e}{d\delta} = \frac{3VE_f}{\omega_s X_s} \cos\delta = \frac{3 \times 6600 / \sqrt{3} \times 3910 \cos 13}{50\pi}$$

$$= 277.26\, N.m / ele.deg.$$

$$\frac{dT_e}{d\delta} = 277.26 \times \frac{4}{2} = 554.5\, N.m / Mech.deg.$$

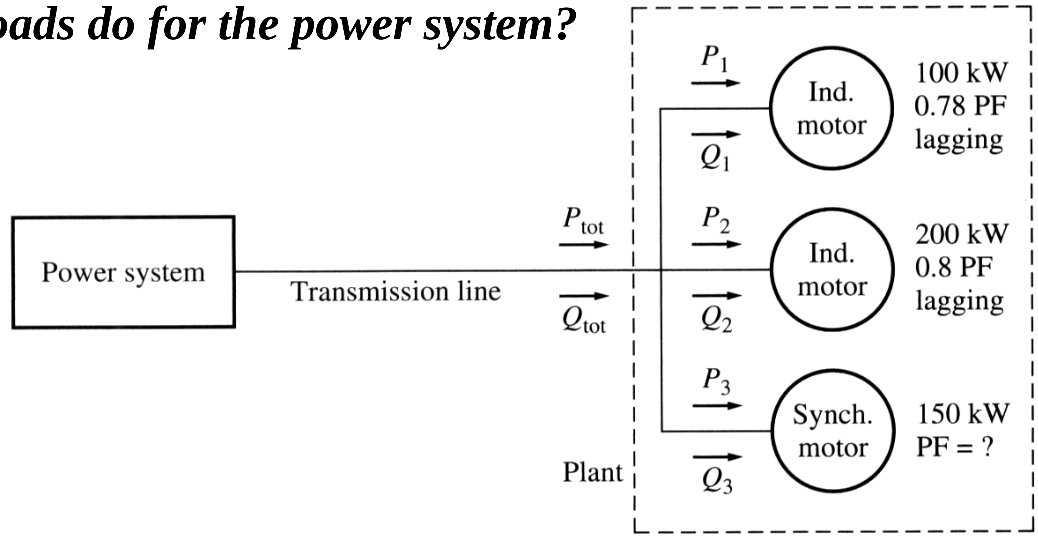
$$f_{UD} = f_{oscillation} = \frac{1}{2\pi}\sqrt{\frac{\frac{dTe}{d\delta}}{J}} (mech.rad / sec) \; or \; Hz$$

$$\frac{1}{1.5} = 0.66 = \frac{1}{2\pi}\sqrt{\frac{554.16}{J}} \quad J = 31.6\, kg.m^2$$

$$The\ stored\ energy\ constant = \frac{stored\ energy\ at\ rated\ speed}{Rated\ Volt\ Ampere}$$

$$= \frac{\frac{1}{2} J \omega_s^2}{MVA} = 0.039 \sec$$

Example 9: Assuming that a load contains a synchronous motor (whose PF can be adjusted) in addition to motors of other types. What does the ability to set the PF of one of the loads do for the power system?



Let us consider a large power system operating at 480 V. Load 1 is an induction motor consuming 100 kW at 0.78 PF lagging, and load 2 is an induction motor consuming 200 kW at 0.8 PF lagging. Load 3 is a synchronous motor whose real power consumption is 150 kW.

a. If the synchronous motor is adjusted to 0.85 PF lagging, what is the line current?

b. If the synchronous motor is adjusted to 0.85 PF leading, what is the line current?

c. Assuming that the line losses are PLL = 3IL2RL, how due these losses compare in the two cases?

Solution:

a. The real power of load 1 is 100 kW, and the reactive power of load 1 is

$$Q_1 = P_1 \tan\theta = 100 \tan\left(\cos^{-1} 0.78\right) = 80.2\,kVAR$$

The real power of load 2 is 200 kW, and the reactive power of load 2 is

$$Q_2 = P_2 \tan\theta = 200\tan\left(\cos^{-1} 0.8\right) = 150\ kVAR$$

The real power of load 3 is 150 kW, and the reactive power of load 3 is

$$Q_3 = P_3 \tan\theta = 150\tan\left(\cos^{-1} 0.85\right) = 93\ kVAR$$

The total real load is $P_{tot} = P_1 + P_2 + P_3 = 100 + 200 + 150 = 450\ kW$

The total reactive load is

$$Q_{tot} = Q_1 + Q_2 + Q_3 = 80.2 + 150 + 93 = 323.2\ kVAR$$

The equivalent system PF is

$$PF = \cos\theta = \cos\left(\tan^{-1}\frac{Q}{P}\right) = \cos\left(\tan^{-1}\frac{323.2}{450}\right) = 0.812\ \ lagging$$

The line current is

$$I_L = \frac{P_{tot}}{\sqrt{3}V_L\cos\theta} = \frac{450\,000}{\sqrt{3}\cdot 480\cdot 0.812} = 667\ A$$

b. The real and reactive powers of loads 1 and 2 are the same. The reactive power of load 3 is

$$Q_3 = P_3 \tan\theta = 150\tan\left(-\cos^{-1} 0.85\right) = -93\ kVAR$$

The total real load is

$$P_{tot} = P_1 + P_2 + P_3 = 100 + 200 + 150 = 450\ kW$$

The total reactive load is

$$Q_{tot} = Q_1 + Q_2 + Q_3 = 80.2 + 150 - 93 = 137.2\ kVAR$$

The equivalent system PF is

$$PF = \cos\theta = \cos\left(\tan^{-1}\frac{Q}{P}\right) = \cos\left(\tan^{-1}\frac{137.2}{450}\right) = 0.957\ \ lagging$$

$$PF = \cos\theta = \cos\left(\tan^{-1}\frac{Q}{P}\right) = \cos\left(\tan^{-1}\frac{137.2}{450}\right) = 0.957\ \ lagging$$

The line current is

$$I_L = \frac{P_{tot}}{\sqrt{3}V_L\cos\theta} = \frac{450\,000}{\sqrt{3}\cdot 480\cdot 0.957} = 566\ A$$

c. The transmission line losses in the first case are

$$P_{LL} = 3I_L^2R_L = 1\,344\,700\ R_L$$

The transmission line losses in the second case are

$$P_{LL} = 3I_L^2R_L = 961\,070\ R_L$$

We notice that the transmission power losses are 28% less in the second case, while the real power supplied to the loads is the same.