Pressure Altitude

In the previous module, we studied the concept of standard atmosphere and the behavior of pressure and density at isothermal and gradient layers. Now, we introduce the concept of pressure and density altitudes.

Pressure altitude is the altitude associated with a given pressure assuming standard atmosphere or standard atmospheric conditions. In other words, pressure altitude is the altitude an airplane would be at, if it was a standard day.

The airplane's altimeters measures the altitude based on the sea level pressure (101325 Pa), for a standard atmosphere. So, when the pilot starts the airplane, he sets the altimeter for this sea level pressure setting.

While pressure altitude provides a correction for pressure, it does not take into consideration the ambient temperature of the air - a factor that, on a hot day when performance of the aircraft is degraded, is far more critical.

Density Altitude

Density altitude is defined as the altitude at which the density of the Standard Atmosphere is same as the density of the air being evaluated. The density altitude is the pressure altitude corrected for the standard temperature.

Solved Example 1:

Calculate the standard atmosphere values of $T, P, and\ \rho$ at a Geo-potential altitude of 15km.

Solution:

From temperature altitude map, $T = 288.16\ K$ (sea level condition)

The calculation will be carried out first from sea level to $11km$ and then from $11km$ to $15km$.

Gradient Region: from $h = 0$ to $h = 11.0km$, with lapse rate of $\alpha = -6.5K/km$. So,

$$P = P_1 \left(\frac{T}{T_1}\right)^{-\frac{g_o}{\alpha R}}$$

$$\rho = \rho_1 \left(\frac{T}{T_1}\right)^{-\left[\frac{g_o}{\alpha R}+1\right]}$$

Using standard atmosphere conditions, we have

$$P_1 = P_s = 1.01325 \times 10^5 N/m^2$$

$$\rho_1 = \rho_s = 1.2250 kg/m^3$$

$$T_1 = T_s = 288.16K$$

The pressure and density at 11 km are then obtained as follows.

$$P_{11} = 0.26 \times 10^4 N/m^2$$

$$\rho_{11} = 0.367 kg/m^3$$

Isothermal region : From $11km$ to $14km$

$$P_{15} = P_{11} \times e^{-\left[\frac{g_o}{RT}\right](h-h_1)}$$

$$\rho_{15} = \rho_{11} \times e^{-\left[\frac{g_o}{RT}\right](h-h_1)} \qquad (1)$$

Here the subscript "11" refers to the values at $h = 11.0km$. The calculations result in pressure and density at $h = 15.0km$ as :

$$P_{15} = 1.2112 \times 10^4 N/m^2$$

$$\rho_{15} = 0.1947 kg/m^3$$

Properties of the atmosphere at high altitude

Altitude, m	Temperature, °C	Pressure, kPa	Gravity g, m/s^2	Speed of Sound, m/s	Density, kg/m^3	Viscosity μ, kg/m·s	Thermal Conductivity, W/m·K
0	15.00	101.33	9.807	340.3	1.225	1.789×10^{-5}	0.0253
200	13.70	98.95	9.806	339.5	1.202	1.783×10^{-5}	0.0252
400	12.40	96.61	9.805	338.8	1.179	1.777×10^{-5}	0.0252
600	11.10	94.32	9.805	338.0	1.156	1.771×10^{-5}	0.0251
800	9.80	92.08	9.804	337.2	1.134	1.764×10^{-5}	0.0250
1000	8.50	89.88	9.804	336.4	1.112	1.758×10^{-5}	0.0249
1200	7.20	87.72	9.803	335.7	1.090	1.752×10^{-5}	0.0248
1400	5.90	85.60	9.802	334.9	1.069	1.745×10^{-5}	0.0247
1600	4.60	83.53	9.802	334.1	1.048	1.739×10^{-5}	0.0245
1800	3.30	81.49	9.801	333.3	1.027	1.732×10^{-5}	0.0244
2000	2.00	79.50	9.800	332.5	1.007	1.726×10^{-5}	0.0243
2200	0.70	77.55	9.800	331.7	0.987	1.720×10^{-5}	0.0242
2400	−0.59	75.63	9.799	331.0	0.967	1.713×10^{-5}	0.0241
2600	−1.89	73.76	9.799	330.2	0.947	1.707×10^{-5}	0.0240
2800	−3.19	71.92	9.798	329.4	0.928	1.700×10^{-5}	0.0239
3000	−4.49	70.12	9.797	328.6	0.909	1.694×10^{-5}	0.0238
3200	−5.79	68.36	9.797	327.8	0.891	1.687×10^{-5}	0.0237
3400	−7.09	66.63	9.796	327.0	0.872	1.681×10^{-5}	0.0236
3600	−8.39	64.94	9.796	326.2	0.854	1.674×10^{-5}	0.0235
3800	−9.69	63.28	9.795	325.4	0.837	1.668×10^{-5}	0.0234
4000	−10.98	61.66	9.794	324.6	0.819	1.661×10^{-5}	0.0233
4200	−12.3	60.07	9.794	323.8	0.802	1.655×10^{-5}	0.0232
4400	−13.6	58.52	9.793	323.0	0.785	1.648×10^{-5}	0.0231
4600	−14.9	57.00	9.793	322.2	0.769	1.642×10^{-5}	0.0230
4800	−16.2	55.51	9.792	321.4	0.752	1.635×10^{-5}	0.0229
5000	−17.5	54.05	9.791	320.5	0.736	1.628×10^{-5}	0.0228
5200	−18.8	52.62	9.791	319.7	0.721	1.622×10^{-5}	0.0227
5400	−20.1	51.23	9.790	318.9	0.705	1.615×10^{-5}	0.0226
5600	−21.4	49.86	9.789	318.1	0.690	1.608×10^{-5}	0.0224
5800	−22.7	48.52	9.785	317.3	0.675	1.602×10^{-5}	0.0223
6000	−24.0	47.22	9.788	316.5	0.660	1.595×10^{-5}	0.0222
6200	−25.3	45.94	9.788	315.6	0.646	1.588×10^{-5}	0.0221
6400	−26.6	44.69	9.787	314.8	0.631	1.582×10^{-5}	0.0220
6600	−27.9	43.47	9.786	314.0	0.617	1.575×10^{-5}	0.0219
6800	−29.2	42.27	9.785	313.1	0.604	1.568×10^{-5}	0.0218
7000	−30.5	41.11	9.785	312.3	0.590	1.561×10^{-5}	0.0217
8000	−36.9	35.65	9.782	308.1	0.526	1.527×10^{-5}	0.0212
9000	−43.4	30.80	9.779	303.8	0.467	1.493×10^{-5}	0.0206
10,000	−49.9	26.50	9.776	299.5	0.414	1.458×10^{-5}	0.0201
12,000	−56.5	19.40	9.770	295.1	0.312	1.422×10^{-5}	0.0195
14,000	−56.5	14.17	9.764	295.1	0.228	1.422×10^{-5}	0.0195
16,000	−56.5	10.53	9.758	295.1	0.166	1.422×10^{-5}	0.0195
18,000	−56.5	7.57	9.751	295.1	0.122	1.422×10^{-5}	0.0195

Source: U.S. Standard Atmosphere Supplements, U.S. Government Printing Office, 1966. Based on year-round mean conditions at 45° latitude and varies with the time of the year and the weather patterns. The conditions at sea level ($z = 0$) are taken to be $P = 101.325$ kPa, $T = 15$°C, $\rho = 1.2250$ kg/m^3, $g = 9.80665$ m^2/s.

Solved Example 2:

If an airplane is flying at an altitude where the actual pressure and temperature are $4.72 \times 10^4 N/m^2$ and $255.7K$ respectively, calculate the pressure, temperature and density altitudes?

Solution:

Using standard atmospheric table, we find that

Pressure altitude $= 6km$ (i.e. altitude corresponding to pressure $= 4.7 \times 10^4 N/m^2$ in standard atmosphere table/figure)

Temperature altitude $= 5km$ (or 38.2 or $59.5km$) (altitude corresponding to Temp $= 255.7K$ is standard atmosphere)

Use equation of state $\rho = P/RT$ to find the density as; $\rho = 0.643 kg/m^3$

And then using the standard atmosphere table, as obtain:

Density altitude $= 6.24km$.

Airspeed

Airspeed is the speed of the aircraft relative to air. In aviation industry, synonyms like indicated air speed (IAS), true air speed (TAS), calibrated air speed(CAS), etc. are used to denote the same. It is important to know the subtle difference among them.

How to measure the speed of the aircraft?
By Using pitot-static tube".

Pitot-static tubes typically face free oncoming airflows to measure the difference of total and static pressure, which is used to find out the velocity of the flight vehicle using equation 2:

$$v = \sqrt{\frac{2(P_o - P_\infty)}{\rho}} \quad \text{..........................2}$$

Various Speeds

1. Indicated Air Speed (IAS):

The speed indicated by the airspeed indicator in the cockpit, which is based on the Pitot - static tube attached to the airplane.

2. Calibrated Air Speed (CAS):

The indicated airspeed correct for the position and instrument errors. In standard atmospheric conditions, this is equal to the True Air Speed (TAS).

3. Equivalent Air Speed (EAS):

The calibrated air speed corrected for adiabatic and compressibility effects. The altitude effects are included in this speed.

To explain the three airspeed, let us assume that an aircraft is in cruise at an altitude of h, where the density is ρ, dynamic pressure is **q** and the corresponding velocity is v. Now if we want to simulate the same dynamic pressure at mean sea level (q_o), the corresponding velocity is known as equivalent air speed.

4. True Air Speed (TAS)

The airspeed of the airplane relative to the undisturbed air.

True Air Speed (TAS) & Equivalent Air Speed (EAS) relation:

Suppose an airplane is flying at an altitude (ha) and experiencing dynamic pressure (Pa).

We define equivalent airspeed as that speed with which the aircraft needs to fly at sea level to duplicate the actual dynamic pressure (ha) at a given altitude.so, we can write;

$$\frac{1}{2}\rho_a V_{TAS}^2 = \frac{1}{2}\rho_o V_{EAS}^2$$

The relationship between true airspeed and indicated airspeed is given by Equation 3.

Hence,

$$v_{TAS} = v_{EAS}\sqrt{\frac{\rho_o}{\rho}} \qquad \text{.............................3}$$

where,

ρ_o - Density of air at sea level

ρ - Density of air at a given altitude

Pitot-Static tube

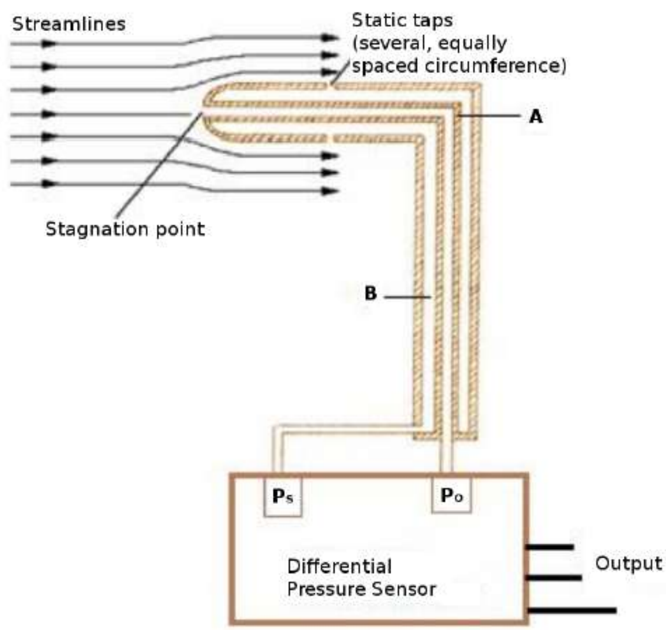

Figure 1: Schematic diagram representing the function of Pitot-Static tube

1. A schematic drawing of a pitot-static tube is presented in the Figure 1. It consists of two concentric tubes, say A and B.

2. The rear ends of these two tubes are connected to a differential pressure sensor.
Typically a pressure sensor is a micro electro-mechanical device that is capable of measuring pressure and provides an equivalent analog output.

3. The front end of the tube A is open to free stream to trap the total pressure (Po) during the flight.

4. Whereas tube B is a closed mouth tube having equally spaced peripheral holes to communicate with the surrounding air. Thus, tube B will be capturing the static pressure (Ps).

5. The output of the pressure sensor is usually in volts (V). In order to use this setup for velocity measurement, one has to calibrate the pressure sensor. Once the calibration chart is available for the specific pressure sensor, we can convert voltage (V) to Pascal (Pa).

Stagnation Pressure or Total Pressure (Po)
It is the pressure measured at a point where the molecules are brought to rest isentropically.

Static Pressure (Ps)
It is the pressure exerted by the fluid due to its random motion.