Chapter Four
RELIABILITY PRINCIPLES

4.1 Basic Definitions

Outage: Describes the state of a component when it is not available to perform its intended function due to some event directly associated with that component. An outage may or may not cause an interruption of service to consumers depending on system configuration.
Forced outage: An outage caused by emergency conditions directly associated with a component that require the component to be taken out of service immediately, either automatically or as soon as switching operations can be performed, or an outage caused by improper operation of equipment or human error.
Scheduled outage: An outage that results when a components is deliberately taken out of service at a selected time, usually for purposes of construction, preventive maintenance, or repair. The key test to determine if an outage should be classified as forced or scheduled is as follows. If it is possible to defer the outage when such deferment is desirable, the outage is a scheduled outage; otherwise, the outage is a forced outage. Deferring an outage may be desirable, for example, to prevent overload of facilities or an interruption of service to consumers.
Partial outage: Describes a component state where the capacity of the component to perform its function is reduced but not completely eliminated.

Transient forced outage: A component outage whose cause is immediately self-clearing so that the affected component can be restored to service either automatically or as soon as a switch or circuit breaker can be reclosed or a fuse replaced. An example of transient forced outage is a lightning flashover which does not permanently disable the flashed component.
Persistent forced outage: A component outage whose cause is not immediately self-clearing but must be corrected by eliminating the hazard or by repairing or replacing the affected component before it can be returned to service. An example of persistent forced outage is a lightning flashover which shatters an insulator, thereby disabling the component until repair or replacement can be made.
Interruption: The loss of service to one or more consumers or other facilities and is the result of one or more component outages, depending on system configuration.
Forced interruption: An interruption caused by a forced outage.
Scheduled interruption: An interruption caused by a scheduled outage.
Momentary interruption: It has a duration limited to the period required to restore service by automatic or supervisor controlled switching operations or by manual switching at locations where an operator is immediately available. Such switching operations are typically completed in a few minutes.
Temporary interruption: It has a duration limited to the period required to restore service by manual switching at locations where an operator is not immediately available. Such switching operations are typically completed within 1-2 hours.
Sustained interruption: It is any interruption not classified as momentary or temporary.

According to an IEEE committee report, the following basic information should be included in an equipment outage report:

1. Type, design, manufacturer, and other descriptions for classification purposes
2. Date of installation, location on system, length in the case of a line
3. Mode of failure (short-circuit, false operation, etc.)
4. Cause of failure (lightning. tree, etc.)
5. Times (both out of service and back in service, rather than outage duration alone), date, meteorological conditions when the failure occurred
6. Type of outage, forced or scheduled, transient or permanent

In 1968, a national organization, the National Electric Reliability Council (NERC), was established to increase the reliability and adequacy of bulk power supply in the electric utility systems of North America.

Table 1 Classification of generic and specific causes of outages

Weather	Miscellaneous	System components	System operation
Blizzard/snow	Airplane/helicopter	Electric and mechanical:	System conditions:
Cold	Animal/bird/snake	Fuel supply	Stability
Flood	Vehicle:	Generating unit failure	High/low voltage
Heat	Automobile/truck	Transformer failure	High/low frequency
Hurricane	Crane	Switchgear failure	Line overload/transformer
Ice	Dig-in	Conductor failure	overload
Lightning	Fire/explosion	Tower, pole attachment	Unbalanced load
Rain	Sabotage/vandalism	Insulation failure:	Neighboring power system
Tornado	Tree	Transmission line	Public appeal:
Wind	Unknown	Substation	Commercial and
Other	Other	Surge arrestor	industrial
		Cable failure	All customers
		Voltage control	Voltage reduction:
		equipment:	0–2 % voltage reduction
		Voltage regulator	Greater than 2–8 %
		Automatic tap changer	voltage reduction
		Capacitor	Rotating blackout
		Reactor	Utility personnel:
		Protection and control:	System operator error
		Relay failure	Powerplant operator error
		Communication signal	Field operator error
		error	Maintenance error
		Supervisory control error	Other

4.2 COMPONENT RELIABILITY:

A component is an entity in a system which is not further subdivided.

Component can be classified into two groups namely:

1- Non-repairable components:

The most important quantity to describe such a component is its lifetime T, a random variable which in turn is determined by its probability distribution.

The cumulative probability distribution CPD of T, $F_{T(t)}$ is defined as:

$$F_{T(t)} = P[T \leq t]$$

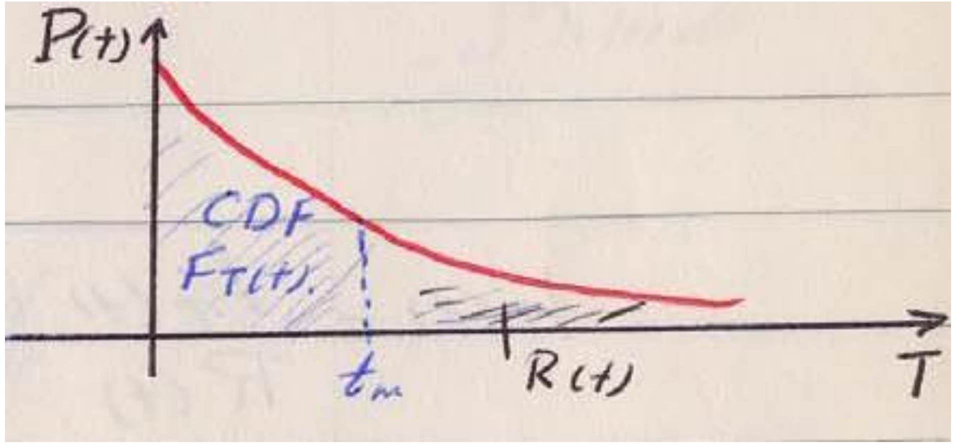
and its probability density function PDF of T, $f_{T(t)}$ as:

$$f_{T(t)} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P[t < T \leq t + \Delta t]$$

where, $$\frac{dy}{dt} = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}$$

$$\therefore f_{T(t)} = \frac{dF_T(t)}{dt}$$

$$\therefore F_{T(t)} = \int_0^t f_T(t)dt$$



According to our definition of reliability of component that it performs this function adequately for the period of time in tended under the operation condition, the reliability R of a component can be expressed as:

$$R = P[T > t_n]$$

where t_n is mission time ($t_n \cong t_m \cong t$), or it's the period of time for which it's intended that the component to perform its function. R is a function of time:

$$\mathrm{R(t)} = P[T > t]$$

From the figure, we get:

$$\mathrm{R(t)} = 1 - F_T(t)$$

The value of R(t) must be between 0 and 1, i.e. R(0)=1 and R(∞)=0.
($F_T(t)$ = Probability of failure, and R(t) = Probability of success)

Hazard Rate Function h(t):

For small Δt, h(t) Δt is the probability that a component that have survived until time t will fail in the next Δt interval or

$$h(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P[failure\ in\ time\ t \rightarrow t+\Delta t \mid \text{working at time t}]$$

$$= \lim_{\Delta t \to 0} \frac{1}{\Delta t} P[\ t < T \leq t+\Delta t \mid T > t]$$

The condition probability can be expressed

$$P[\ t < T < t+\Delta t \mid T > t] = \frac{P[(t < T \leq t+\Delta t) \cap (T > t)\ ]}{P[T > t]}$$

$$h(t) \cong \frac{f_T(t)\,\Delta t}{R(t)}$$

$$\therefore h(t) = \frac{f_T(t)}{R(t)}$$

$$or\ h(t) = \frac{f_T(t)}{1-F(t)}$$

Let R(t) be reliability of function, $\therefore R'(t) = \frac{dR(t)}{dt}$

$$\int \frac{R'(t)}{R(t)} dt = lnR(t)$$

by taken d/dt for both sides: $\frac{R'(t)}{R(t)} = \frac{d\ lnR(t)}{dt}$

$$but\ \ R'(t) = \frac{dR(t)}{dt} = \frac{d(1 - F_T(t))}{dt}$$

$$R'(t) = -\frac{dF_T(t)}{dt} = -f_T(t)$$

$$\frac{d\ lnR(t)}{dt} = \frac{R'(t)}{R(t)} = \frac{-f_T(t)}{1 - F_T(t)} = -h(t)$$

$$or\ \ h(t) = -\frac{d}{dt}\ln R(t)$$

Conversely $-d \ln R(t) = h(t)\, dt$

$$\ln R(t) = -\int_0^t h(t)\ dt$$

$$\therefore\ R(t) = e^{-\int_0^t h(t)\, dt}$$

	$F_T(t)$	$f_T(t)$	$R(t)$	$h(t)$
$F_T(t)$	-	$\int_0^t f_T(t)\ dt$	$1 - R(t)$	$1 - e^{-\int_0^t h(t)\, dt}$
$f_T(t)$	$\frac{d}{dt} F_T(t)$	-	$-\frac{d}{dt} R(t)$	$h(t)\, e^{-\int_0^t h(t)\, dt}$
$R(t)$	$1 - F_T(t)$	$\int_t^\infty f_T(t)\ dt$	-	$e^{-\int_0^t h(t)\, dt}$
$h(t)$	$\frac{\frac{d}{dt} F_T(t)}{1 - F_T(t)}$	$\frac{f_T(t)}{\int_t^\infty f_T(t)\ dt}$	$-\frac{d}{dt} ln\, R(t)$	-

Relations between the functions $F_T(t), f_T(t),\ R(t)\ and\ h(t)$

Failure Rate

In general, there may be some differences between the predicted failure rates and observed failure rates due to the following factors :

1. Definition of failure
2. Actual environment compared with prediction environment
3. Maintainability, support, testing equipment, and special personnel
4. Composition of components and component-failure rates assumed in making the prediction
5. Manufacturing processes including inspection and quality control
6. Distributions of times to failure
7. Independence of component failures

Since every piece of equipment in a system will eventually fail if it is in service for a long period, there is a failure rate associated with each one. For some items, the failure rate is quite significant while for others it could be extremely low.

Failure rate is defined as the number of expected failures per unit in a given time interval. It is just an expected value. In calculating the failure rate of a group of units, the total operating time of the units should be used instead of the chronological time. The formula is

$$\text{Failure rate, } \lambda = \frac{\text{number of failures}}{\text{total operating time of units}}$$

Example

10 transformers were tested for 500 h each, and four transformers failed after the following test time periods: one failed after 50 h , one failed after 150 h , two failed after 400 h .

What is the failure rate for these types of transformers?

Solution: Total operating time of units $= (1 \times 50 + 1 \times 150 + 2 \times 400 + 6 \times 500)$ unit h

$= 4000$ unit h

10-4=6

$$\lambda = \frac{4}{4000} = 0.001 \text{ failures/unit h}$$

Example

Thirty motors were tested for 200 h. Five motors failed during the test. The failures occurred after the following test times:

Motor 1 60 h

Motor 2 71 h

Motor 3 157 h

Motor 4 160 h

Motor 5 170 h

What is the estimated failure rate?

30-5=25

Solution:

Total number of unit operating hours= 60+71+157+160+170+25 x 200

= 5618 unit h

$$\text{Failure rate, } \lambda = \frac{\text{number of failures}}{\text{total unit operating time}}$$

= 5/5168 unit h

= 0:00092 failures/h

Derivation the expression of Reliability using number of components as a random variable:

Consider the case in which a fixed number **No** of identical components are tested
Lets **Ns(t)** – Number of surviving components at time t
Nf(t) – Number of failed components at time t

At any time t, the Reliability R(t) is given by

$$R(t) = \frac{N_s(t)}{N_o} = \frac{N_o - N_f(t)}{N_o} = 1 - \frac{N_f(t)}{N_o}$$

Therefore,

$$\frac{d\,R(t)}{dt} = -\frac{1}{N_o}\frac{d\,N_f(t)}{dt}$$

$$as\ \ dt \to 0 \quad , then \frac{1}{N_o}\frac{d\,N_f(t)}{dt} \ \ is\ the\ instantaneous\ failure\ density\ f_T(t)$$

$$\frac{d\,R(t)}{dt} = -f_T(t)$$

Let λ(t) = instantaneous hazard rate $= \dfrac{\frac{1}{N_o}\frac{d\,N_f(t)}{dt}}{\frac{N_s}{N_o}}$

$$\lambda(t) = \frac{\left.\dfrac{d\,N_f(t)}{dt}\right/ N_o}{N_s / N_o}$$

$$\lambda(t) = \frac{f(t)}{R(t)} = -\frac{dR(t)/dt}{R(t)}$$

Note: f(t) is the failure density function and $f(t) = -\dfrac{dR(t)}{dt}$

$$\int_0^t f(t)dt = \int_0^{R(t)} -d\,R(t) = R(0) - R(t) = 1 - R(t)$$

$$\lambda(t) = -\frac{dR(t)/dt}{R(t)}$$

$$-\lambda(t)\,dt = -\frac{d\,R(t)}{R(t)}$$

$$\int -\lambda(t)\ dt = \int \frac{1}{R(t)} dR(t) = \ln R(t)$$

$$\therefore R(t) = e^{-\int_0^t \lambda(t)dt}$$

Assume λ(t) is constant = λ $\Longrightarrow$ $R(t) = e^{-\int_0^t \lambda \, dt}$

$$\therefore R(t) = e^{-\lambda t}$$

Note: This is the first term of Poisson Distribution

$\frac{(\lambda t)^r}{r!} e^{-\lambda t}$ at r = 0 $\Longrightarrow$ $e^{-\lambda t}$ Reliability.

If λ(t) is constant $R(t) = e^{-\lambda t}$

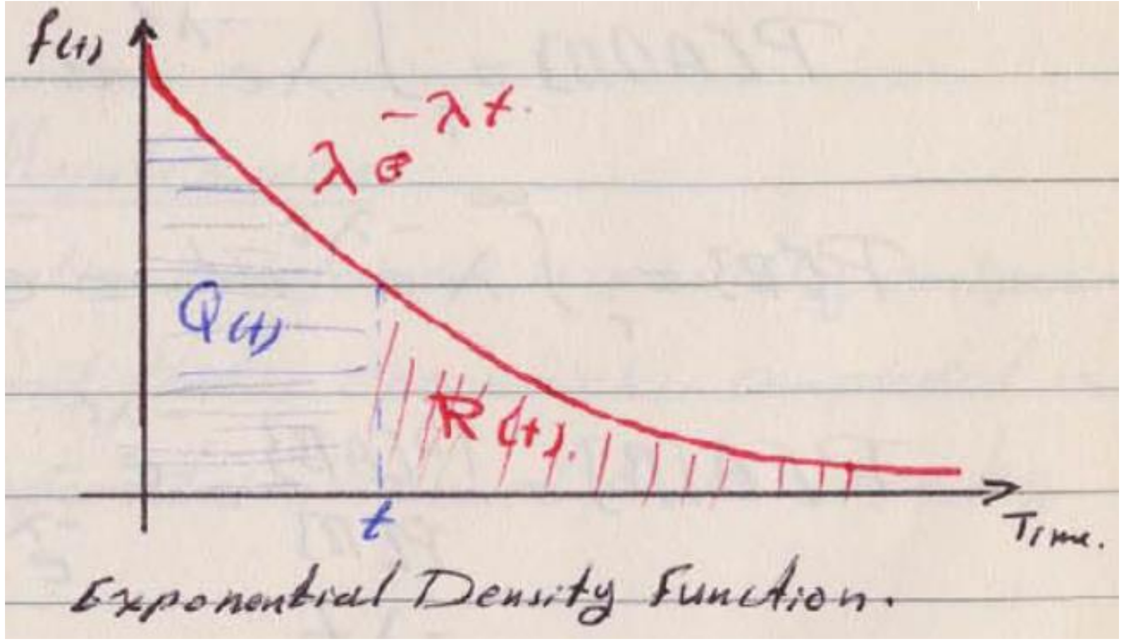
the failure density function $f(t) = -\frac{d\,R(t)}{dt}$

$$\therefore f(t) = \lambda \, e^{-\lambda t}$$

$$h(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

From this curve

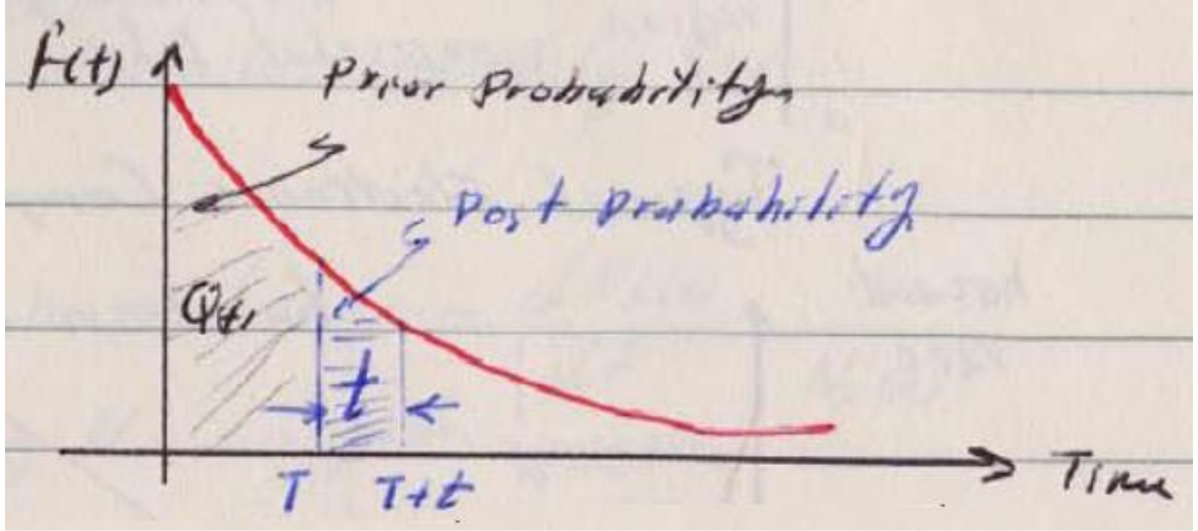
$$Q(t) = \int_0^t \lambda\, e^{-\lambda t} dt \quad and \quad R(t) = \int_t^\infty \lambda\, e^{-\lambda t} dt$$



The component for the time T is the event B
$\therefore P[B]$ is the probability of survived up to time T.
The event A is the failure of the component during time t.
$\therefore P[A]$ is the probability of failure during time t.
$P[A/B]$ is the probability of failure during time t, given that the component has survived up to time T [this Q(t), i.e. $Q(t) = P[A/B]$.
$P[A \cap B]$ is the probability of surviving up to time T and failing during time t, T+t ,:

$$P[A \cap B] = \int_T^{T+t} \lambda\, e^{-\lambda t} dt \;=\; e^{-\lambda T} - e^{-\lambda (T+t)}$$

$$P[B] = \int_T^\infty \lambda\, e^{-\lambda t} dt \;=\; e^{-\lambda T}$$



$$P[A|B] = \frac{P[A \cap B]}{P[B]} = \frac{e^{-\lambda T} - e^{-\lambda(T+t)}}{e^{-\lambda T}}$$

$$Q(t) = 1 - e^{-\lambda t}$$

$$If\ \lambda t \ << 1, then\ Q(t) = 1 - [1 - \lambda t + \frac{(\lambda t)^2}{2!} - \cdots\ ]$$

$$Q(t) = \lambda t$$

$$R(t) = 1 - \lambda t \qquad for\ \ \lambda < 0.01$$

is accurate to at least four decimal place

Mean Time to Failure (MTTF)

MTTF is a third useful reliability measure for non-repairable components in addition to R(t) and λ(t) or h(t), it is given by

$$MTTF = \int_0^\infty t.f(t)dt$$

$$MTTF = -\int_0^\infty t.\frac{d\,R(t)}{dt}\,dt$$

Integrating by parts we get

$$MTTF = -[t.R(t)]_0^\infty + \int_0^\infty R(t)dt$$

$$MTTF = \int_0^\infty R(t)dt$$

Since for R(0)=1 and at t= ∞ R(∞) →0

For constant rate of failure λ(t) or h(t) = λ

$$MTTF = \int_0^\infty e^{-\lambda t}dt = \frac{1}{\lambda}$$

$$MTTF = \frac{1}{\lambda}$$

Example

Two hundred capacitors were installed and at the end of each year, the number of surviving units was tallied.

End of	Number of Units Remaining
Year 1	196
Year 2	188
Year 3	179
Year 4	175
Year 5	169

Based on these figures, what is the reliability of the capacitors for 5 years?

The annual reliability of Year 4 ? Assuming the reliability function is exponential, that is, $R(t) = e^{-\lambda t}$, what is the failure rate for this formula?

Solution:

Reliability for 5 years = 169/200 = 0.845

$$\text{Annual reliability of Year 4} = \frac{\text{number of units surviving at the end of Year 4}}{\text{number of units at the beginning of Year 4}}$$

$$= 175/179$$

$$= 0.9777$$

$$\text{For 5 years}, R(t) = e^{-\lambda 5} \text{ (from formula)}$$

$$0.845 = e^{-5\lambda}$$

$$\lambda = 0.0337$$

Example

Ten thousand new oil circuit reclosers (OCRs) are put in service. They have a constant failure rate of 0.1 per year. How many units of the original 10,000 will still be in service after 10 years? How many of the original will fail in Year 10?

Solution:

Probability of survival is given by

$$R(t) = e^{-\lambda t}$$

In 10 years, probability of survival

$$R(10) = e^{-0.1\times 10} = e^{-1.0} = 0.3679$$

Out of 10,000 original units

10,000x0.3679=3679 should survive

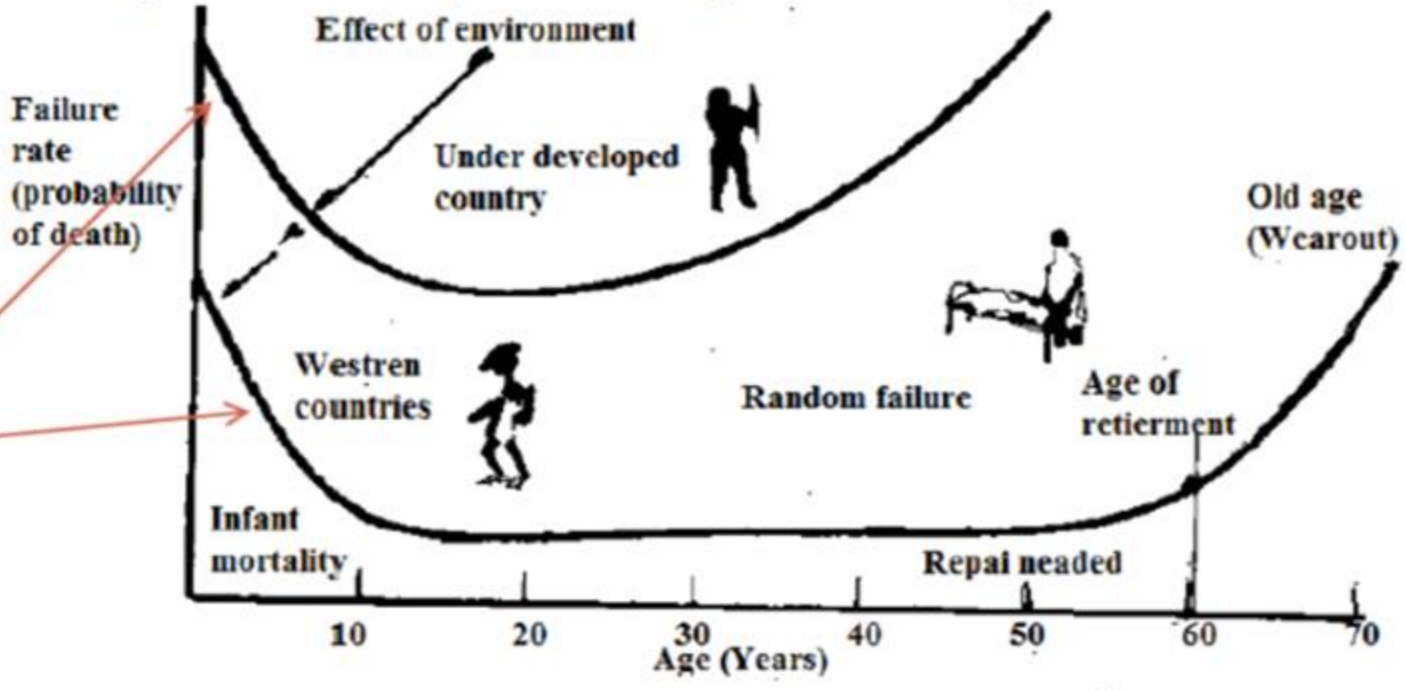
Number of failures in Year 10

$$
\begin{aligned}
&= (\text{number of survivors after Year 9}) - (\text{number of survivors after Year 10}) \\
&= 10,000 \times e^{-0.1 \times 9} - 3679 \\
&= 1000 \times e^{-0.9} - 3679 \\
&= 4066 - 3679 \\
&= 387
\end{aligned}
$$

Concept of Bathtub Curve

The life of a man has the following three major distinguishable periods as shown in Fig.

1. Infant mortality period
2. Useful life period
3. Wear-out period



In the western countries the frailer rate is less than under developed countries

Bathtub curve failure rate versus time for human life.

Also the life of equipment has similar three major periods as shown in Fig.4.2:

1. Infant mortality period
2. Useful life period
3. Wear-out period

- In the infant mortality period, the failure rate is high due to the presence of weak spots from the manufacturing process such as poor workmanship, substandard components, and so on. As these weaknesses are manifested one by one by the stress of operation, the failure rate keeps decreasing until a low constant level is reached.

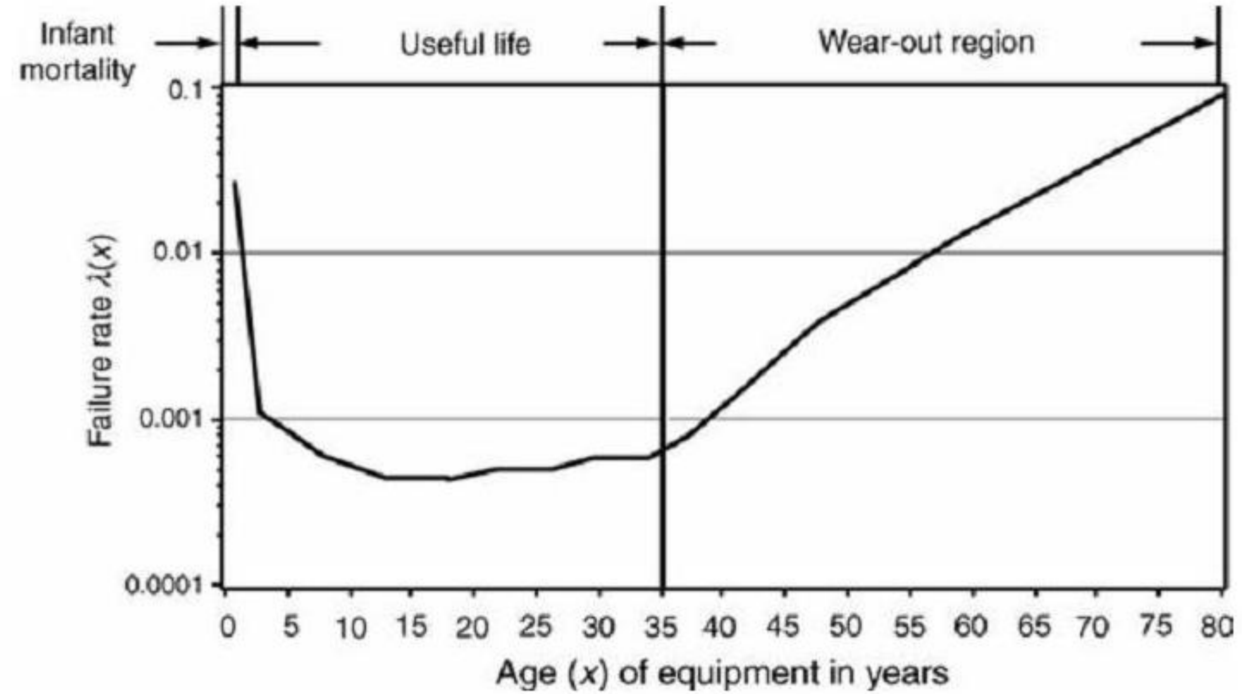
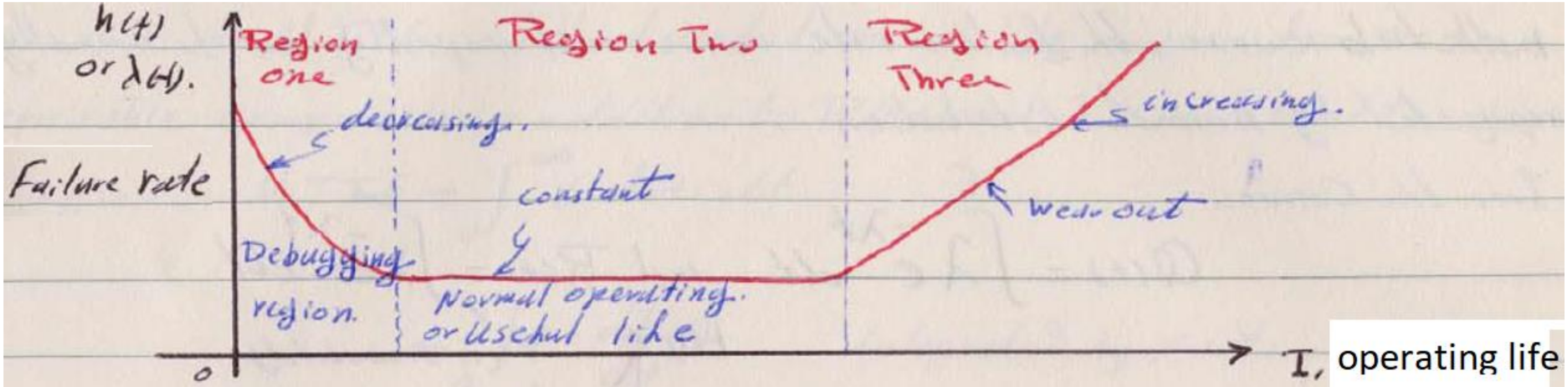


Fig.4.2. Bathtub curve failure rate versus time for components.

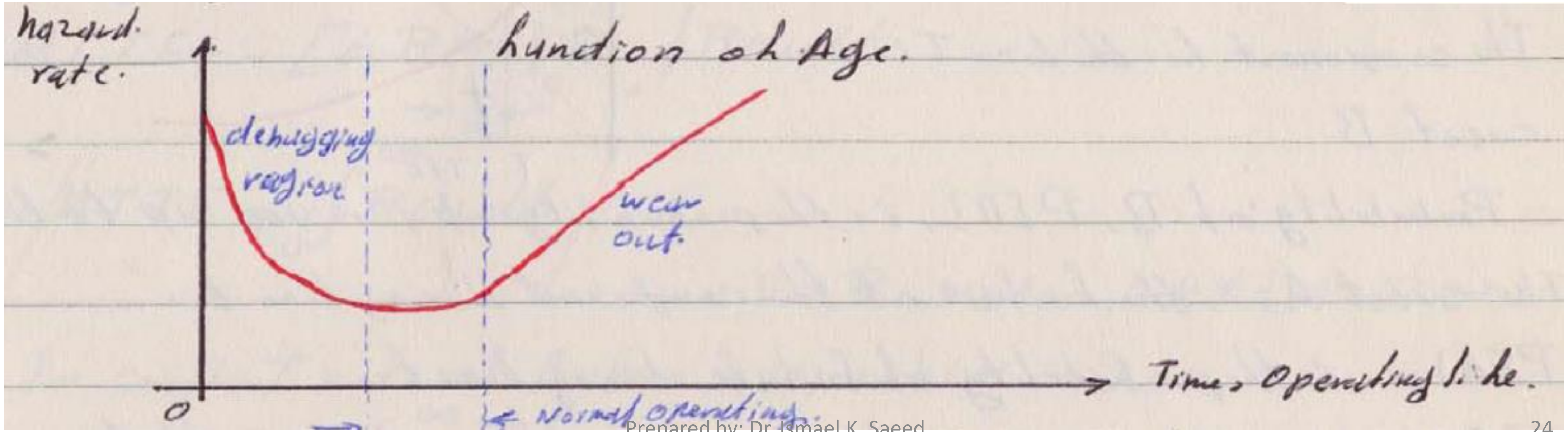
- The equipment then enters the useful life period where failures are due to chance and occur at random times. Failures in this period are also independent of the age of the equipment.
- When the components of the equipment start to wear out. From this time on, the failure rate rises rather rapidly due to deterioration.

Most reliability work deals with the useful life period when the failure rate is constant and the exponential distribution applies, one of the several possible distributions.

Typical figure of λ(t) or h(t) is shown below



Typical Electronic Component Failure Rate as a Function Of Age



Typical Mechanical Component Failure Rate as a Function Of Age

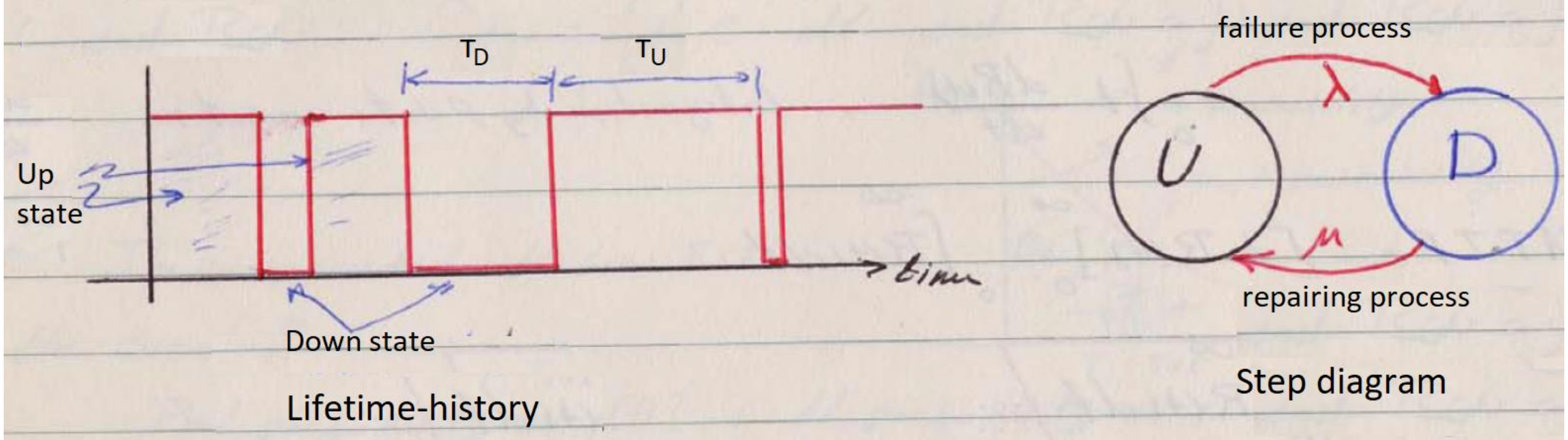
Components with preventive maintenance

The mean objective of preventive maintenance is possibly postponing failure. The ideal maintenance is assumed that is one which is completed in zero time. The operating condition of component is assumed to be better by this replacement (i.e. as good as new).

Normally we apply preventive maintenance for component with increasing hazard functions where the performance of preventive maintenance is beneficial and its costs to increase the main time to failure.

2- Repairable Components (Normal Repair)

In the normal repair, the repair time is tended as another random variable together with the one representing the operating time. By the both random variables the life process of a repairable component is described.



From the lifetime diagram its obvious that the lifetime history of repairable component is determine by two distributions:

1- $F_U(t)$ = Cumulative Density Function CDF of up time

2- $F_D(t)$ = CDF of down time

let x_t be state at time t.

The probability of being in the up state

$$P_U(t) = P[Up\ at\ time\ t]$$

$$P_U(t) = P[x_t = U]$$

The probability of being in the down state

$$P_D(t) = P[Down\ at\ time\ t]$$

$$P_D(t) = P[x_t = D]$$

Failure density:

$$L(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \ P[\ failure\ in\ (\ t\ , t + \Delta t\ )]$$

$$L(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \ P[(x_{t+\Delta t} = D) \cap (x_t = U)]$$

Intensity of transitions from U to D

$$q_{UD} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \ P[\ failure\ in\ (\ t\ , t + \Delta t\ )\ |\ working\ at\ t]$$

$$q_{UD} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \ P[(x_{t+\Delta t} = D)\ |\ (x_t = U)]$$

Mean Up time or mean time to failure (MTTF)

$$M_U = \int_0^{\infty} t.\frac{dF_U}{dt}dt$$

Mean Down time

$$M_D = \int_0^{\infty} t.\frac{dF_D}{dt}dt$$

Mean time between failures (MTBF) or Mean cycle time

$$MTBF = M_U + M_D$$

Availability (Availability is the probability of success instead Reliability sometime used**)**

When a system fails, it will be out of service for some time until it is repaired or replaced. Even for systems with spare units, the system can be “down” if a failure occurs when no more spares are available. The percentage of time that the system is functioning is called the availability of the system. It is usually expressed as

$$\text{Availability} = A = \frac{\text{total hours of operation in 1 year}}{8760}$$

$$\text{Unavailability} = \bar{A} = \frac{\text{total hours of down time in 1 year}}{8760}$$

Since on average, it takes a time interval equal to the MTTF for the system to fail and a time interval equal to the MTTR for the system to be operational again, **the availability**, defined as uptime / (uptime+ downtime), can be

expressed as

$$A = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}}$$

MTTF mean time to failures

MTTR: Time interval for the system to be operational again

and unavailability can be expressed as

$$\bar{A} = \frac{\text{MTTR}}{\text{MTTF} + \text{MTTR}}$$

$$A = \frac{M_U}{M_U + M_D} \qquad and \quad \bar{A} = \frac{M_D}{M_U + M_D}$$

$$\bar{A} = 1 - A$$

where A is Availability and $\bar{A}$ is Unavailability

The function $L(t), P(t)\ (or\ A), q_{UD}$ are related from the definition of q_{UD} as bellow

$$q_{UD}(t) = \frac{P[(x_{t+\Delta t} = D) \cap (x_t = U)]}{P[x_t = U]} \approx \frac{L(t)\,\Delta t}{P_U(t)}$$

and therefore, $$L(t) = P_U(t)\, q_{UD}(t)$$

The $q_{UD}(t)$ function is an extension of the hazard function $h(t)$. In general case $F_U(t)$ and $F_D(t)$ can assume any form, but it consider both the Up and Down times to be exponential distribution, and when only the long term behavior of the component as $t \to \infty$ is of interest.

Exponential Up and Down times

In this case of exponential Up and Down time

$$F_U(t) = \lambda e^{-\lambda t}$$

$$F_D(t) = \mu e^{-\mu t}$$

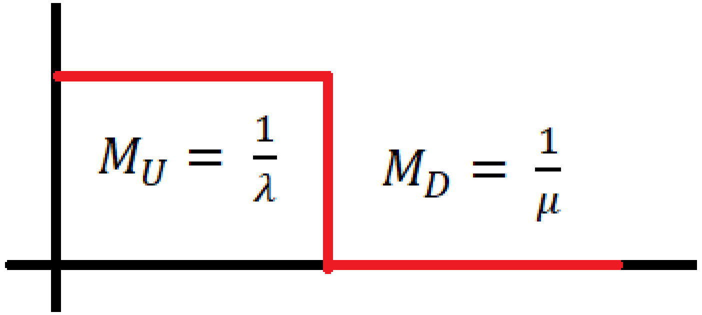
with $M_U = \frac{1}{\lambda}$ and $M_D = \frac{1}{\mu}$

its easy to verify that the Availability and Unavailability indices are now given by the expressions

$$A = \frac{M_U}{M_U + M_D} = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda} + \frac{1}{\mu}}$$

$$A = \frac{\mu}{\mu + \lambda}$$

$$and \quad \bar{A} = \frac{M_D}{M_U + M_D} = \frac{\frac{1}{\mu}}{\frac{1}{\lambda} + \frac{1}{\mu}}$$



$$\bar{A} = \frac{\lambda}{\lambda + \mu}$$

In the long run, the probability of being in the up state is the ratio of the mean up time to the sum of the both mean up and down times, i.e.

$$\underset{as\ t \to \infty}{P_U(t)} \to A \qquad , \text{ at any time } P_U(t) + P_D(t) = 1$$

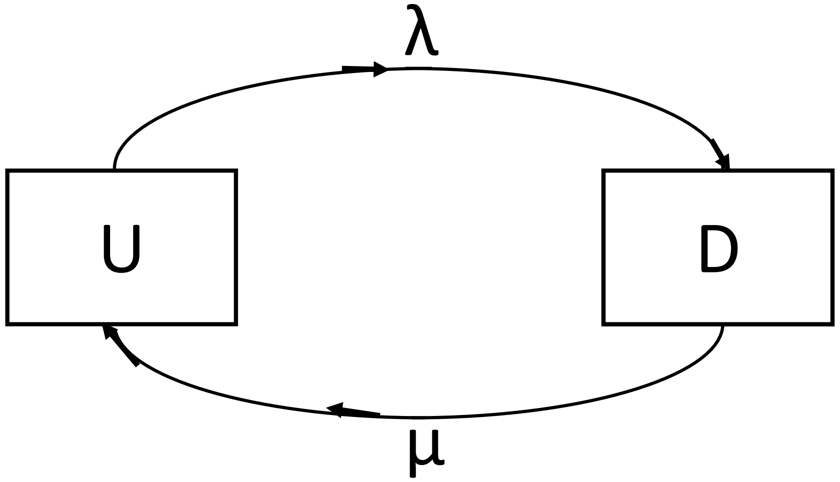
$$\underset{as\ t \to \infty}{P_D(t)} \to \bar{A}$$

$L(t)$, the failure density can be considered

$$L(t) = \frac{1}{M_U + M_D} \qquad \text{for } t \to \infty$$

For state space

transition matrix is



$$A = \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix}$$

$$and \quad A^n = (-1)^{n-1}(\lambda+\mu)^{n-1} A$$

$$P(t) = P_o \, e^{At}$$

$$e^{At} = I + \sum_{n=1}^{\infty} (-1)^{n-1}(\lambda+\mu)^{n-1} A \, \frac{t^n}{n!}$$

the initial condition, give state at P_o is [1, 0]

$$[P_o(t), P_1(t)] = [1,0][I + \sum_{n=1}^{\infty} (-1)^{n-1}(\lambda+\mu)^{n-1} A \, \frac{t^n}{n!}]$$

$$[P_o(t), P_1(t)] = 1 + \frac{1}{\lambda+\mu}[1 - e^{-(\lambda+\mu)t}][-\lambda \quad \lambda]$$

$$\therefore P_o(t) = P_U(t) = 1 - \frac{\lambda}{\lambda+\mu}[1 - e^{-(\lambda+\mu)t}]$$

$$P_U(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t}$$

$$and \quad P_1(t) = P_D(t) = \frac{\lambda}{\lambda+\mu} - \frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t}$$

the failure density $L(t) = P_U(t)\, q_{UD}(t)$

$$q_{UD}(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P\left[x_{(t, t+\Delta t)} = D \mid x_t = U\right]$$

$$q_{UD}(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} . \lambda\, \Delta t = \lambda$$

$$\therefore \quad L(t) = \frac{\lambda\mu}{\lambda+\mu} + \frac{\lambda\mu}{\lambda+\mu} e^{-(\lambda+\mu)t}$$

for long run, $t \to \infty$

$$P_U(t) = \frac{\mu}{\lambda+\mu} = A \ , \qquad P_D(t) = \frac{\lambda}{\lambda+\mu} = \bar{A}$$

$$L(t) = \frac{\lambda\mu}{\lambda+\mu} \qquad since, \qquad M_D = \frac{1}{\mu} \ , \quad M_U = \frac{1}{\lambda}$$

$$L(t) = \frac{\frac{1}{M_U}\frac{1}{M_D}}{\frac{1}{M_U} + \frac{1}{M_D}} = \frac{1}{M_U + M_D}$$

$$L(t) = P_U(t)\, q_{UD}(t)$$

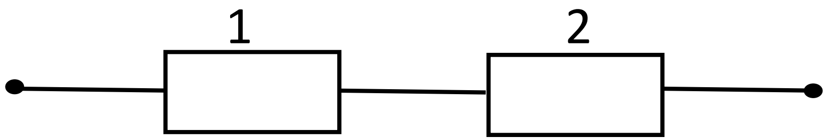
$$q_{UD}(t) = \frac{L(t)}{P_U(t)} \qquad at \ \ t \rightarrow \infty$$

$$q_{UD}(t) = \frac{\frac{\lambda\mu}{\lambda + \mu}}{\frac{\mu}{\lambda + \mu}} = \lambda$$

$$q_{UD}(t) = \lambda$$

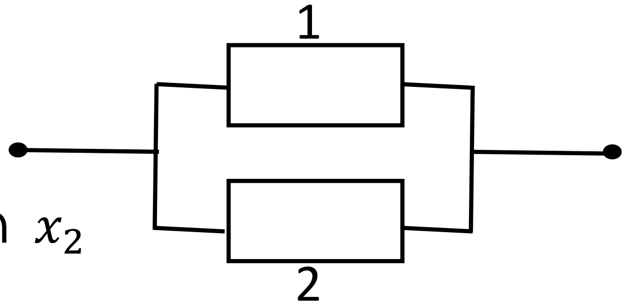
Let S be the event that the system is working and $\bar{S}$ that it is not. And x_i denote the event that component i is working and $\bar{x}_i$ that it is not.

1- Series connection



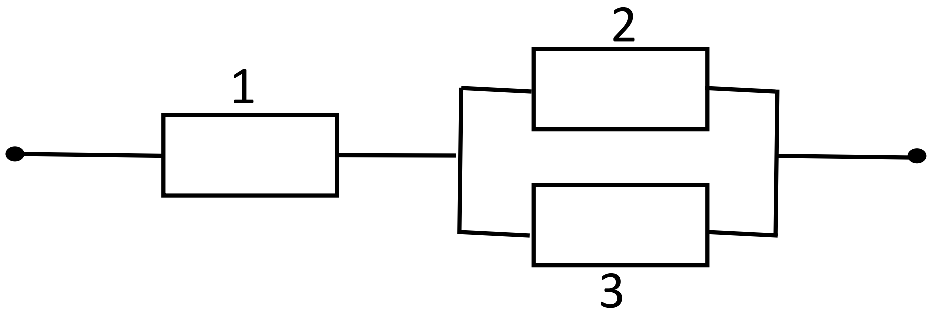
$$S = x_1 \cap \ x_2 \qquad and \qquad \bar{S} = x_1 \cup \ x_2$$

2- Parallel connection



$$S = x_1 \cup \ x_2 \qquad and \qquad \bar{S} = x_1 \cap \ x_2$$

3- Series-Parallel connection



$$S = x_1 \cap (\ x_2 \ \cup \ x_3\ )$$

and $\bar{S} = x_1 \cup (\ x_2 \ \cap \ x_3\ )$

Using the Boolean algebra to reduce the system, where **.** is AND gate and + is OR gate.

$$S = x_1 \,.\, (\ x_2 + \ x_3\ ) = x_1 \,.\, x_2 + \ x_1 \,.\, x_3$$

Example: Reduce the following system

$$S = a + \left(b \, . \, (a + (b \, . \, (c + d))) \right)$$

$$S = a + \left(b \, . \, (a + (b \, . c \; + b \, . d)) \right)$$

$$S = a + (b \, . \, (a + b \, . c \; + b \, . d))$$

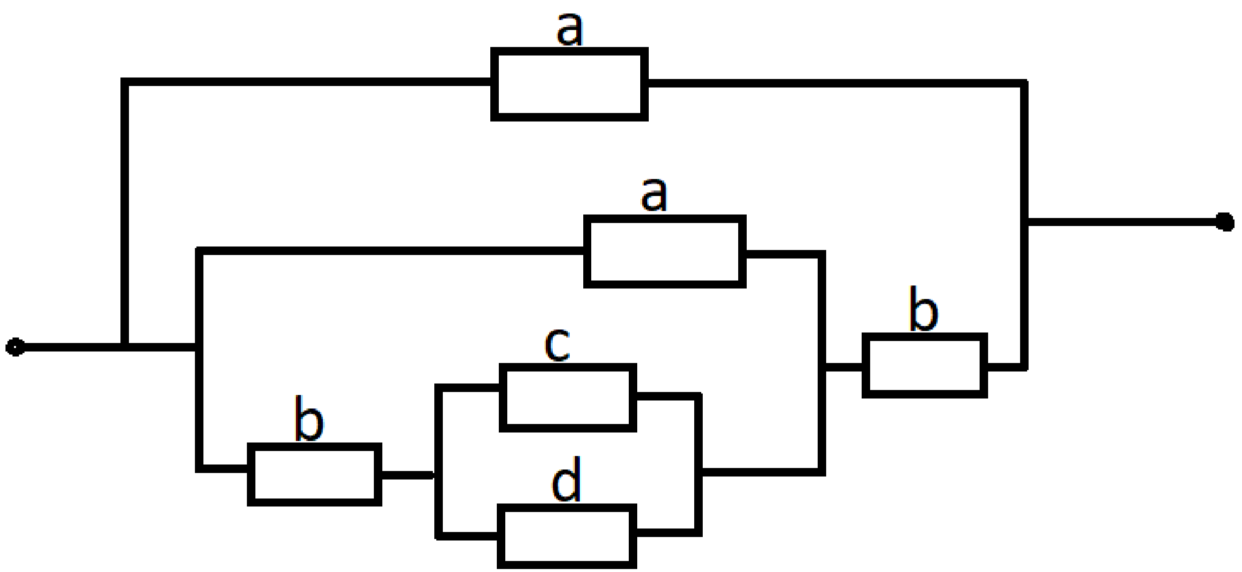
$$S = a + (b \, . a + b \, . b \, . c \; + b \, . b \, . d)$$

$$S = a + b \, . a + b \, . b \, . c \; + b \, . b \, . d$$

$$S = a + b \, . a + b \, . c \; + b \, . d$$

$$S = a \, (1 + b \,) + b \, (c \; + d \,)$$

$$S = a \; + b \, (c \; + d \,)$$



$$since \;\; a \; + a = a \quad , \qquad a \, . a = a \qquad and \quad a + 1 = 1$$

The system will be equivalent to