

Voltage Regulation of Synchronous Generator

Introduction:

The SG when driven at synchronous speed, and when the load is varied depending on the power factor, the voltage decrease or increase, if the voltage E_f (open circuit voltage or no-load voltage or excitation voltage) and (V) is the terminal voltage, then the difference between them in percent is called the voltage regulation (V.R)%.

$$V.R\% = \frac{E_f - V}{V} \times 100$$

In small machine voltage regulation may be calculated directly but in case of large synchronous may be calculated by open circuit test (OCC) and short circuit test (SCC) by three methods of cylindrical type alternator:

- 1- The synchronous impedance method***
- 2- The Ampere-turn method***
- 3- The Poiter (ZPF) method.***

16.1 The Synchronous Impedance Method:

To estimate synchronous impedance (Z_s) of the machine, X_s is calculated with R_a .

The Test requirements:

1-The effective resistance per phase (R_a): by using DC current

and converting to ac by multiply the skin effect factor.

2-The O.C.C (open circuit curve) between no-load induced

voltage and excitation or field current (I_f) at rated speed.

3- The S.C.C (short circuit curve) which is drawn between the armature current and the field current at rated speed and till 1.5 times the full load current.

It is as straight line passing through the origin point. Both curves are plotted on the same field current on X-axis. The mmf in S.C.C is pure demagnetizing.

Advantages:

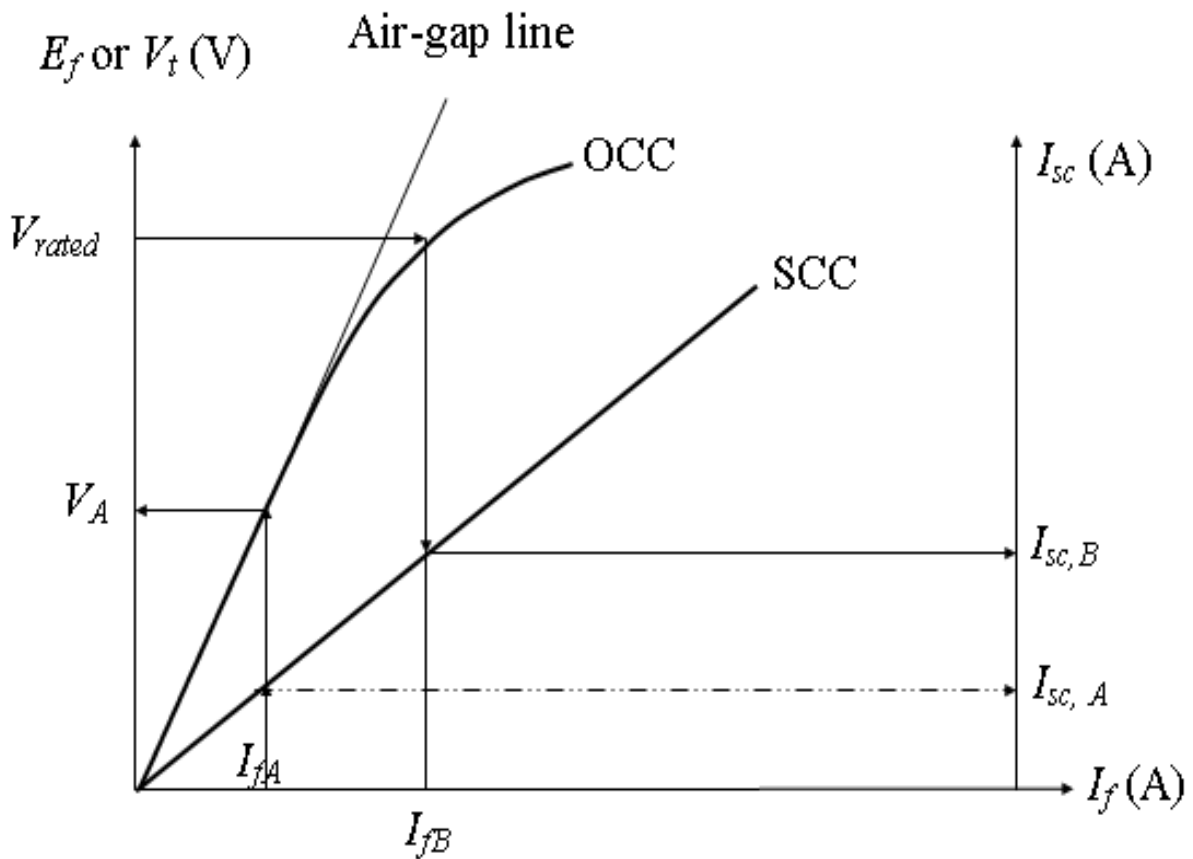
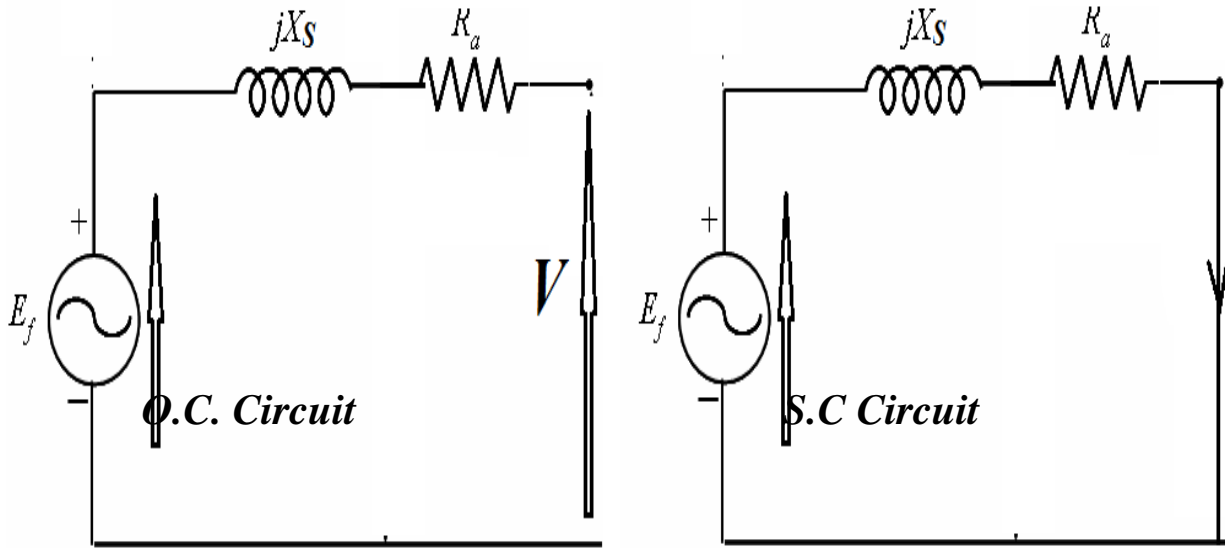
- **Simple no load tests (for obtaining OCC and SCC) are to be conducted**
- **Calculation procedure is much simpler**

Disadvantages:

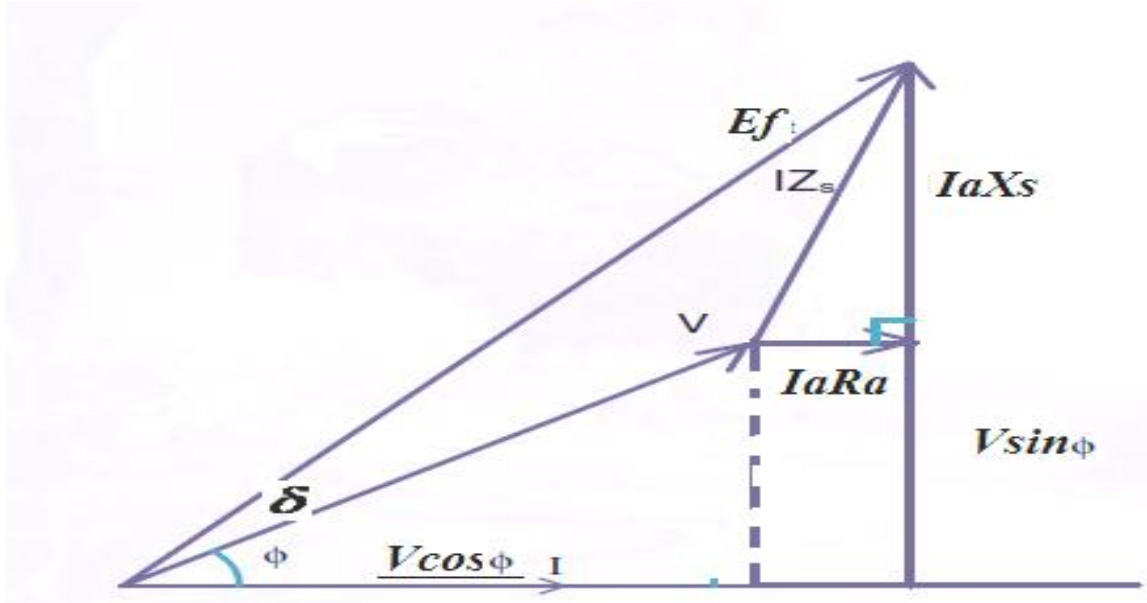
- **The value of voltage regulation obtained by this method is always higher than the actual value.**

A drawback of this method is that the internal generated voltage E_A is measured during the OCC, where the machine can be saturated for large field currents, while the armature current is measured in SCC, where the core is unsaturated. Therefore, this approach is accurate for unsaturated cores only.

The approximate value of synchronous reactance varies with the degree of saturation of the OCC.

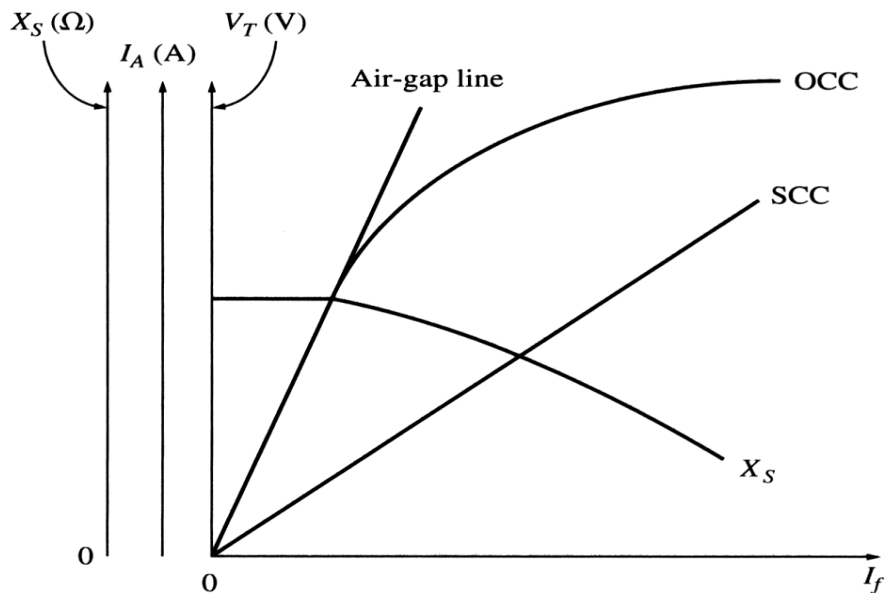
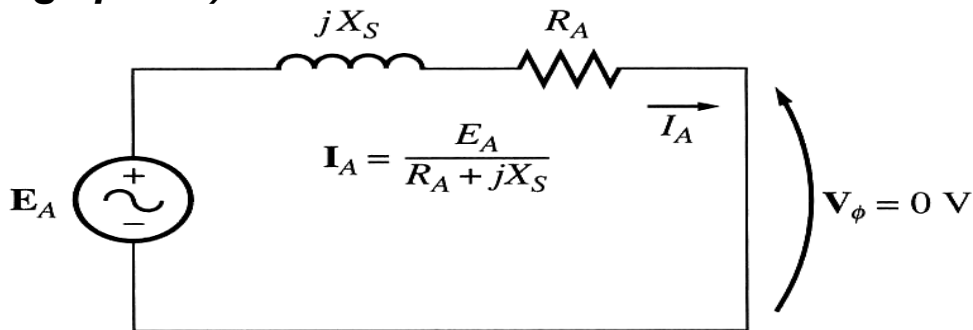


$$Z_s = \frac{E_f \text{ (open circuit voltage from OCC curve)}}{I_{sc} \text{ (Short circuit current from SCC curve)}}$$



$$E_f = \sqrt{(V \cos \phi + I_a R_a)^2 + (V \sin \phi + I_a X_s)^2}$$

(Note: The preferred point for calculating X_s it is the rated voltage point).



Example: A 50KVA, 415V, 50Hz, Y-connected synchronous generator has an effective resistance of $0.2\Omega/\text{phase}$. A field current of 8A causes 0f 415V on open circuit and 185A on short circuit. Calculate 1- Z_s , 2- X_s , 3- Voltage regulation at 0.8 p.f lagging and 4- The voltage regulation at 0.75 full load with 0.9 p.f leading.

Solution:

$$V/\text{phase} = 415/\sqrt{3} = 239.6V$$

$$1 - Z_s = \frac{239.6}{185} = 1.261\Omega$$

$$2 - X_s = \sqrt{1.361^2 - 0.2^2} = 1.245\Omega$$

$$3 - S = \sqrt{3}V_t I_t \Rightarrow 50000 = \sqrt{3} \cdot 415 \cdot I_t \Rightarrow I_t = 69.56A$$

$$\begin{aligned} E_f &= V + I_a Z_s \\ &= 239.6 + 69.56 \angle -36.8^\circ \cdot 1.261 \angle 80.98^\circ \\ &= 308 \angle 11.31^\circ V, \quad / E_f / = 308V \text{ \& } \delta = 11.31^\circ \end{aligned}$$

or

$$\begin{aligned} E_f &= \sqrt{(239.6 \cos 36.8 + 69.56 \cdot 0.2)^2 + (239.6 \sin 36.8 + 69.6 \cdot 1.245)^2} \\ &= 308V \end{aligned}$$

$$V.R\% = \frac{308 - 239.6}{239.6} \cdot 100 = 28.5\%$$

$$\begin{aligned} 4 - E_f &= V + 0.75 I_a Z_s \\ &= 239.6 + 0.75 \cdot 69.56 \angle 25.8^\circ \cdot 1.261 \angle 80.98^\circ \\ &= 229.47 \angle 14.93^\circ V, \quad / E_f / = 229.47V \text{ \& } \delta = 14.93^\circ \end{aligned}$$

or

$$\begin{aligned} E_f &= \sqrt{(239.6 \cos 25.8 + 0.75 \cdot 69.56 \cdot 0.2)^2 + (239.6 \sin 25.8 - 69.6 \cdot 1.245)^2} \\ &= 229.5V \end{aligned}$$

$$V.R\% = \frac{229.47 - 239.6}{239.6} \cdot 100 = 4.22\%$$

Example 7.2: A 480 V, 60 Hz, Y-connected six-pole synchronous generator has a per-phase synchronous reactance of 1.0 Ω. Its full-load armature current is 60 A at 0.8 PF lagging. Its friction and windage losses are 1.5 kW and core losses are 1.0 kW at 60 Hz at full load. Neglect armature resistance. The field current has been adjusted such that the no-load terminal voltage is 480 V.

a. What is the terminal voltage of the generator and its V.R% if

1. It is loaded with the rated current at 0.8 PF lagging;
2. It is loaded with the rated current at 1.0 PF;
3. It is loaded with the rated current at 0.8 PF leading.

b. What is the efficiency of this generator (ignoring the unknown electrical losses) when it is operating at the rated current and 0.8 PF lagging?

c. How much shaft torque must be applied by the prime mover at the full load? how large is the induced counter torque?

Solution: a/1-

$$V_t = \frac{480}{\sqrt{3}} = 277V$$

$$E_f \angle \delta = V \angle 0 + jI_a X_s$$

$$277 \angle \delta = V_t + 1 \angle 90 \times 60 \angle -36.87$$

$$277 \angle \delta = V_t + 60 \angle 53.13$$

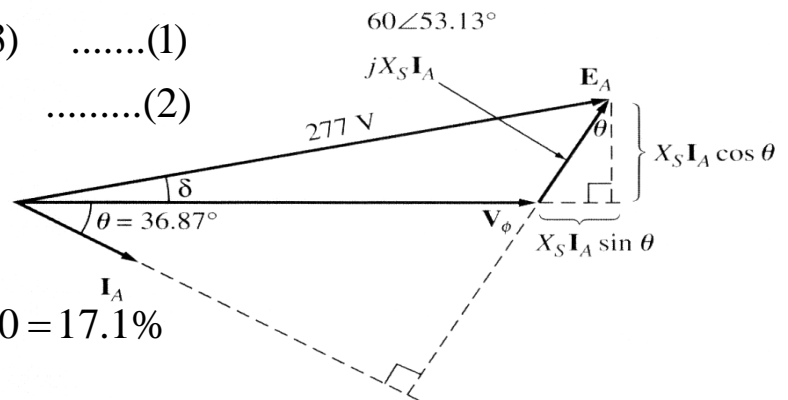
$$\mathbf{a-} 277 \cos \delta = V + 60 \cos(53.13) \quad \dots\dots(1)$$

$$277 \sin \delta = 60 \sin(53.13) \quad \dots\dots(2)$$

$$\therefore \delta = 10^\circ$$

$$V_t = 236.8V$$

$$V.R\% = \frac{277 - 236.8}{236.8} \times 100 = 17.1\%$$



a/2-

$$V_t = \frac{480}{\sqrt{3}} = 277V$$

$$E_F \angle \delta = V \angle 0 + jI_a X_s$$

$$277 \angle \delta = V_t + 1 \angle 90 \times 60 \angle 0$$

$$277 \angle \delta = V_t + 60 \angle 90$$

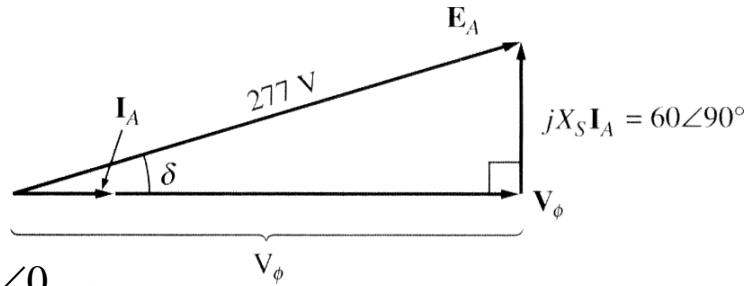
$$277 \cos \delta = V \quad \dots\dots(1)$$

$$277 \sin \delta = 60 \sin(90) \quad \dots\dots(2)$$

$$\therefore \delta = 12.5$$

$$V_t = 270.4V$$

$$V.R\% = \frac{277 - 270.4}{270.4} \times 100 = 2.6\%$$



a/3-

$$V_t = \frac{480}{\sqrt{3}} = 277V$$

$$E_F \angle \delta = V \angle 0 + jI_a X_s$$

$$277 \angle \delta = V_t + 1 \angle 90 \times 60 \angle 36.87$$

$$277 \angle \delta = V_t + 60 \angle 126.87$$

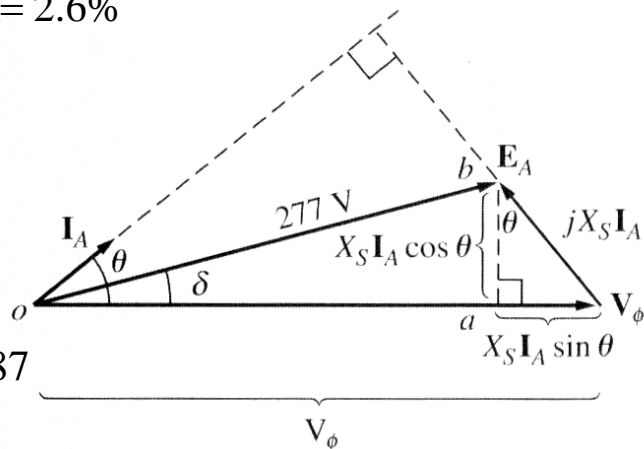
$$277 \cos \delta = V + 60 \cos(126.87) \quad \dots\dots(1)$$

$$277 \sin \delta = 60 \sin(126.87) \quad \dots\dots(2)$$

$$\therefore \delta = 10^\circ$$

$$V_t = 308.8V$$

$$V.R\% = \frac{277 - 308.8}{308.8} \times 100 = -10.3\%$$



b-

$$\begin{aligned}P_{out} &= \sqrt{3} V_l I_a \cos \phi \\ &= \sqrt{3} \times \sqrt{3} \times 236.8 \times 60 \times \cos 36.87 \\ &= 34.1 \text{ kW}\end{aligned}$$

$$\begin{aligned}P_{inp} &= P_{out} + \text{Loss} \\ &= 34.1 \text{ kW} + 1 \text{ kW} + 1.5 \text{ kW} \\ &= 36.6 \text{ kW}\end{aligned}$$

$$\eta\% = \frac{34.1}{36.6} \times 100 = 93.2\%$$

c/-

$$\text{The input torque} = \frac{36.6 \text{ kW}}{125.7} = 291.2 \text{ N.m}$$

$$\text{The induced counter torque by the generator} = \frac{34.1 \text{ kW}}{125.7} = 271.3 \text{ N.m}$$

16.2 Ampere Turn (MMF) method:

The ampere-turn /MMF method is the converse of the EMF method in the sense that instead of having the phasor addition of various voltage drops/EMFs, here the phasor addition of MMF required for the voltage drops are carried out. Further the effect of saturation is also taken care of it.

Data required for MMF method are:

- Effective resistance per phase of the 3-phase winding R***
- Open circuit characteristic (OCC) at rated speed/frequency***
- Short circuit characteristic (SCC) at rated speed/frequency***

Compared to the EMF method, MMF method, involves more number of complex calculation steps. Further the OCC is referred twice and SCC is referred once while predetermining the voltage regulation for each load condition.

The steps for calculating the voltage regulation:

1-Field current (A.T.) required to produce $V+I_aR_a$ on no load from OCC. We assume it OA.

2-Field current (A.T.) required to overcome the demagnetizing effect of armature reaction on full load from SCC.

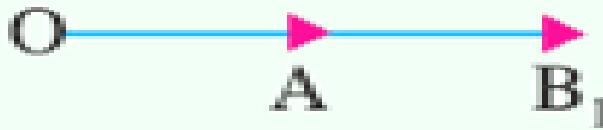
“ The demagnetizing armature A.T. on full load are equal and opposite to the field A.T. required produce full load current on short circuit”.

The procedure of V.R% Calculation:

i- The field (I_{f1}) A.T.=OA necessary to produce normal voltage as obtained “ $V+I_aR_a$ “.

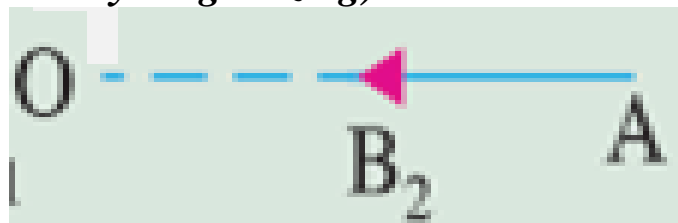
ii- The Field (I_{f2}) A.T. necessary to neutralize or overcome the armature reaction = AB_1 .

**Then OB_1 = Total field current A.T. required to produce E_f . (Zero power factor lagging pure inductive load).
(wholly demagnetizing)**



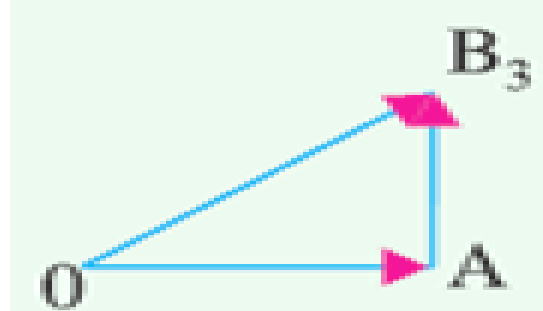
iii- The Field (I_{f2}) A.T. necessary to neutralize or overcome the armature reaction = AB_2 .

**Then OB_2 = Total field current A.T. required to produce E_f . (Zero power factor leading pure capacitive load).
(wholly Magnetizing)**

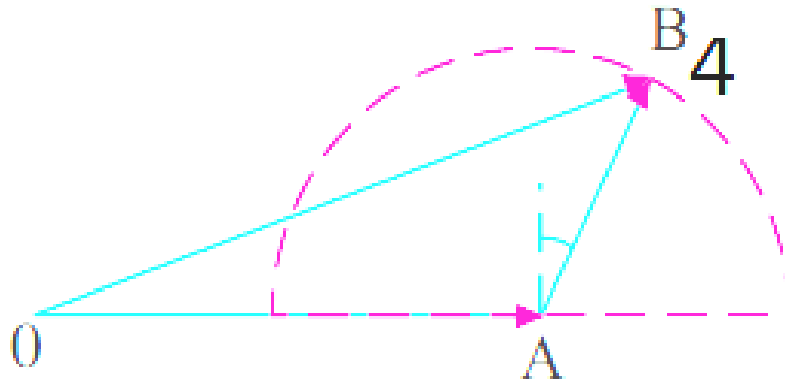


iv- The Field (I_{f2}) A.T. necessary to neutralize or overcome the armature reaction = AB_3 .
Then OB_3 = Total field current A.T. required to produce E_f . (Pure R)(cross magnetizing).

$$OB_3 = \sqrt{OA^2 + AB^2}$$



v- The Field (I_{f2}) A.T. necessary to neutralize or overcome the armature reaction = AB_4 .
Then OB_4 = Total field current A.T. required to produce E_f . (R+L)(demagnetizing).



AB_4 is drawn by an angle $90+\phi$

$$I_f(\text{Total}) = OB_4 = \sqrt{I_{f1}^2 + I_{f2}^2 + 2I_{f1}I_{f2} \cos(90 - \phi - \gamma)}$$

For unity and Lagging power factor.

$I_{f1} = OA$

$I_{f2} = AB_{1,2,3,4,5}$

1----- for pure L

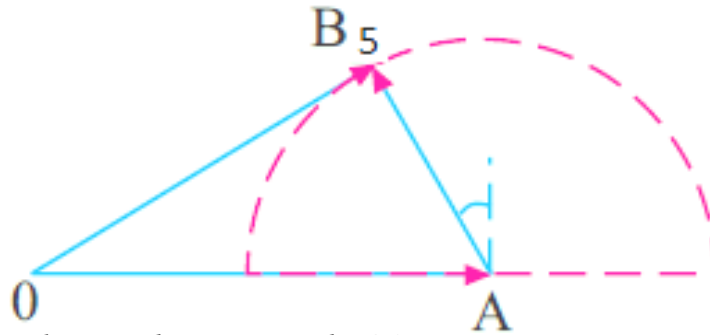
2----- for pure C

3 ----- for pure R

4 ----- for R+L

5----- for R+C

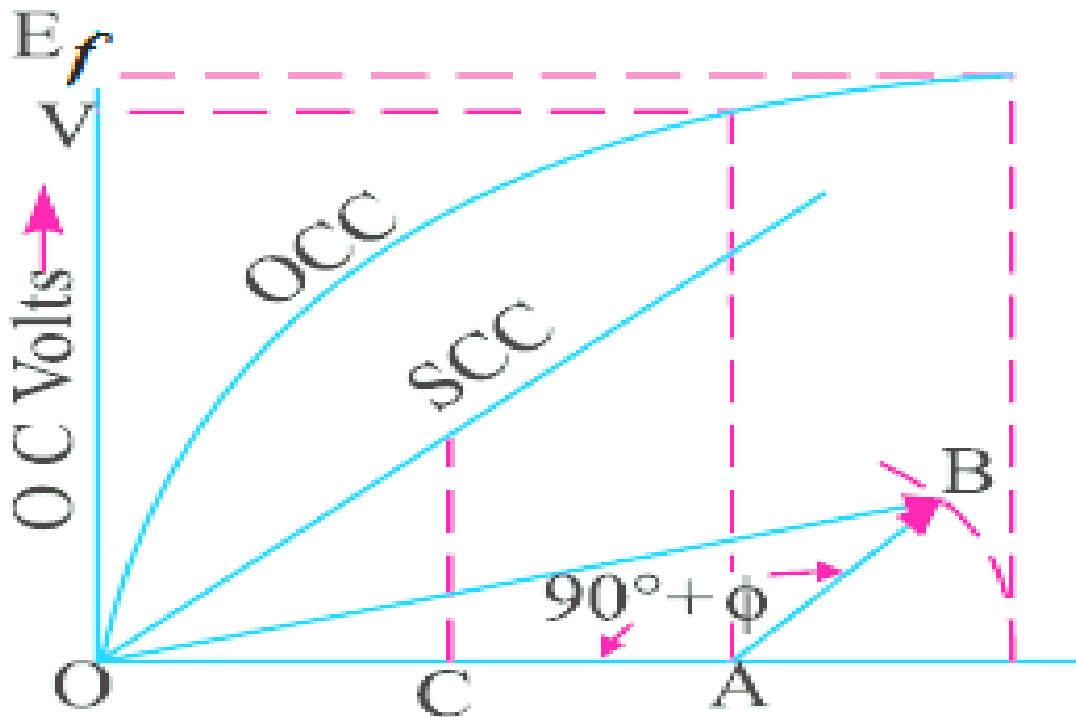
**vi-The Field (I_{f2}) A.T. necessary to neutralize or overcome the armature reaction = AB_5 .
Then OB_5 = Total field current A.T. required to produce E_f . ($R+C$)(magnetizing).**



AB_5 is drawn by an angle $90-\phi$

$$I_{f(Total)} = OB_5 = \sqrt{I_{f_1}^2 + I_{f_2}^2 - 2I_{f_1}I_{f_2} \cos(90 - \phi + \gamma)}$$

For leading power factor.



Example 1 The following test results are obtained on a 6,600-V alternator:

Open-circuit voltage : 3,100 4,900 6,600 7,500 8,300

Field current (amps) : 16 25 37.5 50 70

A field current of 20 A is found necessary to circulate full-load current on short-circuit of the armature. Calculate by the ampere-turn method, full-load regulation at 0.8 p.f.

Solution.

It is seen from the given data that for the normal voltage of 6,600 V, the field current needed is 37.5 A.

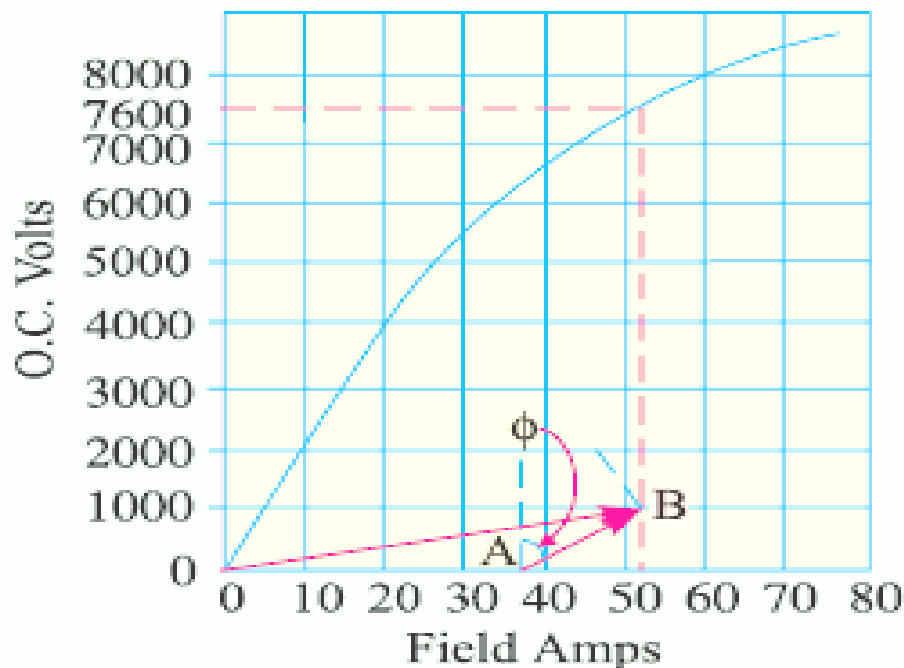
Field-current for full-load current, on short-circuit, is given as 20 A.

In Fig, OA represents 37.5 A. The vector AB , which represents 20 A, is vectorially added to OA at $(90^\circ + 36^\circ 52') = 126^\circ 52'$. Vector OB represents the excitation necessary to produce a terminal p.d. of 6,600 V at 0.8 p.f. lagging on **full-load**

$$OB = \sqrt{37.5^2 + 20^2 + 2 \times 37.5 \times 20 \times \cos 53^\circ 8'} = 52 \text{ A}$$

The generated e.m.f. E_0 corresponding to this excitation, as found from O.C.C. of Fig. is 7,600 V.

$$\text{Percentage regulation} = \frac{E_0 - V}{V} \times 100 = \frac{7,600 - 6,600}{6,600} \times 100 = 15.16\%$$



EXAMPLE . A 3-phase, star-connected, 1000 kVA, 2000 V, 50 Hz alternator gave the following open-circuit and short-circuit test readings :

Field current	A	10	20	25	30	40	50
O.C. voltage	V	800	1500	1760	2000	2350	2600
S.C. armature current	A		200	250	300		

The armature effective resistance per phase is 0.2 Ω.

Draw the characteristic curves and determine the full-load percentage regulation at (a) 0.8 power factor lagging, (b) 0.8 power factor leading.

SOLUTION. The O.C.C. and S.C.C. are shown in Fig.

The phase voltage in volts are

$$\frac{800}{\sqrt{3}}, \quad \frac{1500}{\sqrt{3}}, \quad \frac{1760}{\sqrt{3}}, \quad \frac{2000}{\sqrt{3}}, \quad \frac{2350}{\sqrt{3}}, \quad \frac{2600}{\sqrt{3}};$$

or 462, 866, 1016, 1155, 1357, 1501.

$$\text{Full-load phase voltage } V_p = \frac{2000}{\sqrt{3}} = 1155 \text{ V}$$

$$\text{kVA} = \frac{\sqrt{3} V_L I_{fl}}{1000}$$

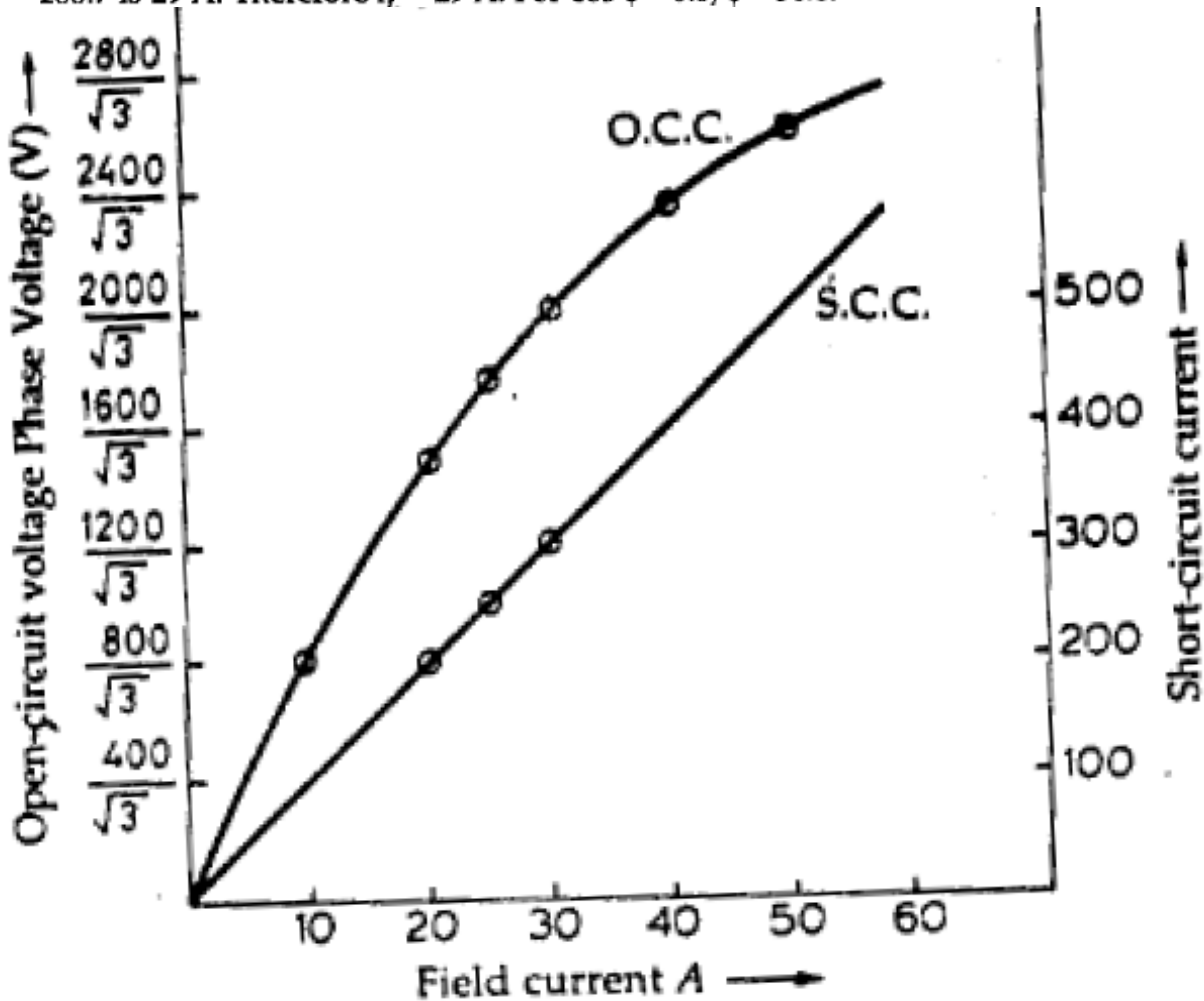
$$1000 = \frac{\sqrt{3} \times 2000 \times I_{fl}}{1000}, \quad I_{fl} = I_a = 288.7 \text{ A}$$

(a) Lagging power factor of 0.8

$$\begin{aligned} E' &= V_p + I_a R_a = 1155 + (288.7 \angle -\cos^{-1} 0.8) \times 0.2 \\ &= 1155 + (57.74 \times 0.8 - j 57.74 \times 0.6) \\ &= 1155 + 46.2 - j 34.44 \\ &= 1201.2 - j 34.44 = 1201.7 \angle -1.6^\circ \text{ V} \\ \gamma &= -1.6^\circ \end{aligned}$$

From the O.C.C., the field current required to produce the voltage of 1201.7 V is 32 A. Therefore $I_{f1} = 32$ A.

From the S.C.C., the field current required to produce full-load current of 288.7 is 29 A. Therefore $I_f = 29$ A. For $\cos \phi = 0.8$, $\phi = 36.87^\circ$



$$\begin{aligned}
 I_f(\text{Total}) &= \sqrt{I_{f1}^2 + I_{f2}^2 + 2I_{f1}I_{f2} \cos(90 - \phi - \gamma)} \\
 &= \sqrt{32^2 + 29^2 + 2 \times 32 \times 29 \times \cos(90 - 36.87 + 1.6)} \\
 &= 54.2 \text{ A}
 \end{aligned}$$

The corresponding voltage to this field current is 1559V.

$$\text{V.R}\% = \frac{1559 - 1155}{1155} \times 100 = 34.97\%$$

(b) Leading power factor of 0.8

$$\begin{aligned}
 E' &= V_p + I_a R_a \\
 &= 1155 + (288.7 \angle + \cos^{-1} 0.8) \times 0.2 \\
 &= 1155 + 46.2 + j 34.44 \\
 &= 1201.2 + j 34.44 = 1201.7 \angle + 1.6^\circ \text{ V.}
 \end{aligned}$$

$$\begin{aligned}
 I_f(\text{Total}) &= \sqrt{I_{f_1}^2 + I_{f_2}^2 - 2I_{f_1}I_{f_2} \cos(90 - \phi + \gamma)} \\
 &= \sqrt{32^2 + 29^2 - 2 \times 32 \times 29 \times \cos(90 - 36.87 + 1.6)} \\
 &= 28.16 \text{ A}
 \end{aligned}$$

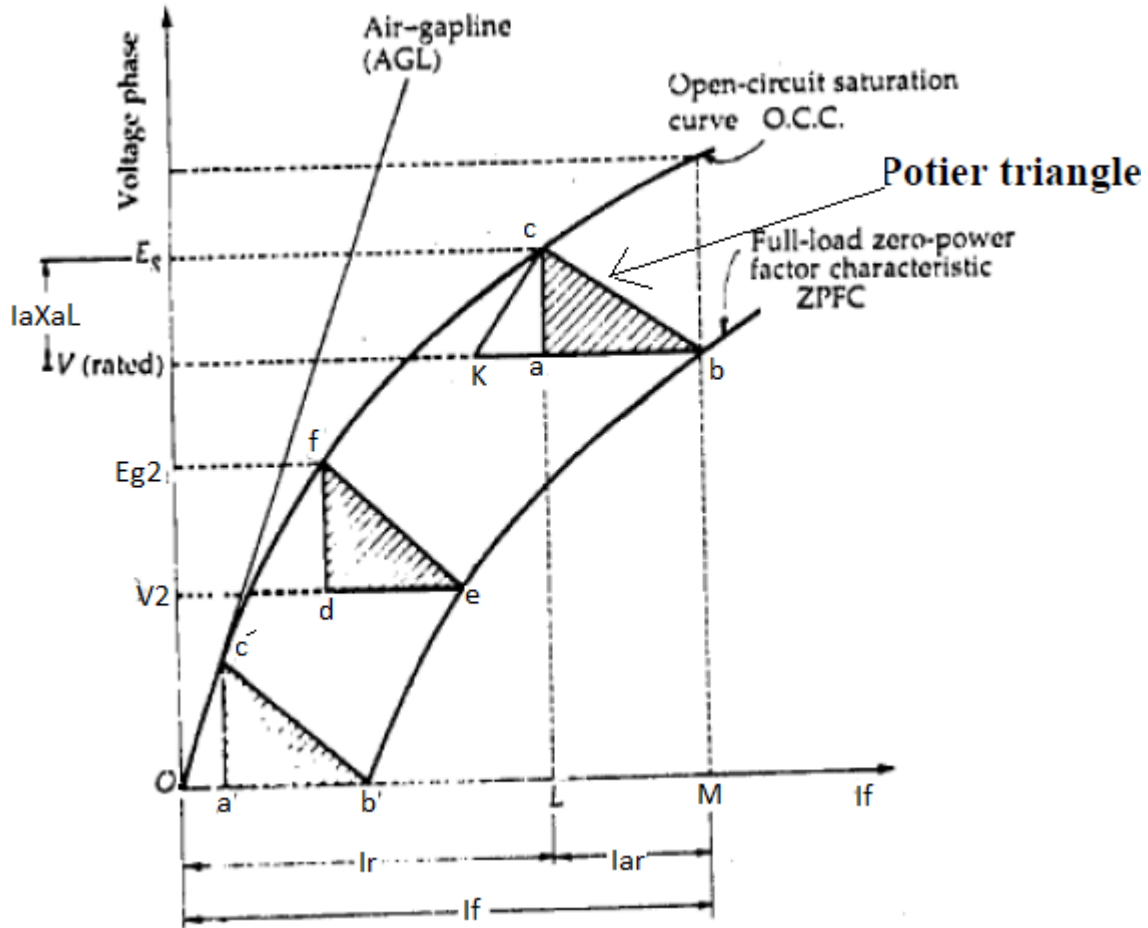
The corresponding voltage to this field current is 1098V.

$$V.R\% = \frac{1098 - 1155}{1155} \times 100 = -5.02\%$$

16.3 Poiter Method or Zero Power Factor Method:

Zero Power Factor Curve(ZPFC):

The ZPFC of an alternator is a curve of the armature voltage per phase plotted against the field current obtained by operating the machine with constant rated armature current at synchronous speed and zero power factor lagging. The alternator is loaded by means of a reactor. Its shape like OCC displaced downwards and to right.



1- The field current OL would result in a generated voltage ($E_g=Lc$) from the no load saturation curve.

$$E_g = V + I_a X_{al}$$

2-The vertical distance ac must be equal to the leakage reactance voltage drop $I_a X_{al}$ where I_a is the rated armature current.

$$X_{al} = \frac{\text{Voltage } ac \text{ per phase}}{\text{Rated armature current}}$$

The Triangle formed by the vertices a , b & c is called the Potier triangle.

The construction of ZPFC:

- i- Take a point b on the ZPFC its prefer to be upon the knee curve.**
- ii- Draw bk equal to b'o (b' is the point for zero voltage and full load current). Ob' =short circuit current excitation.**
- iii-From k draw draw kc parallel to the air gap line oc' to intersect with OCC in c.**
- iv-Drop the perpendicular ca on to bk.**
- v- To scale Voltage ca is the leakage reactance drop $I_a X_{al}$ and ab is the armature reaction mmf or field current equivalent to it at rated current.**

$$X_p = \frac{\text{Voltage } ac \text{ per phase}}{\text{ZPFC Rated armature current}}$$

For cylindrical rotor machine x_p is approximately equal to leakage reactance x_{al} . In salient pole $X_p=3 *X_{al}$.

Zero power factor method to determine the voltage regulation:

The following procedure to obtaining the V.R. the phasor diagram required;

1- OA= V= terminal phase voltage at full load its taken as the reference.

2- OB= I_a = full load current lagging armature behind V by an angle of ϕ which is the power factor of the load.

3-AC= voltage drop on the R_a .

4-CD= $I_a X_{al}$ =leakage reactance voltage drop. Its perpendicular to AC join OD, its equal to the voltage of E_g .

5- I_r is the field current corresponding to the voltage E_g .

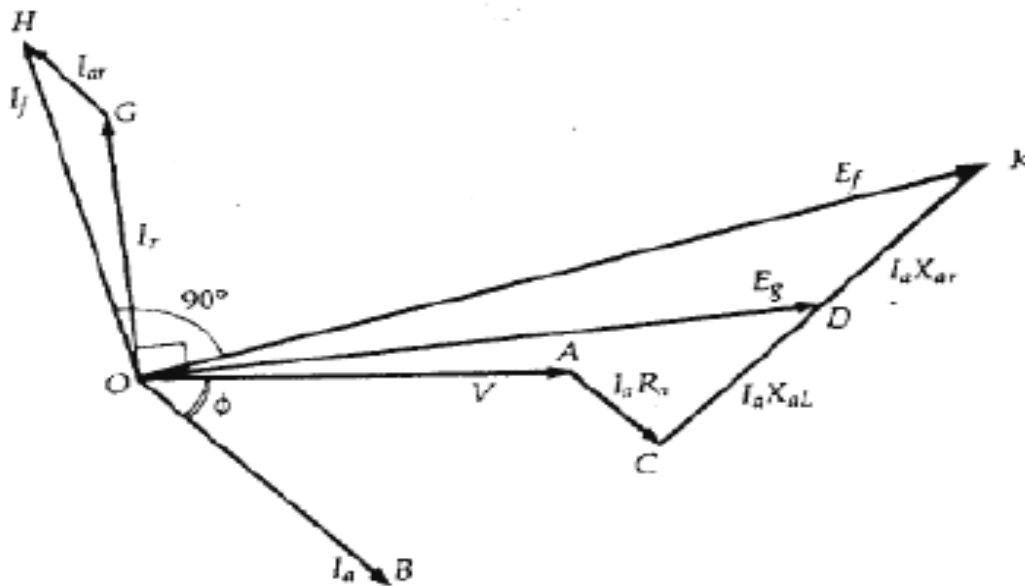
6-OG is perpendicular onto OD.

7-Gh is parallel to OB to represent the field current equivalent to full load armature current I_{ar} .

8-OH is equal to total field current equivalent to the $OK = E_f$.

9-The voltage E_f is determined is perpendicular onto OH.

10-DK is represent the voltage drop due to the armature reaction X_m .



EXAMPLE . A 5000 kVA, 6600 V, 3-phase, star-connected alternator has a resistance of 0.75Ω per phase. Estimate by zero power factor method the regulation for a load of 500 A at power factor (a) unity, (b) 0.9 leading, (c) 0.71 lagging, from the following open-circuit and full load, zero power factor curves :

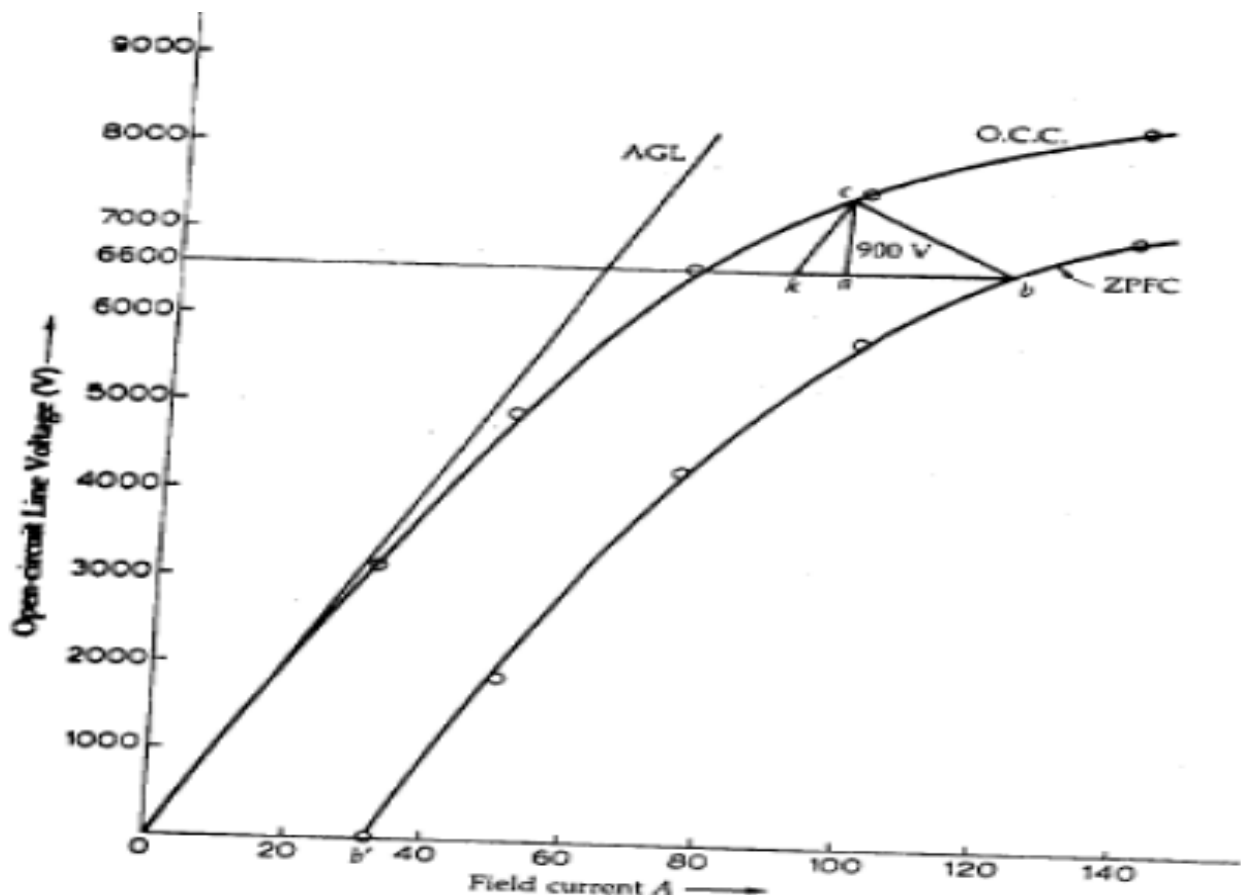
Field current, A	Open-circuit terminal voltage, V	Saturation curve, zero p.f., V
32	3100	0
50	4900	1850
75	6600	4250
100	7500	5800
140	8300	7000

SOLUTION. The O.C.C. and the ZPFC are plotted

Draw a horizontal line at rated line voltage of 6600 V to meet the ZPFC at *b*. On this line take $bk = Ob' = 32$ A.

Ob' is the field current required to circulate full-load current on S.C.

Draw a line kc parallel to Oc' (the initial slope of the O.C.C.) to meet the O.C.C. at *c*. Draw the perpendicular ca on the line kb . Hence abc is the Potier's triangle. In this triangle,



$$ab = \text{field current required to overcome armature reaction on load} \\ = I_{ar} = 25 \text{ A}$$

$$ac = 900 \text{ V (line-to-line)} = \frac{900^\circ}{\sqrt{3}} \text{ V per phase}$$

∴ Leakage impedance voltage drop

$$I_a X_L = \frac{900}{\sqrt{3}} \text{ V per phase} \\ = 579.6 \text{ V, } I_a = 500 \text{ A}$$

$$X_L = \frac{900}{\sqrt{3} \times 500} = 1.039 \Omega$$

Taking I_a as reference phasor: $I_a = I_a \angle 0^\circ = 500 \angle 0^\circ \text{ A} = 500 + j 0$

(a) Unity power factor

$$V_p = V_p \angle 0^\circ = \frac{6600}{\sqrt{3}} \angle 0^\circ = 3810.6$$

$$E_{sp} = V_p + I_a Z_L = V_p + I_a (R_a + j X_L) = V_p + I_a R_a + j I_a X_L \\ = 3810.6 + 500 \times 0.075 + j 519.6 \\ = 3848.1 + j 519.6 = 3883 \angle 7.69^\circ \text{ V}$$

$$E_{gf} = \sqrt{3} \times 3883 = 6725 \text{ V}$$

From the O.C.C., the field current corresponding to the line voltage of 6725 V is 78 A.

$$I_f(\text{Total}) = \sqrt{I_{f_1}^2 + I_{f_2}^2 + 2I_{f_1}I_{f_2} \cos(90 - \phi - \gamma)} \\ = \sqrt{78^2 + 25^2 + 2 \times 78 \times 25 \times \cos(90 - 7.69)} \\ = 85 \text{ A}$$

From the O.C.C., corresponding to a field current of 85 A, the voltage $E_{fp} = 7000 \text{ V (line-line)}$

$$E_{fp} = \frac{7000}{\sqrt{3}} \text{ V} = 4041.6 \text{ V}$$

$$\therefore \text{ voltage regulation} = \frac{E_{fp} - V_p}{V_p} \times 100 \\ = \frac{4041.6 - 3810.6}{3810.6} \times 100 = 6.06\%$$

b- 0.9 Leading

$$E_{gp} = 3810.6 + 500 \angle 25.84 \times (0.075 + j1.039)$$

$$= 3650 \angle 7.7^\circ \text{ V}$$

The corresponding line voltage = $\sqrt{3} \times 3650 = 6321.8 \text{ V}$

From OCC the corresponding field current is 71A

$$I_f(\text{Total}) = \sqrt{I_{f1}^2 + I_{f2}^2 - 2I_{f1}I_{f2} \cos(90 - \phi + \gamma)}$$

$$= \sqrt{71^2 + 25^2 - 2 \times 71 \times 25 \times \cos(90 - 25.8 + 7.7)}$$

$$= 67.55 \text{ A}$$

From the O.C.C., corresponding to a field current of 67.6 A, the voltage $E_{fl} = 6000 \text{ V (line-to-line)}$

$$\text{Corresponding phase voltage } E_{fp} = \frac{6000}{\sqrt{3}} = 3464 \text{ V}$$

$$\therefore \text{ voltage regulation} = \frac{E_{fp} - V_p}{V_p} \times 100$$

$$= \frac{3464 - 3810.6}{3810.6} \times 100 = -9.1\%$$

c- 0.71 Lagging

$$E_{gp} = 3810.6 + 500 \angle -44.77 \times (0.075 + j1.039)$$

$$= 4218 \angle 4.7^\circ \text{ V}$$

The corresponding line voltage = $\sqrt{3} \times 4218 = 7304 \text{ V}$

From OCC the corresponding field current is 95A

$$I_f(\text{Total}) = \sqrt{I_{f1}^2 + I_{f2}^2 - 2I_{f1}I_{f2} \cos(90 - \phi + \gamma)}$$

$$= \sqrt{95^2 + 25^2 + 2 \times 95 \times 25 \times \cos(90 - 44.77 - 4.7)}$$

$$= 115 \text{ A}$$

7900V is corresponding to 115A from OCC.

$$V.R\% = \frac{7900 - 6600}{6600} \times 100 = 19.7\%$$

H.W . The table gives data for open-circuit and load zero power factor tests on a 6-pole, 440 V, 50 Hz, 3-phase star-connected alternator. The effective ohmic resistance between any two terminals of the armature is 0.3Ω .

Field current (A)	2	4	6	7	8	10	12	14	16	18
O.C.terminal voltage (V)	156	288	396	440	474	530	568	592	—	—
S.C.line current (A)	11	22	34	40	46	57	69	80		
Zero p.f. terminal voltage (V)	—	—	—	0	80	206	314	398	460	504

Find the regulation at full-load current of 40 A at 0.8 power factor lagging using

- synchronous impedance method,
- mmf method,
- Potier-triangle method