ADVANCED POWER SYSTEM ANALYSIS

Advanced power system analysis is a field of electrical engineering that deals with:

1. **Power System Overview and Linear Graph Theory**
2. **Modeling and Simulation**
3. **Load Flow Analysis**
4. **Fault Analysis**
5. **Power System Stability**
6. **Optimization**
7. **Economic Dispatch**
8. **Smart Grids**
9. **Voltage and Reactive Power Control**

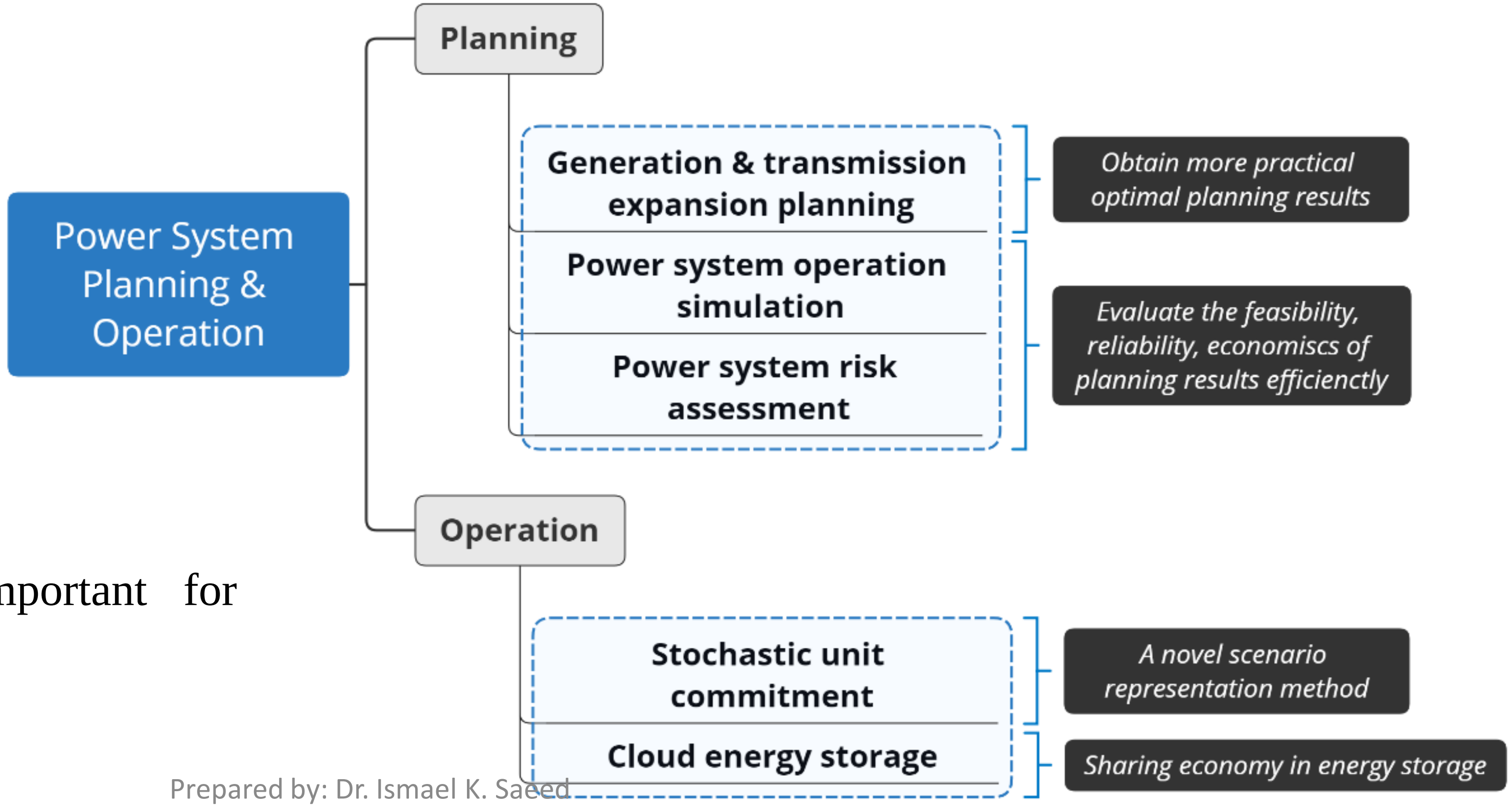
Unit One: Power system overview and linear graph theory

Need for system planning and operational studies

Planning and operation of power system operational planning covers the whole period ranging from the incremental stage of system development. The system operation engineers at various points like area, space, regional & national load dispatch of power. Power system planning and operational analysis covers the maintenance of generation, transmission and distribution facilities.

Steps:

- Planning of power system
- Implementation of the plans
- Monitoring system
- Compare plans with the results
- If no undesirable deviation occurs, then directly go to planning of system
- If undesirable deviation occurs then take corrective action and then go to planning of the system



The following analysis are very important for planning and operation of power system

(a). Load flow analysis

(b). Short circuit analysis

(c). Stability analysis

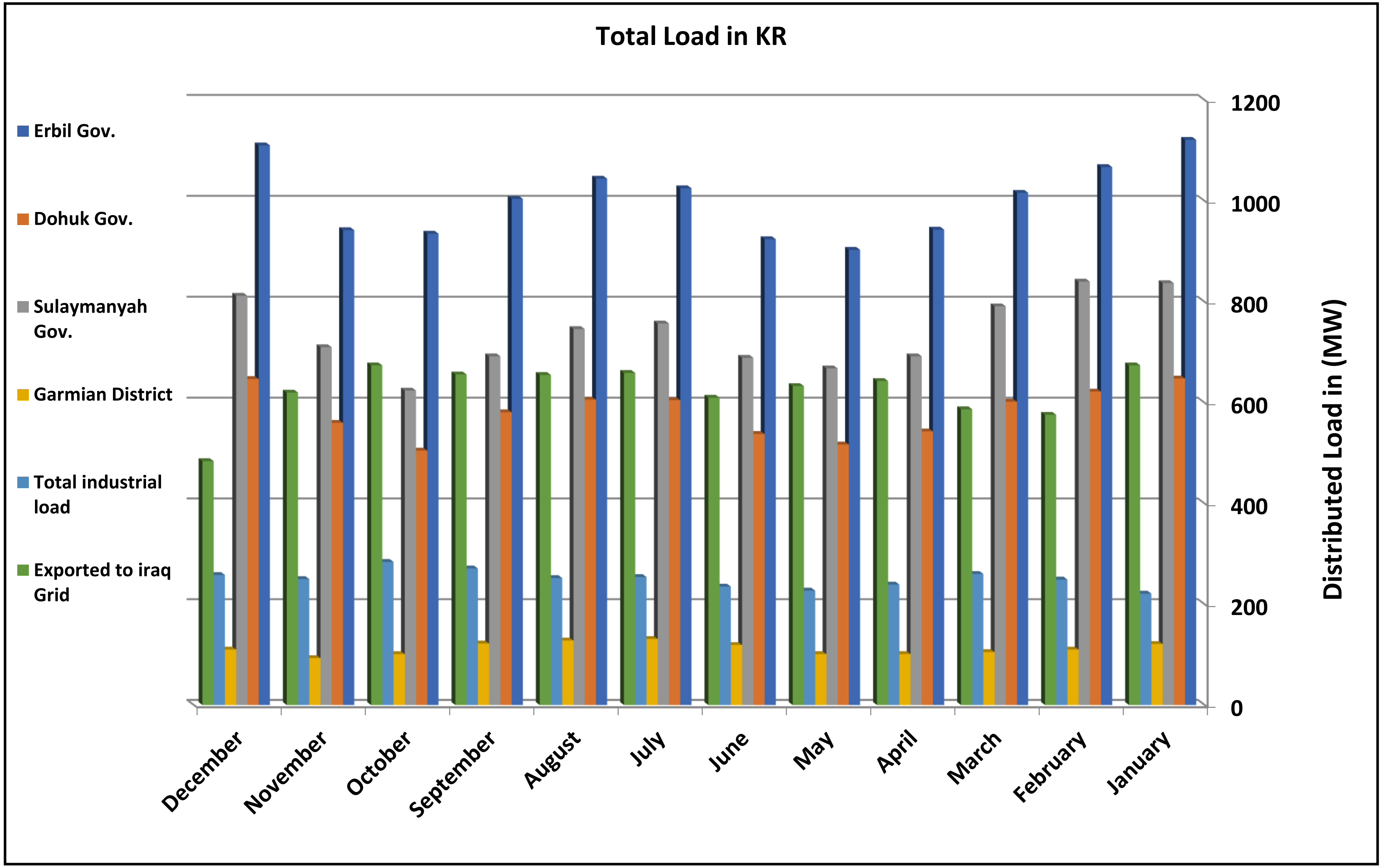
Power scenario in Kurdistan region

Electricity infrastructure in Kurdistan region has been developed by United Nations Development Program (UNDP) based on Electricity Network Rehabilitation Program (ENRP) during 1997 to 2003. During the period, many electricity generation facilities including small generation facilities for hospitals/public utilities have been constructed and transmission/distribution networks have been expanded.

Japan's Government has provided Official Development Assistant such as the gratis fund aid in 2005-2006 and the five-loan fund cooperation in 2007-2009 as well. It is greatly necessary to construct the transmission lines connecting to substations. The trend of electricity demand can be seen as; The peak load in the Kurdistan region in 2012 was 3,279MW while 1,171MW in 2005, which means the peak load became about three times in 7 years.

A 132 kV power system of KR network consists of 280 buses, 123 loads and 284 branches with total power generation of 3535.0513 MW and 3455.6566 MW as peak load for the month of July 2020. It still needs to have further more supports for the transmission and substation sector in Kurdistan region.

RMEK constructed 400kV transmission lines and substations as the power sector reconstruction projects in the Kurdistan region. New grid construction of the 400 kV electricity system network is conducive to the improvement of the tight electricity situation and power supply reliability, which results in regional social economic improvements and an enhanced quality of life.



Total distributed load in KRPS for the year of 2022

Prepared by: Dr. Ismael K. Saeed

INCIDENCE AND NETWORK MATRICES

The solution of a given linear network problem requires the formation of a set of equations describing the response of the network. The mathematical model so derived, must describe the characteristics of the individual network components, as well as the relationship which governs the interconnection of the individual components. In the bus frame of reference the variables are the node voltages and node currents.

The formulation of the network equations in different frames of reference requires the knowledge of graph theory. Elementary graph theory concepts are presented here, followed by development of network equations in the bus frame of reference.

Important Terms

The geometrical interconnection of the various branches of a network is called the ***topology*** of the network. The connection of the network topology shown by replacing all its elements by lines is called a ***graph***. **A *linear graph*** consists of a set of objects called ***nodes*** and another set called ***elements*** such that each element is identified with an ordered pair of nodes. An *element* is defined as any line segment of the graph irrespective of the characteristics of the components involved. A graph in which a direction is assigned to each element is called an ***oriented graph*** or a ***directed graph***.

It is to be noted that the directions of currents in various elements are arbitrarily assigned and the network equations are derived, consistent with the assigned directions. Elements are indicated by numbers and the nodes by encircled numbers.

The ground node is taken as the reference node. In electric networks the convention is to use associated directions for the voltage drops. This means the voltage drop in a branch is taken to be in the direction of the current through the branch. Hence, we need not mark the voltage polarities in the oriented graph.

Connected Graph : This is a graph where at least one path (disregarding orientation) exists between any two nodes of the graph. A representative power system and its oriented graph are as shown in Fig 1, with:

e = number of elements = 6

n = number of nodes = 4

b = number of branches = n-1 = 3

l = number of links = e-b = 3

Tree = T(1,2,3) and

Co-tree = T(4,5,6)

Sub-graph : sG is a sub-graph of G if the following conditions are satisfied:

- sG is itself a graph
- Every node of sG is also a node of G
- Every branch of sG is a branch of G

For eg., sG(1,2,3), sG(1,4,6), sG(2), sG(4,5,6), sG(3,4),.. are all valid sub-graphs of the oriented graph of Fig.1c.

Loop : A sub-graph L of a graph G is a loop if

- L is a connected sub-graph of G
- Precisely two and not more/less than two branches are incident on each node in L

In Fig 1c, the set{1,2,4} forms a loop, while the set{1,2,3,4,5} is not a valid, although the set(1,3,4,5) is a valid loop. The KVL (Kirchhoff's Voltage Law) for the loop is stated as follows: *In any lumped network, the algebraic sum of the branch voltages around any of the loops is zero.*

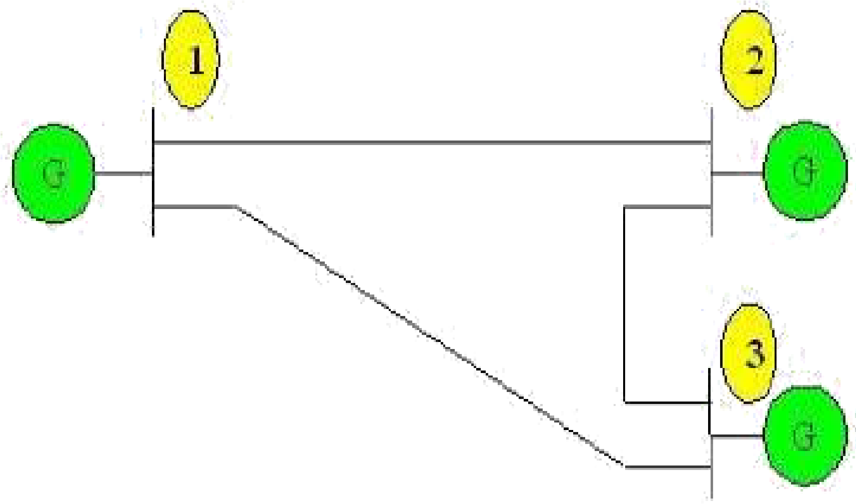


Fig 1a. Single line diagram of a power system

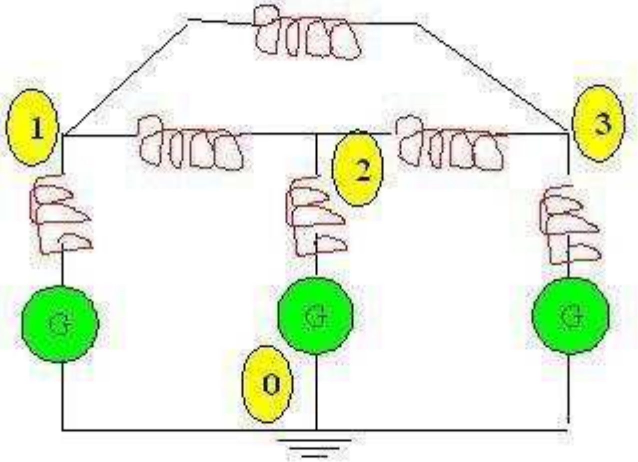


Fig 1b. Reactance diagram

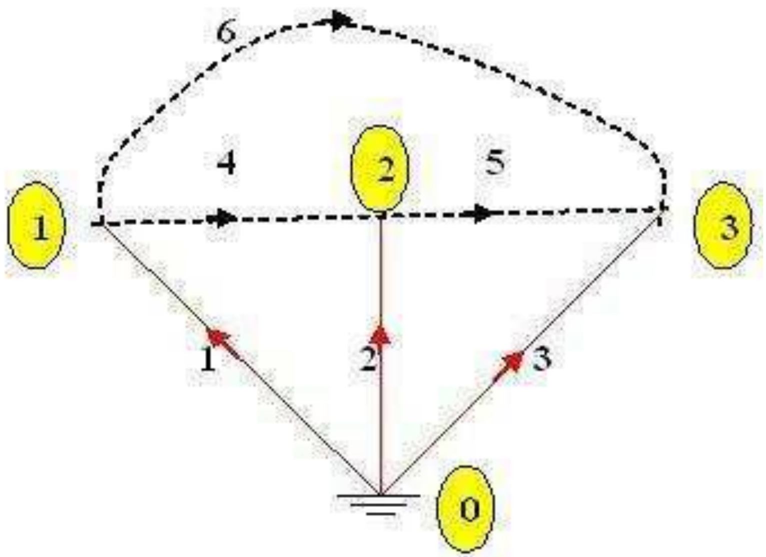


Fig 1c. Oriented Graph

Cutset : It is a set of branches of a connected graph G which satisfies the following conditions :

- The removal of all branches of the cutset causes the remaining graph to have two separate unconnected sub-graphs.
- The removal of all but one of the branches of the set, leaves the remaining graph connected.

Referring to Fig 1c, the set {3,5,6} constitutes a cutset since removal of them isolates node 3 from rest of the network, thus dividing the graph into two unconnected subgraphs. However, the set(2,4,6) is not a valid cutset! The KCL (Kirchhoff's Current Law) for the cutset is stated as follows: *In any lumped network, the algebraic sum of all the branch currents traversing through the given cutset branches is zero.*

Tree: It is a connected sub-graph containing all the nodes of the graph G, but without any closed paths (loops). There is one and only one path between every pair of nodes in a tree. The elements of the tree are called twigs or branches. In a graph with *n* nodes,

The number of branches: b = n-1 **(1)**

For the graph of Fig 1c, some of the possible trees could be T(1,2,3), T(1,4,6), T(2,4,5), T(2,5,6), etc.

Co-Tree : The set of branches of the original graph G, not included in the tree is called the *co-tree*. The co-tree could be connected or non-connected, closed or open.

The branches of the co-tree are called *links*. By convention, the tree elements are shown as solid lines while the co-tree elements are shown by dotted lines as shown in Fig.1c for tree T(1,2,3). With e as the total number of elements,

The number of links: $l = e - b = e - n + 1$ **(2)**

For the graph of Fig 1c, the co-tree graphs corresponding to the various tree graphs are as shown in the table below:

Tree	T(1,2,3)	T(1,4,6)	T(2,4,5)	T(2,5,6)
Co-Tree	T(4,5,6)	T(2,3,5)	T(1,3,6)	T(1,3,4)

Basic loops: When a link is added to a tree it forms a closed path or a loop. Addition of each subsequent link forms the corresponding loop. A loop containing only one link and remaining branches is called a *basic loop* or a *fundamental loop*. These loops are defined for a particular tree. Since each link is associated with a basic loop, the number of basic loops is equal to the number of links.

Basic cut-sets: Cut-sets which contain only one branch and remaining links are called *basic cutsets* or fundamental cut-sets. The basic cut-sets are defined for a particular tree. Since each branch is associated with a basic cut-set, the number of basic cut-sets is equal to the number of branches.

Examples on Basics of LG Theory:

Obtain the oriented graph for the system shown in Fig. 2. Select any four possible trees. For a selected tree show the basic loops and basic cut-sets.

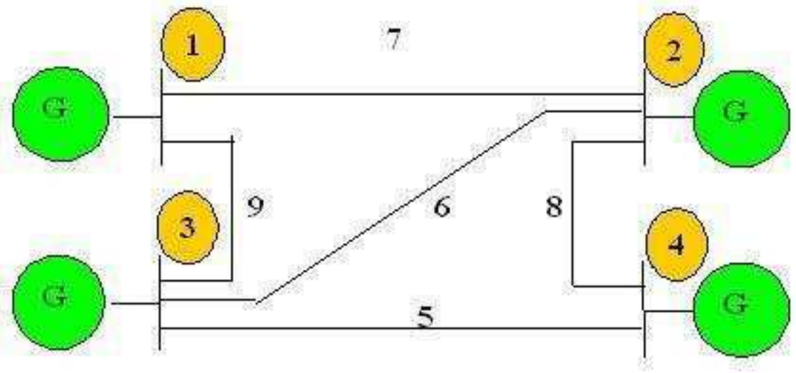


Fig. 2a. Single line diagram of Example System

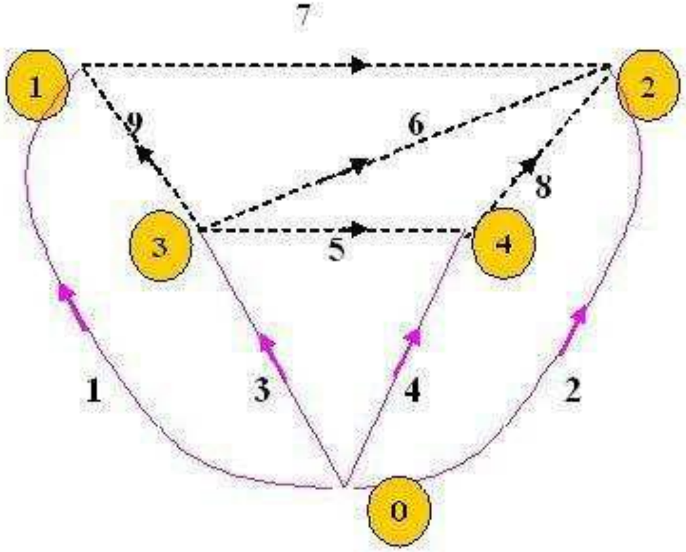


Fig. 2b. Oriented Graph of Fig. 2a.

For the system given, the oriented graph is as shown in figure 2b. some of the valid Tree graphs could be T(1,2,3,4), T(3,4,8,9), T(1,2,5,6), T(4,5,6,7), etc. The basic cut-sets (A,B,C,D) and basic loops (E,F,G,H,I) corresponding to the oriented graph of Fig.2a and tree, T(1,2,3,4) are as shown in Figure 2c and Fig.2d respectively.

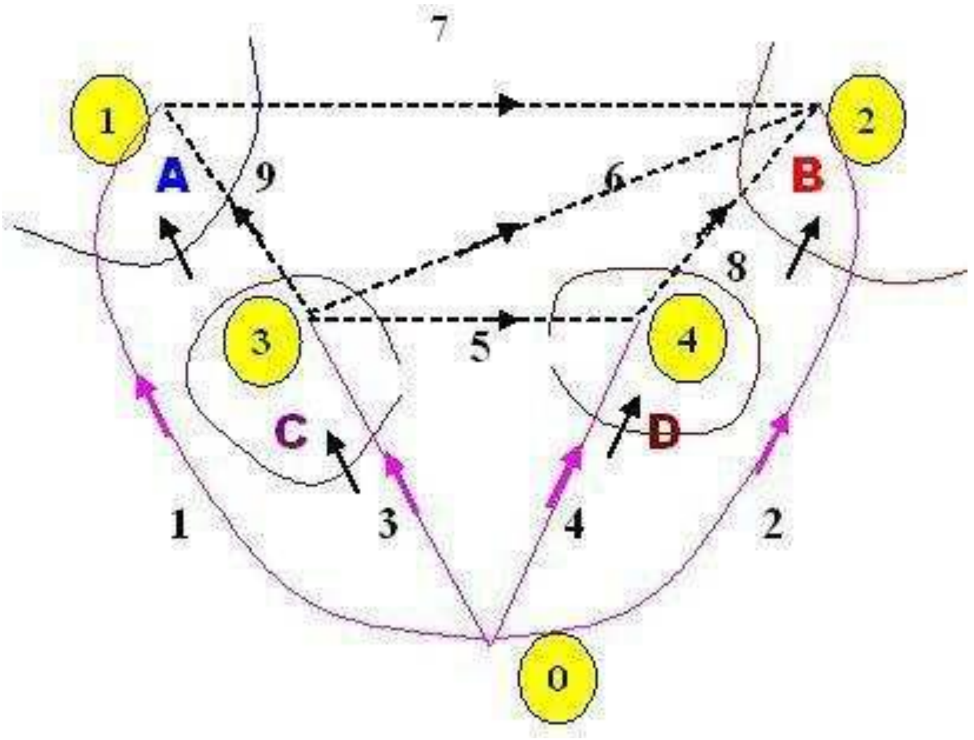


Fig. 2c. Basic Cutsets of Fig. 2a.

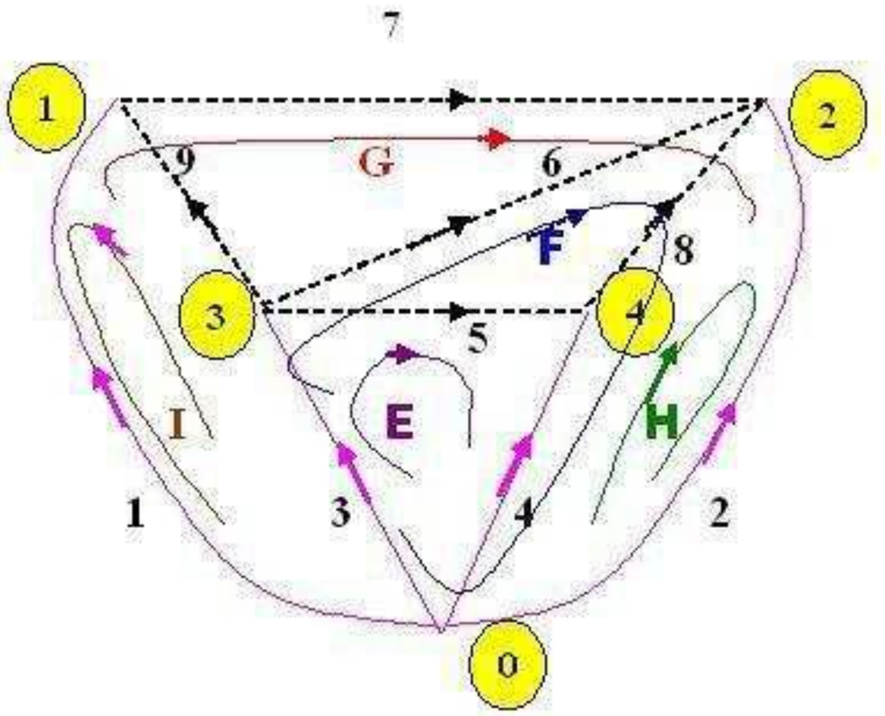


Fig. 2d. Basic Loops of Fig. 2a.

Incidence Matrices

Element–node incidence matrix:

The incidence of branches to nodes in a connected graph is given by the element-node incidence matrix, *A* . An element aij of *A* is defined as under:

a_{ij} = 1 if the branch -i is incident to and oriented away from the node-j.

= -1 if the branch-i is incident to and oriented towards the node-j.

= 0 if the branch-i is not at all incident on the node-j.

Thus the dimension of is $e \times n$, where e is the *A* number of elements and *n* is the number of nodes in the network. For example, consider again the sample system with its oriented graph as in fig. 1c. the corresponding element-node incidence matrix, is obtained as under:

Nodes / Elements	0	1	2	3
1	1	-1	0	0
2	1	0	-1	0
3	1	0	0	-1
4	0	1	-1	0
5	0	0	1	-1
6	0	1	0	-1

It is to be noted that the first column and first row are not part of the actual matrix and they only indicate the element number node number respectively as shown. Further, the sum of every row is found to be equal to zero always. Hence, the rank of the matrix is less than *n*. Thus in general, the matrix *A* satisfies the identity:

$$\sum_{j=1}^{n} a_{ij} = 0 \quad \forall\ i = 1,2,\ldots..e.$$

Bus incidence matrix: A

By selecting any one of the nodes of the connected graph as the reference node, the corresponding column is deleted from *A* to obtain the bus incidence matrix, A. The dimensions of A are e × *(n-1)* and the rank is *n-1*. In the above example, selecting node-0 as reference node, the matrix A is obtained by deleting the column corresponding to node-0, as under:

A=

Elements \ Buses	1	2	3
1	-1		
2		-1	
3			-1
4	1	-1	
5		1	-1
6	1		-1

=

A_b	Branches
A_l	Links

It may be observed that for a selected tree, say, T(1,2,3), the bus incidence matrix can be so arranged that the branch elements occupy the top portion of the A-matrix followed by the link elements. Then, the matrix-A can be partitioned into two sub
matrices A_b and A_l as shown, where,

(i) A_b is of dimension (bxb) corresponding to the branches and
(ii) A_l is of dimension (lxb) corresponding to links.
A is a rectangular matrix, hence it is singular. Ab is a non-singular square matrix of dimension-b. Since A gives the incidence of various elements on the nodes with their direction of incidence, the KCL for the nodes can be written as

$$A^T i = 0 \qquad \textbf{(4)}$$

where A^T is the transpose of matrix A and i is the vector of branch currents. Similarly for the branch voltages we can write,

$$v = A\, E_{bus} \qquad \textbf{(5)}$$

PRIMITIVE NETWORKS

The complete behaviour of the network can be obtained from the knowledge of the behaviour of the individual elements which make the network, along with the incidence matrices. An element in an electrical network is completely characterized by the relationship between the current through the element and the voltage across it.

General representation of a network element: In general, a network element may contain active or passive components. Figure 3 represents the alternative impedance and admittance forms of representation of a general network component.

The network performance can be represented by using either the impedance or the admittance form of representation. With respect to the element, p-q, let,

v_{pq} = voltage across the element p-q,

e_{pq} = source voltage in series with the element pq,

i_{pq}= current through the element p-q,

j_{pq}= source current in shunt with the element pq,

z_{pq}= self impedance of the element p-q and

y_{pq}= self admittance of the element p-q.

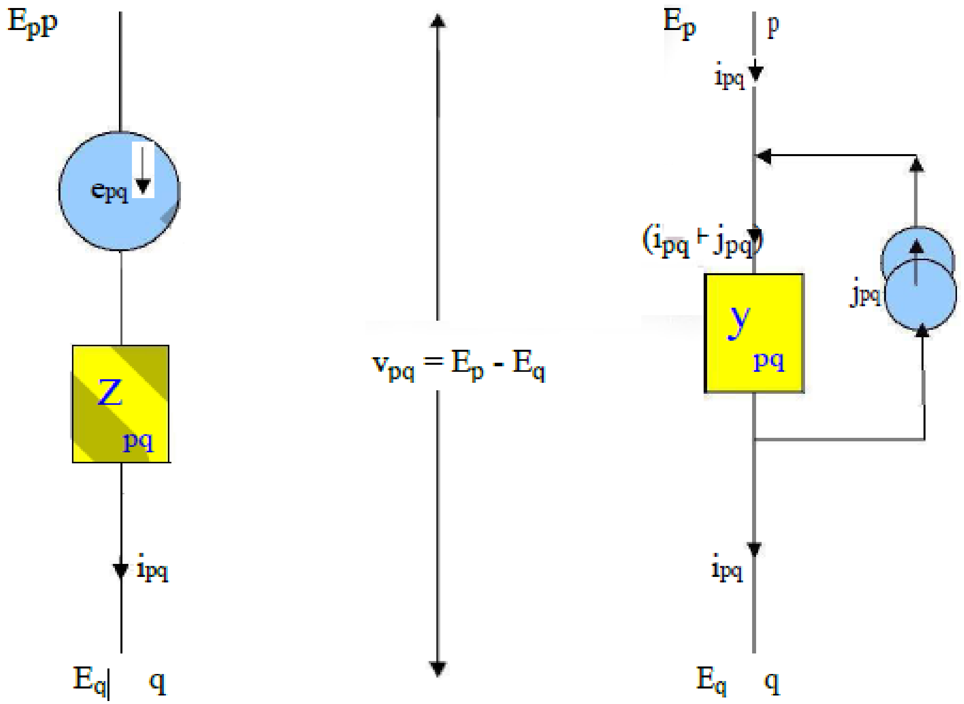


Fig.3 Representation of a primitive network element (a) Impedance form (b) Admittance form

Performance equation: Each element p-q has two variables, vpq and ipq. The performance of the given element p-q can be expressed by the performance equations as under:

$v_{pq} + e_{pq} = z_{pq} i_{pq}$ (in its impedance form)

$i_{pq} + j_{pq} = y_{pq} v_{pq}$ (in its admittance form) **(6)**

Thus the parallel source current j_{pq} in admittance form can be related to the series source voltage, e_{pq} in impedance form as per the identity:

$j_{pq} = - y_{pq} e_{pq}$ **(7)**

A set of non-connected elements of a given system is defined as a ***primitive Network*** *and a*n element in it is a fundamental element that is not connected to any other element.

In the equations above, if the variables and parameters are replaced by the corresponding vectors and matrices, referring to the complete set of elements present in a given system, then, we get the performance equations of the primitive network in the form as under:

$v + e = [z]\, i$

$i + j = [y]\, v$ **(8)**

Primitive network matrices:

A diagonal element in the matrices, [z] or [y] is the self impedance $z_{pq\text{-}pq}$ or self admittance $y_{pq\text{-}pq}$. An off-diagonal element is the mutual impedance $z_{pq\text{-}rs}$ or mutual admittance $y_{pq\text{-}rs}$, the value present as a mutual coupling between the elements p-q and r-s. The primitive network admittance matrix [y] can be obtained also by inverting the primitive impedance matrix [z]. Further, if there are no mutually coupled elements in the given system, then both the matrices, [z] and [y] are diagonal. In such cases, the self impedances are just equal to the reciprocal of the corresponding values of self admittances, and vice-versa.

Examples on Primitive Networks:

Given that the self impedances of the elements of a network referred by the bus incidence matrix given below are equal to: $Z_1=Z_2=0.2$, $Z_3=0.25$, $Z_4=Z_5=0.1$ and $Z_6=0.4$ units, draw the corresponding oriented graph, and find the primitive network matrices. Neglect mutual values between the elements.

A=

Elements \ Buses	1	2	3
1	-1	0	0
2	0	-1	0
3	0	0	-1
4	1	-1	0
5	0	1	-1
6	1	0	-1

Solution:

The element node incidence matrix, can be A obtained from the given A matrix, by pre-augmenting to it an extra column corresponding to the reference node, as under.

$$\hat{A} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

Based on the conventional definitions of the elements of A , the oriented graph can be formed as under:

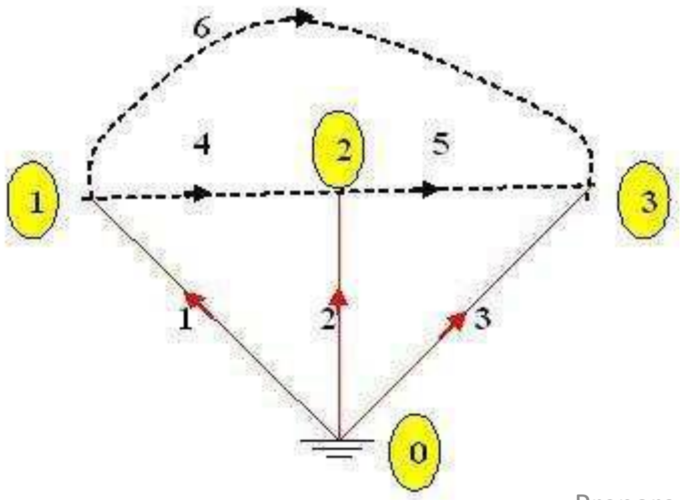


Fig. 4 Oriented Graph

Thus the primitive network matrices are square, symmetric and diagonal matrices of order e=no. of elements = 6. They are obtained as follows.

[z]=

0.2	0	0	0	0	0
0	0.2	0	0	0	0
0	0	0.25	0	0	0
0	0	0	0.1	0	0
0	0	0	0	0.1	0
0	0	0	0	0	0.4

and

[y]=

5	0	0	0	0	0
0	5	0	0	0	0
0	0	4	0	0	0
0	0	0	10	0	0
0	0	0	0	10	0
0	0	0	0	0	2.5

FORMATION OF Y_{BUS} AND Z_{BUS}

The bus admittance matrix, Y_{BUS} plays a very important role in computer aided power system analysis. It can be formed in practice by either of the methods as under:

1. Rule of Inspection
2. Singular Transformation
3. Non-Singular Transformation
4. Z_{BUS} Building Algorithms, etc.

The performance equations of a given power system can be considered in three different frames of reference as discussed below:

Frames of Reference:

Bus Frame of Reference: There are b independent equations (b = no. of buses) relating the bus vectors of currents and voltages through the bus impedance matrix and bus admittance matrix:

$E_{BUS} = Z_{BUS} I_{BUS}$

$I_{BUS} = Y_{BUS} E_{BUS}$ **(9)**

Branch Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$E_{BR} = Z_{BR} I_{BR}$

$I_{BR} = Y_{BR} E_{BR}$ **(10)**

Loop Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$E_{LOOP} = Z_{LOOP} I_{LOOP}$

$I_{LOOP} = Y_{LOOP} E_{LOOP}$ **(11)**

Of the various network matrices referred above, the bus admittance matrix (Y_{BUS}) and the bus impedance matrix (Z_{BUS}) are determined for a given power system by the rule of inspection as explained next.

Rule of Inspection

Consider the 3-node admittance network as shown in figure5. Using the basic branch relation: I = YV, for all the elemental currents and applying Kirchhoff's Current Law principle at the nodal points, we get the relations as under:

$$\text{At node 1: } I_1 = Y_1V_1 + Y_3 (V_1 - V_3) + Y_6 (V_1 - V_2)$$
$$\text{At node 2: } I_2 = Y_2V_2 + Y_5 (V_2 - V_3) + Y_6 (V_2 - V_1)$$
$$\text{At node 3: } 0 = Y_3 (V_3 - V_1) + Y_4V_3 + Y_5 (V_3 - V_2) \qquad \textbf{(12)}$$

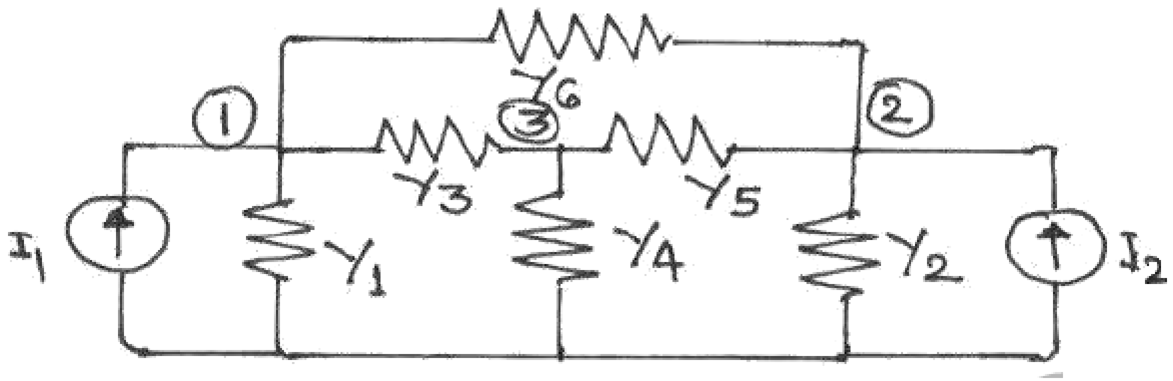


Fig. 5 Example System for finding YBUS

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{vmatrix} (Y_1+Y_3+Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix} \qquad \textbf{(13)}$$

In other words, the relation of equation (9) can be represented in the form

$$[I_{BUS}] = [Y_{BUS}][E_{BUS}] \quad \textbf{(14)}$$

Where, Y_{BUS} is the bus admittance matrix, I_{BUS} & E_{BUS} are the bus current and bus voltage vectors respectively.

By observing the elements of the bus admittance matrix, Y_{BUS} of equation (13), it is observed that the matrix elements can as well be obtained by a simple inspection of the given system diagram:

Diagonal elements: A diagonal element (Y_{ii}) of the bus admittance matrix,
Y_{BUS}, is equal to the sum total of the admittance values of all the elements incident at the bus/node i,

Off Diagonal elements**:** An off-diagonal element (Y_{ij}) of the bus admittance matrix, Y_{BUS}, is equal to the negative of the admittance value of the connecting element present between the buses i and j, if any.

This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

$$Y_{ii} = \Sigma\, y_{ij} \quad (j = 1,2,\ldots\ldots n)$$

$$Y_{ij} = -\,y_{ij} \quad (j = 1,2,\ldots\ldots n) \quad \textbf{(15)}$$

For i = 1,2,….n, n = no. of buses of the given system, y_{ij} is the admittance of element connected between buses i and j and y_{ii} is the admittance of element connected between bus i and ground (reference bus).

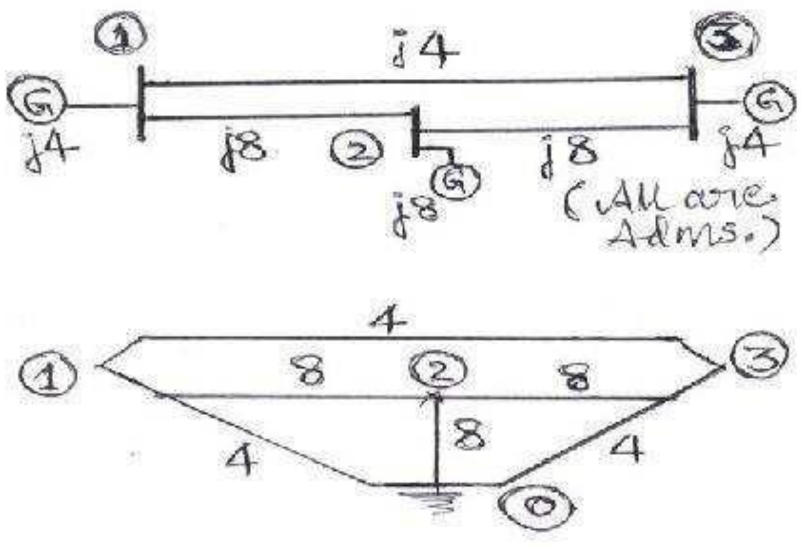
Bus impedance matrix

In cases where, the bus impedance matrix is also required, it cannot be formed by direct inspection of the given system diagram. However, the bus admittance matrix determined by the rule of inspection following the steps explained above, can be inverted to obtain the bus impedance matrix, since the two matrices are inter-invertible.

Note: It is to be noted that the rule of inspection can be applied only to those power systems that do not have any mutually coupled elements.

Examples on Rule of Inspection:

Obtain the bus admittance matrix for the admittance network shown aside by the rule of inspection



Singular Transformations

The primitive network matrices are the most basic matrices and depend purely on the impedance or admittance of the individual elements. However, they do not contain any information about the behaviour of the interconnected network variables. Hence, it is necessary to transform the primitive matrices into more meaningful matrices which can relate variables of the interconnected network.

Bus admittance matrix, Y_{BUS} and Bus impedance matrix, Z_{BUS}

In the bus frame of reference, the performance of the interconnected network is described by *n* independent nodal equations, where *n* is the total number of buses (n+1 nodes are present, out of which one of them is designated as the reference node). For example a 5-bus system will have 5 external buses and 1 ground/ ref. bus). The performance equation relating the bus voltages to bus current injections in bus frame of reference in admittance form is given by

$I_{BUS} = Y_{BUS} E_{BUS}$

$$Y_{BUS} = A^t [y] A \qquad (17)$$

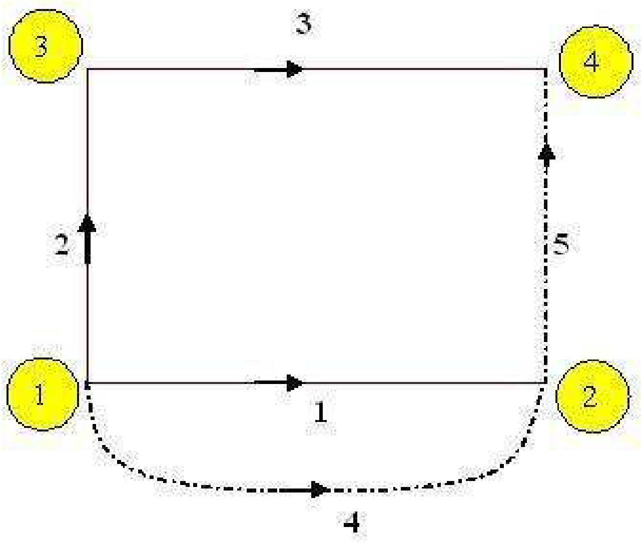
The bus incidence matrix is rectangular and hence singular. Hence, (16) gives a singular transformation of the primitive admittance matrix [y]. The bus impedance matrix is given by ,

$$Z_{BUS} = Y_{BUS}^{-1} \qquad (18)$$

Example on Singular Transformation:

For the network of Figure, form the primitive matrices [z] & [y] and obtain the bus admittance matrix by singular transformation. Choose a Tree T(1,2,3). The data is given in Table.

Elements	Self impedance	Mutual impedance
1	j 0.6	-
2	j 0.5	j 0.1(with element 1)
3	j 0.5	-
4	j 0.4	j 0.2(with element 1)
5	j 0.2	-



Solution:

The bus incidence matrix is formed taking node 1 as the reference bus.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

The primitive incidence matrix is given by,

$$[z] = \begin{bmatrix} j0.6 & j0.1 & 0.0 & j0.2 & 0.0 \\ j0.1 & j0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & j0.5 & 0.0 & 0.0 \\ j0.2 & 0.0 & 0.0 & j0.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & j0.2 \end{bmatrix}$$

The primitive admittance matrix $[y] = [z]^{-1}$ and given by,

$$[y]=\begin{bmatrix} -j2.0833 & j0.4167 & 0.0 & j1.0417 & 0.0 \\ j0.4167 & -j2.0833 & 0.0 & -j0.2083 & 0.0 \\ 0.0 & 0.0 & -j2.0 & 0.0 & 0.0 \\ j1.0417 & -j0.2083 & 0.0 & -j3.0208 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -j5.0 \end{bmatrix}$$

The bus admittance matrix by singular transformation is obtained as

$$Y_{BUS} = A^t [y] A = \begin{bmatrix} -j8.0208 & j0.2083 & j5.0 \\ j0.2083 & -j4.0833 & j2.0 \\ j5.0 & j2.0 & -j7.0 \end{bmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1} = \begin{bmatrix} j0.2713 & j0.1264 & j0.2299 \\ j0.1264 & j0.3437 & j0.1885 \\ j0.2299 & j0.1885 & j0.3609 \end{bmatrix}$$

Node Elimination by Matrix Algebra

Nodes can be eliminated by the matrix manipulation of the standard node equations. However, *only those nodes at which current does not enter or leave the network can be considered for such elimination*. Such nodes can be eliminated either in one group or by taking the eligible nodes one after the other for elimination.

CASE-A: Simultaneous Elimination of Nodes:

Consider the performance equation of the given network in bus frame of reference in admittance form for a n-bus system, given by:

$$\mathbf{I}_{BUS} = \mathbf{Y}_{BUS}\,\mathbf{E}_{BUS} \qquad \textbf{(19)}$$

Now, of the n buses present in the system, let p buses be considered for node elimination so that the reduced system after elimination of p nodes would be retained with m (= n-p) nodes only. Hence the corresponding performance equation would be similar to (19), except that the coefficient matrix would be of order m now, i.e.,

$$\mathbf{I}_{BUS} = \mathbf{Y}_{BUS}^{new}\,\mathbf{E}_{BUS} \qquad \textbf{(20)}$$

Where $\mathbf{Y}_{BUS}^{new}$ is the bus admittance matrix of the reduced network and the vectors I_{BUS} and E_{BUS} are of order m. It is assumed in (19) that I_{BUS} and E_{BUS} are obtained with their elements arranged such that the elements associated with p nodes to be eliminated are in the lower portion of the vectors. Then the elements of Y_{BUS} also get located accordingly so that (19) after matrix partitioning yields,

$$\begin{vmatrix} I_{BUS-m} \\ I_{BUS-p} \end{vmatrix} = \begin{array}{c} \\ m \\ p \end{array}\!\!\begin{vmatrix} \overset{m}{Y_A} & \overset{p}{Y_B} \\ Y_C & Y_D \end{vmatrix} \begin{vmatrix} E_{BUS-m} \\ E_{BUS-p} \end{vmatrix} \qquad (21)$$

Where the self and mutual values of Y_A and Y_D are those identified only with the nodes to be retained and removed respectively and $Y_C=Y_B$ is composed of only the corresponding mutual admittance values, that are common to the nodes m and p.

Now, for the p nodes to be eliminated, it is necessary that, each element of the vector I_{BUS-p} should be zero. Thus we have from (21):

$$I_{BUS-m} = Y_A E_{BUS-m} + Y_B E_{BUS-p}$$
$$I_{BUS-p} = Y_C E_{BUS-m} + Y_D E_{BUS-p} = 0 \qquad (22)$$

Solving, $$E_{BUS-p} = -Y_D^{-1} Y_C E_{BUS-m} \qquad (23)$$

Thus, by simplification, we obtain an expression similar to (20) as,

$$I_{BUS-m} = \{Y_A - Y_B Y_D^{-1} Y_C\} E_{BUS-m} \qquad (24)$$

Thus by comparing (20) and (24), we get an expression for the new bus admittance matrix in terms of the sub-matrices of the original bus admittance matrix as:

$$Y_{BUS}^{new} = \{Y_A - Y_B Y_D^{-1} Y_C\} \qquad \textbf{(25)}$$

This expression enables us to construct the given network with only the necessary nodes retained and all the unwanted nodes/buses eliminated. However, it can be observed from (25) that the expression involves finding the inverse of the sub-matrix Y_D (of order p). This would be computationally very tedious if p, the nodes to be eliminated is very large, especially for real practical systems. In such cases, it is more advantageous to eliminate the unwanted nodes from the given network by considering one node only at a time for elimination, as discussed next.

CASE-B: Separate Elimination of Nodes:

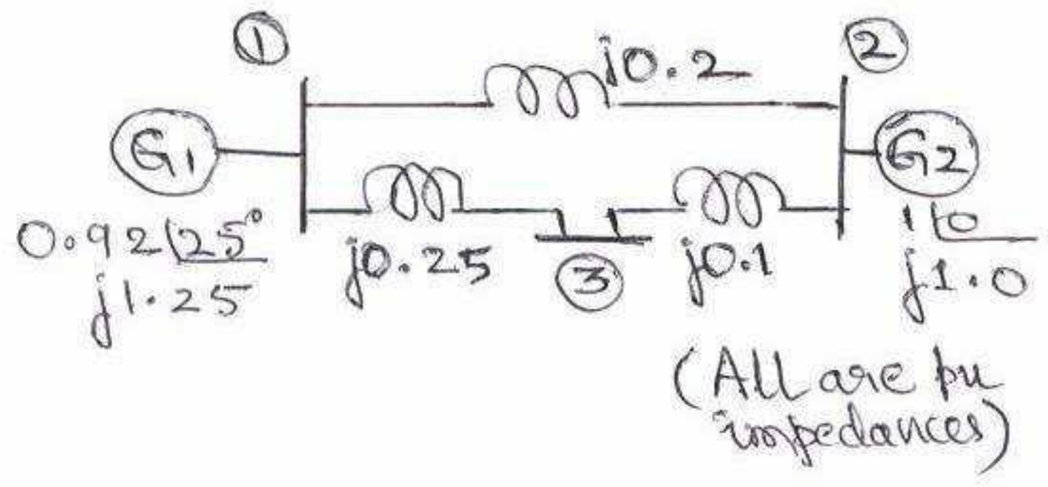
Here again, the system buses are to be renumbered, if necessary, such that the node to be removed always happens to be the last numbered one. The sub-matrix Y_D then would be a single element matrix and hence it inverse would be just equal to its own reciprocal value. Thus the generalized algorithmic equation for finding the elements of the new bus admittance matrix can be obtained from (24) as,

$$Y_{ij}^{new} = Y_{ij}^{old} - Y_{in} Y_{nj} / Y_{nn} \; \forall \; i,j = 1,2,\ldots\ldots n. \qquad \textbf{(26)}$$

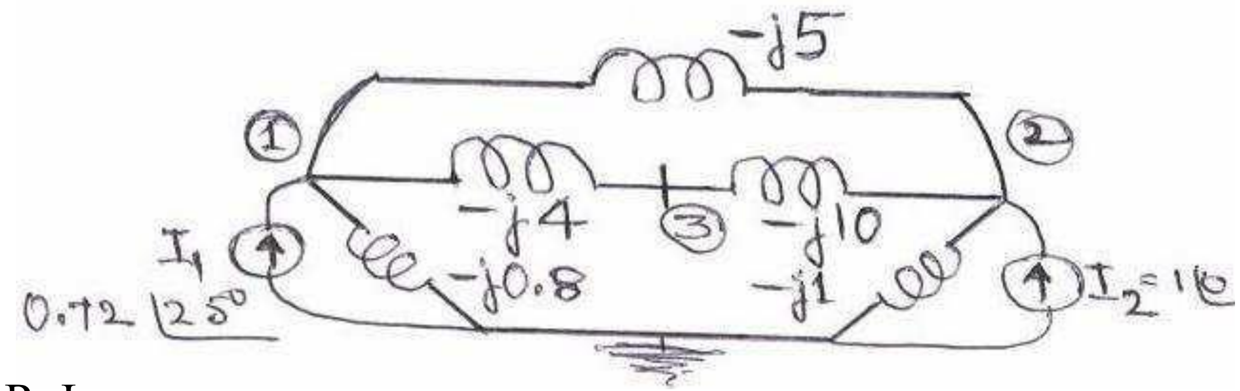
Each element of the original matrix must therefore be modified as per (25). Further, this procedure of eliminating the last numbered node from the given system of n nodes is to be iteratively repeated p times, so as to eliminate all the unnecessary p nodes from the original system.

Examples on Node elimination:

Obtain Y_{BUS} for the impedance network shown below by the rule of inspection. Also, determine Y_{BUS} for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.



The admittance equivalent network is as follows:



The bus admittance matrix is obtained by RoI as:

$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$

The reduced matrix after elimination of node 3 from the given system is determined as per the equation:

$$Y_{BUS}^{new} = \{Y_A - Y_B Y_D^{-1} Y_C\}$$

$Y_{BUS}^{new} =$

n/n	1	2
1	-j8.66	j7.86
2	j7.86	-j8.66

Alternatively,

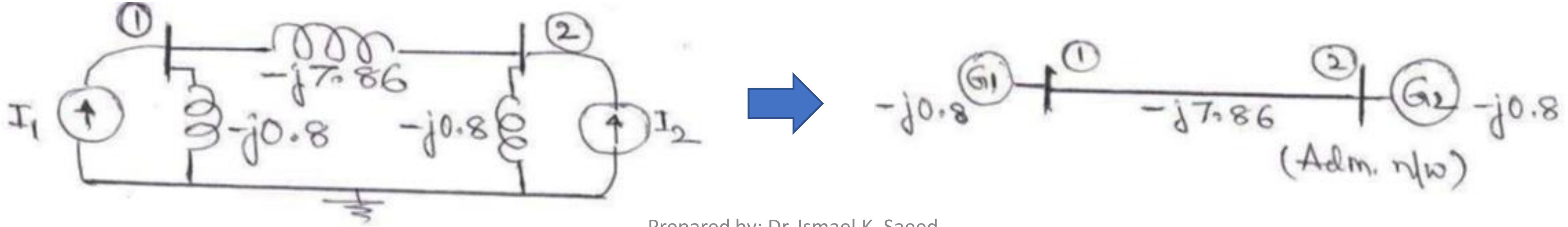
$$Y_{ij}^{new} = Y_{ij}^{old} - Y_{in} Y_{nj} / Y_{nn} \; \forall \; i,j = 1,2,\ldots\ldots n.$$

$$Y_{11} = Y_{11} - Y_{13} Y_{31} / Y_{33} = -j8.66$$

$$Y_{22} = Y_{22} - Y_{23} Y_{32} / Y_{33} = -j8.66$$

$$Y_{12} = Y_{21} = Y_{12} - Y_{13} Y_{32} / Y_{33} = j7.86$$

Thus the reduced network can be obtained again by the rule of inspection as shown be low.



Z_{BUS} Building Algorithms

Formation of Bus Impedance Matrix

The bus impedance matrix is the inverse of the bus admittance matrix. An alternative method is possible, based on an algorithm to form the bus impedance matrix directly from system parameters and the coded bus numbers. The bus impedance matrix is formed adding one element at a time to a partial network of the given system. The performance equation of the network in bus frame of reference in impedance form using the currents as independent variables is given in matrix form by

$$\bar{\mathbf{E}}_{\mathbf{BUS}} = [\,\mathbf{Z}_{\mathbf{BUS}}]\,\bar{\mathbf{I}}_{\mathbf{BUS}} \qquad \mathbf{(27)}$$

When expanded so as to refer to a *n* bus system, (27) will be of the form

$$\begin{aligned} E_1 &= Z_{11} I_1 + Z_{12} I_2 + \ldots\ldots + Z_{1k} I_k \ldots + Z_{1n} I_n \\ E_k &= Z_{k1} I_1 + Z_{k2} I_2 + \ldots\ldots + Z_{kk} I_k + \ldots + Z_{kn} I_n \\ E_n &= Z_{n1} I_1 + Z_{n2} I_2 + \ldots\ldots + Z_{nk} I_k + \ldots + Z_{nn} I_n \end{aligned} \qquad \mathbf{(28)}$$

Now assume that the bus impedance matrix Z_{bus} is known for a partial network of *m* buses and a known reference bus. Thus, Z_{bus} of the partial network is of dimension $m \times m$. If now a new element is added between buses *p* and *q* we have the following two possibilities:

(i) *p* is an existing bus in the partial network and *q* is a new bus; in this case *p-q* is a **branch** added to the p-network as shown in Fig a, and

(ii) both *p* and *q* are buses existing in the partial network; in this case *p-q* is a **link** added to the p-network as shown in Fig b.

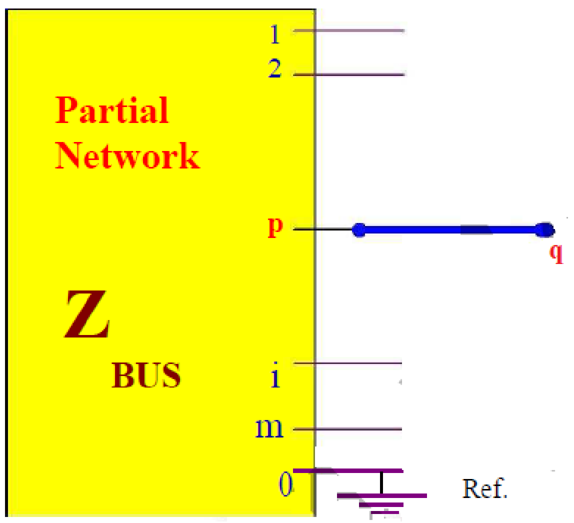


Fig a. Addition of branch p-q

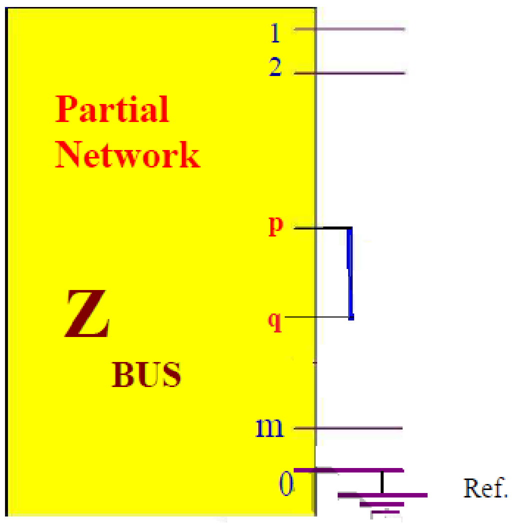


Fig b. Addition of link p-q

If the added element ia a branch, p-q, then the new bus impedance matrix would be of order m+1, and the analysis is confined to finding only the elements of the new row and column (corresponding to bus-q) introduced into the original matrix. If the added element ia a link, p-q, then the new bus impedance matrix will remain unaltered with regard to its order. However, all the elements of the original matrix are updated to take account of the effect of the link added.

Addition of a Branch

Consider now the performance equation of the network in impedance form with the added branch p-q, given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \\ E_p \\ \\ E_m \\ \\ E_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{1P} & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & Z_{2P} & Z_{2m} & Z_{2q} \\ \\ Z_{P1} & Z_{P2} & Z_{PP} & Z_{pm} & Z_{pq} \\ \\ Z_{m1} & Z_{m2} & Z_{mp} & Z_{mm} & Z_{mq} \\ \\ Z_{q1} & Z_{q2} & Z_{qp} & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \\ I_p \\ \\ I_m \\ \\ I_q \end{bmatrix} \qquad \textbf{(29)}$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have **Vector $y_{pq\text{-}rs}$ is not equal to zero** and **$Z_{ij} = Z_{ji}$** $\forall$ i,j=1,2,…m,q

To find Z_{qi}:

The elements of last row-q and last column-q are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig. Since all other bus currents are zero, we have from (29) that

$E_k = Z_{ki} I_i = Z_{ki}$ $\forall$ k = 1, 2,…i…....p,….m, q **(30)**

Hence, $E_q = Z_{qi}$; $E_p = Z_{pi}$ ………

Also, $E_q = E_p - v_{pq}$; so that $Z_{qi} = Z_{pi} - v_{pq}$ $\forall$ i =1, 2,…i…....p,….m, $\neq$q **(31)**

To find v_{pq}:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pq} \\ i_{rs} \end{bmatrix} = \begin{bmatrix} y_{pq,pq} & y_{pq,rs} \\ y_{rs,pq} & y_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pq} \\ v_{rs} \end{bmatrix} \qquad (32)$$

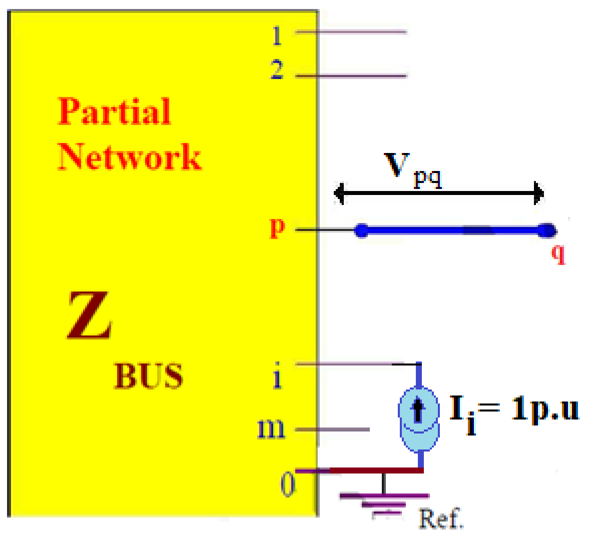


Fig. Calculation for Z_{qi}

where i_{pq} is current through element *p-q*

i_{rs} is vector of currents through elements of the partial network

v_{pq} is voltage across element *p-q*

$y_{pq,pq}$ is self – admittance of the added element

$y_{pq,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

v_{rs} is vector of voltage across elements of partial network.

$y_{rs,pq}$ is transpose of $y_{pq,rs}$.

$y_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-q, is zero, $i_{pq} = 0$. We thus have

$$i_{pq} = y_{pq,pq}\, v_{pq} + y_{pq,rs}\, v_{rs} = 0$$

$$v_{pq} = \frac{-y_{pq,rs}}{y_{pq,pq}} v_{rs}$$

$$v_{pq} = \frac{-y_{pq,rs}}{y_{pq,pq}} (E_r - E_s) \qquad \textbf{(33)}$$

Using (30) and (33) in (31), we get

$$Z_{qi} = Z_{qi} + \frac{-y_{pq,rs}}{y_{pq,pq}} (Z_{ri} - Z_{si}) \qquad i = 1,2......m;\ i \neq q \qquad \textbf{(34)}$$

To find Z_{qq}

The element Z_{qq} can be computed by injecting a current of 1pu at bus-q, I_q = 1.0 pu. As before, we have the relations as under:

$$E_k = Z_{kq} I_q = Z_{kq} \qquad \forall\ k = 1,2,\ldots i\ldots\ldots p,\ldots m,\ q \qquad \textbf{(35)}$$

Hence, $E_q = Z_{qq}$; $E_p = Z_{pq}$; Also, $E_q = E_p - v_{pq}$; so that $Z_{qq} = Z_{pq} - v_{pq}$ **(36)**

Since now the current in the added element is $i_{pq} = -I_q = -1.0$, we have

$$i_{pq} = y_{pq,pq}\, v_{pq} + y_{pq,rs}\, v_{rs} = -1$$

$$v_{pq} = -1 + \frac{-y_{pq,rs}}{y_{pq,pq}} v_{rs}$$

$$v_{pq} = -1 + \frac{-y_{pq,rs}}{y_{pq,pq}} (E_r - E_s) \quad \textbf{(37)}$$

Using (35) and (37) in (36), we get

$$Z_{qq} = Z_{pq} + [1 + y_{pq,rs} (Z_{rq} - Z_{sq})]/y_{pq,pq} \quad \textbf{(38)}$$

Special Cases

The following special cases of analysis concerning Z_{BUS} building can be considered with respect to the addition of branch to a p-network.

Case (a): If there is no mutual coupling then elements of $y_{pq,rs}$ are zero. Further, if p is the reference node, then E_p=0. thus,

$Z_{pi} = 0 \quad i = 1,2......m : i \neq q$

And $Z_{pq} = 0.$

Hence, from (34) (38) $Z_{qi} = 0 \quad i = 1,2.......m; i \neq q$

And $Z_{qq} = z_{pq,pq}$

Case (b): If there is no mutual coupling and if p is not the ref. bus, then, from (34) and (38), we again have,

$$Z_{qi} = Z_{pi} \qquad i = 1,2.......m; i \neq q$$

$$Z_{qq} = Z_{pq} + z_{pq,pq} \quad \textbf{(39)}$$

Addition of a Link

Consider now the performance equation of the network in impedance form with the added link p-l, (p-l being a fictitious branch and l being a fictitious node) given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \\ E_p \\ \\ E_m \\ \\ E_l \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{1P} & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & Z_{2P} & Z_{2m} & Z_{2q} \\ \\ Z_{P1} & Z_{P2} & Z_{PP} & Z_{pm} & Z_{pq} \\ \\ Z_{m1} & Z_{m2} & Z_{mp} & Z_{mm} & Z_{mq} \\ \\ Z_{l1} & Z_{l2} & Z_{lp} & Z_{lm} & Z_{ll} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \\ I_p \\ \\ I_m \\ \\ I_l \end{bmatrix} \qquad \textbf{(40)}$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have **Vector $y_{pq\text{-}rs}$ is not equal to zero** and **$Z_{ij} = Z_{ji}$** $\forall$ i,j=1,2,…m,l.

To find Z_{li}:

The elements of last row-*l* and last column-*l* are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Figure. Further, the current in the added element is made zero by connecting a voltage source, e_l in series with element p-q, as shown. Since all other bus currents are zero, we have from (40) that

$$E_k = Z_{ki} I_i = Z_{ki} \qquad \forall\ k = 1, 2, \ldots i \ldots\ldots p, \ldots m, l \qquad \textbf{(41)}$$

Hence, $e_l = E_l = Z_{li}$; $E_p = Z_{pi}$; $E_q = Z_{qi}$ ………

Also, $e_l = E_p - E_q - v_{pq}$;

So that $Z_{li} = Z_{pi} - Z_{qi} - v_{pq}$ $\forall$ i=1,2,…i….p,...q,….m, ≠l **(42)**

To find v_{pq}:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pl} \\ i_{rs} \end{bmatrix} = \begin{bmatrix} y_{pl,pl} & y_{pl,rs} \\ y_{rs,pl} & y_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pl} \\ v_{rs} \end{bmatrix} \quad \textbf{(43)}$$

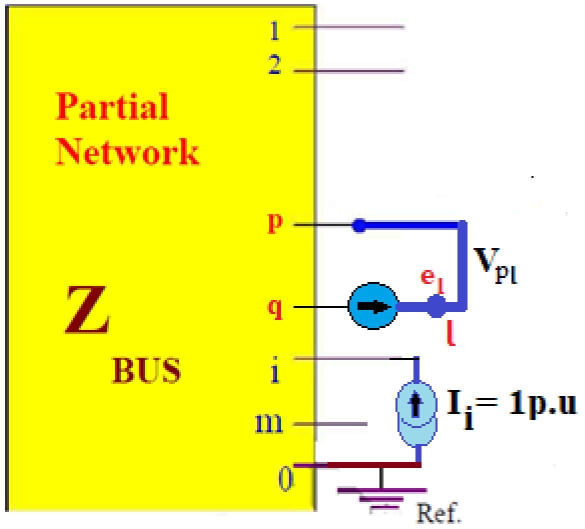


Fig. Calculation for Z_{li}

Since the current in the added branch *p-l*, is zero, $i_{pl} = 0$. We thus have from (43),

$$i_{pl} = y_{pl,pl} v_{pl} + y_{pl,rs} v_{rs} = 0$$

Solving, $$v_{pl} = -y_{pl,rs}\, v_{rs}/y_{pl,pl}$$

or $$v_{pl} = -y_{pl,rs}\,(E_r - E_s)/y_{pl,pl} \qquad \textbf{(44)}$$

However, $$y_{pl,rs} = y_{pq,rs}$$

$$y_{pl,pl} = y_{pq,pq} \qquad \textbf{(45)}$$

Using (41), (44) and (45) in (42), we get

$$Z_{li} = Z_{pi} - Z_{qi} + y_{pq,rs}\,(Z_{ri} - Z_{si})/y_{pq,pq} \qquad \textbf{(46)}$$

$i = 1,2......m;\ i \neq l$

To find Z_{ll}:

The element Z_{ll} can be computed by injecting a current of 1pu at bus-l, $I_l = 1.0$ pu.

As before, we have the relations as under:

$E_k = Z_{kl}\, I_l = Z_{kl}$ $\quad \forall\ k = 1, 2,\dots i\dots\dots p,\dots q,\dots .m, l$ **(47)**

Hence, $e_l = E_l = Z_{ll}$; $E_p = Z_{pl}$;

Also, $e_l = E_p - E_q - v_{pl}$;

So that $Z_{ll} = Z_{pl} - Z_{ql} - v_{pl}$ $\quad \forall\ i=1,2,\dots i\dots p,...q,\dots .m, \neq l$ **(48)**

Since now the current in the added element is $i_{pl} = -I_l = -1.0$, we have from (43)

$$i_{pl} = y_{pl,pl} v_{pl} + y_{pl,rs} v_{rs} = -1$$

Solving, $$v_{pl} = -1 - y_{pl,rs}\, v_{rs} / y_{pl,pl}$$

or $$v_{pl} = -1 - y_{pl,rs}\,(E_r - E_s)/y_{pl,pl} \qquad \textbf{(49)}$$

However, $$y_{pl,rs} = y_{pq,rs}$$

$$y_{pl,pl} = y_{pq,pq} \qquad \textbf{(50)}$$

And

Using (47), (49) and (50) in (48), we get

$$Z_{ll} = Z_{pl} - Z_{ql} + [1 + y_{pq,rs}(Z_{rl} - Z_{sl})/y_{pq,pq} \qquad \textbf{(51)}$$

The following special cases of analysis concerning Z_{BUS} building can be considered with respect to the addition of link to a p-network.

Case (c): If there is no mutual coupling, then elements of $y_{pq,rs}$ are zero. Further, if p is the reference node, then E_p=0. thus,

$$Z_{li} = -Z_{qi} \qquad i = 1,2....m; i \neq l$$

$$Z_{ll} = -Z_{ql} + z_{pq,pq} \qquad \textbf{(52)}$$

From (52), it is thus observed that, when a link is added to a ref. bus, then the situation is similar to adding a branch to a fictitious bus and hence the following steps are followed:

1. The element is added similar to addition of a branch (case-b) to obtain the new matrix of order m+1.
2. The extra fictitious node, *l* is eliminated using the node elimination algorithm.

Case (d): If there is no mutual coupling, then elements of $y_{pq,rs}$ are zero. Further, if p is not the reference node, then

$$Z_{li} = Z_{pi} - Z_{qi}$$

$$Z_{ll} = Z_{pl} - Z_{ql} + z_{pq,pq}$$

$$Z_{ll} = Z_{pp} + Z_{qq} - 2Z_{pq} + z_{pq,pq} \qquad \textbf{(53)}$$

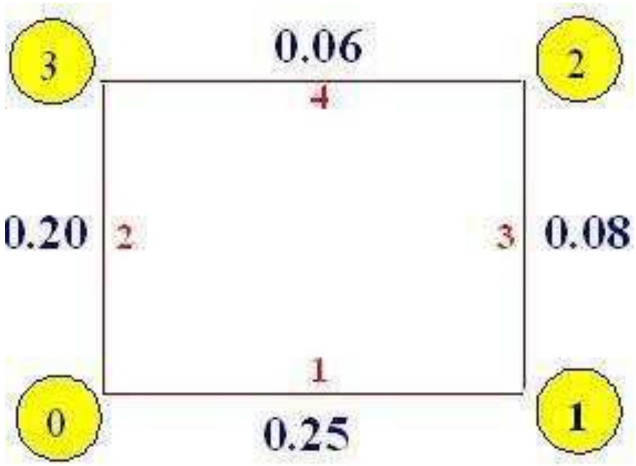
Examples on Z_{BUS} building

For the positive sequence network data shown in table below, obtain Z_{BUS} by building procedure.

Sl. No.	p-q (nodes)	Pos. seq. reactance in pu
1	0-1	0.25
2	0-3	0.20
3	1-2	0.08
4	2-3	0.06

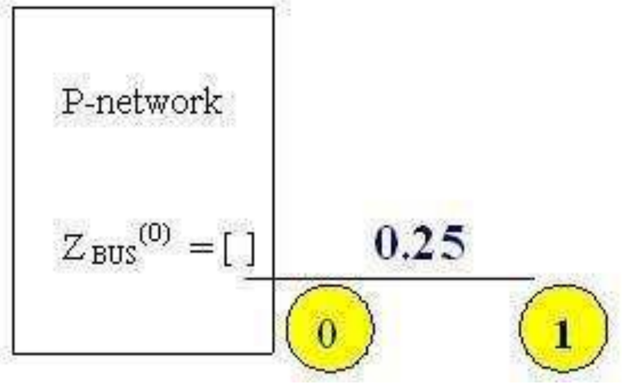
Solution:

The given network is as shown below with the data marked on it. Assume the elements to be added as per the given sequence: 0-1, 0-3, 1-2, and 2-3.



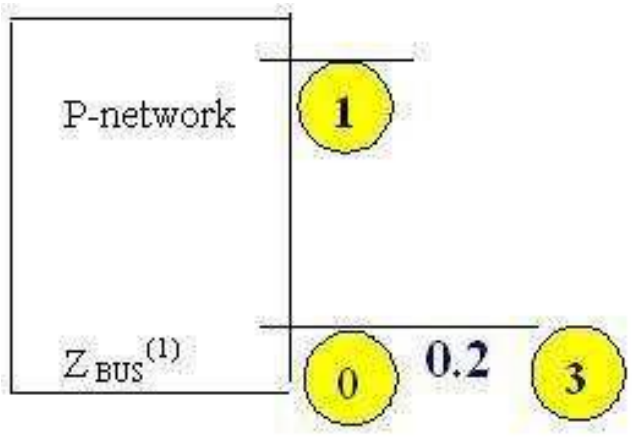
Consider building Z_{BUS} as per the various stages of building through the consideration of the corresponding partial networks as under:

Step-1: Add element–1 of impedance 0.25 pu from the external node-1 (q=1) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



$$Z_{BUS}^{(1)} = \begin{array}{c|c|} & 1 \\ 1 & 0.25 \end{array}$$

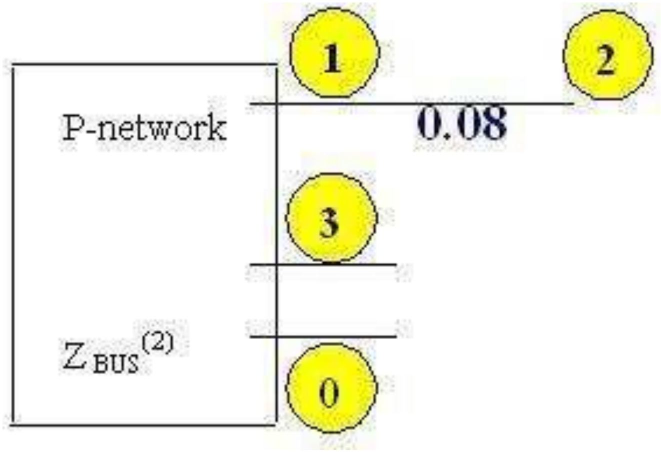
Step-2: Add element–2 of impedance 0.2 pu from the external node-3 (q=3) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



$$Z_{BUS}^{(2)} =$$

	1	3
1	0.25	0
3	0	0.2

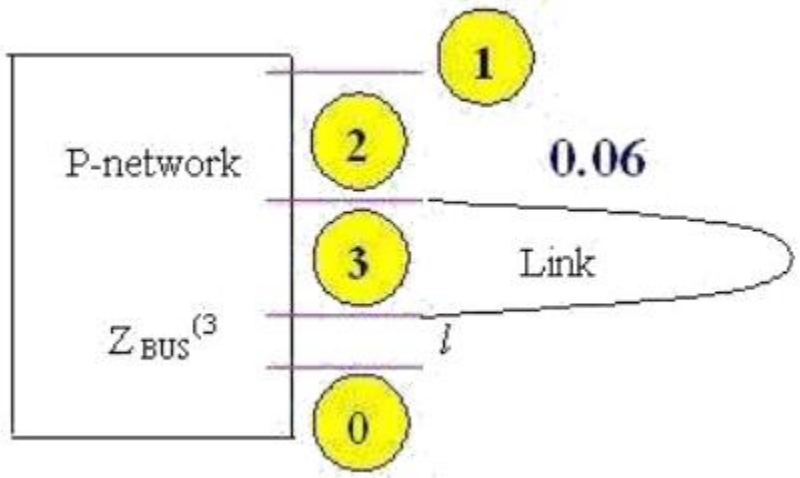
Step-3: Add element–3 of impedance 0.08 pu from the external node-2 (q=2) to internal node-1 (p=1). (Case-b), as shown in the partial network;



$$Z_{BUS}^{(3)} =$$

	1	3	2
1	0.25	0	0.25
3	0	0.2	0
2	0.25	0	0.33

Step-4: Add element–4 of impedance 0.06 pu between the two internal nodes, node-2 (p=2) to node-3 (q=3). (Case-d), as shown in the partial network:



$Z_{BUS}^{(4)} =$

	1	3	2	l
1	0.25	0	0.25	0.25
3	0	0.2	0	-0.2
2	0.25	0	0.33	0.33
l	0.25	-0.2	0.33	0.59

The fictitious node *l* is eliminated further to arrive at the final impedance matrix as under:

$Z_{BUS}^{(final)} =$

	1	2	3
1	0.1441	0.0847	0.1100
2	0.0847	0.1322	0.1120
3	0.1100	0.1120	0.1454