CHAPTER ONE

1.1 Introduction:

The main objective of the study of the strength of materials is to provide the engineer with the means of analyzing and designing various machines and load bearing structures.

1.2 Stress

Stress is the internal resistance offered by a unit area of the material from which a member is made to an externally applied load as shown in fig(1.1). Direct or normal stress σ is calculated using the following equation:

$$\sigma = \frac{\text{Applied Load}}{\text{Original Cross sectionalArea resisting the Force}} = \frac{P}{A}$$

And has a unit of **Pa** or $\boldsymbol{N/m^2}$.

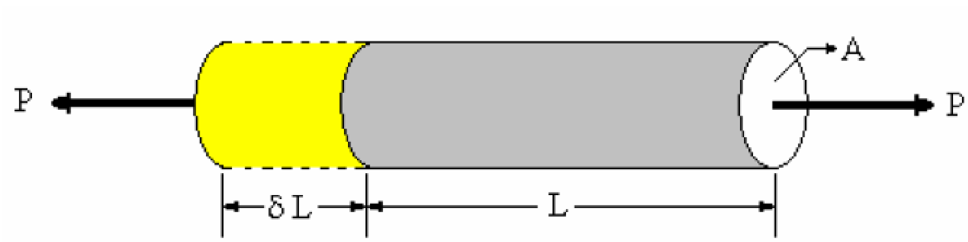


Fig.(1.1)

1.3 Strain

No material is perfectly rigid. Under the action of forces a rubber undergoes changes in shape and size. This phenomenon is very well known to all since in case of rubber, even for small forces deformations are quite large. Actually all materials including steel, cast iron, brass, concrete, etc. undergo similar deformation when loaded. But the deformations are very small and hence we cannot see them with naked eye. There are instruments like extensometer, electric strain gauges which can measure extension of magnitude 1/100th, 1/1000th of a millimeter. There are machines like universal testing machines in which bars of different materials can be subjected to accurately known forces of magnitude as high as 1000 kN. The studies have shown that the bars extend under tensile force and shorten under compressive forces as shown in Fig.1.2.

The change in length per unit length is known as linear strain.

Thus, linear strain $\boldsymbol{\varepsilon} = \frac{\text{Change in Length}}{\text{Original Length}} = \frac{\boldsymbol{\Delta L}}{\boldsymbol{L}} = \frac{\boldsymbol{\delta L}}{\boldsymbol{L}}$

Percentage Strain= $\frac{\Delta L}{L} \times \mathbf{100}\%$

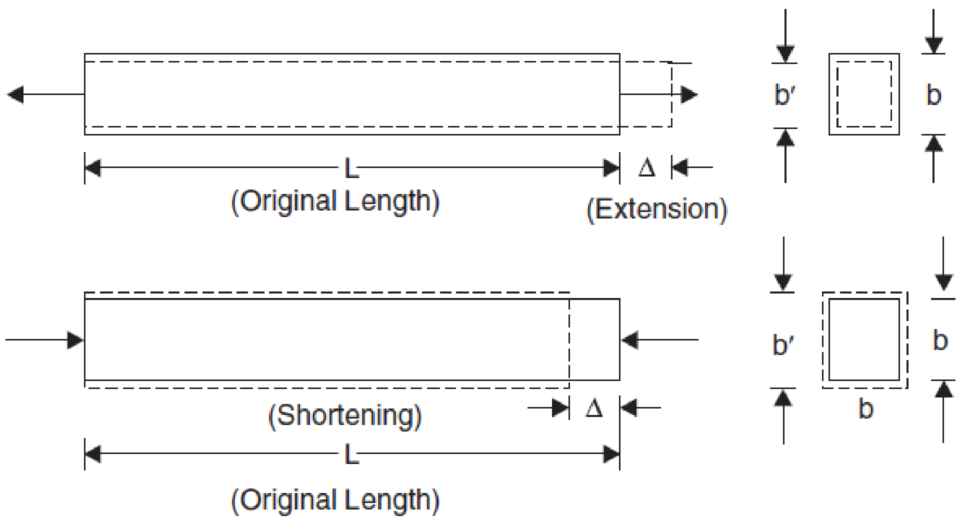


Fig. 1.2

When changes in longitudinal direction is taking place changes in lateral direction also take place. The nature of these changes in lateral direction are exactly opposite to that of changes in longitudinal direction *i.e.*, if extension is taking place in longitudinal direction, the shortening of lateral dimension takes place and if shortening is taking place in longitudinal direction extension takes place in lateral directions (See Fig. 1.2).

The lateral strain may be defined as changes in the lateral dimension per unit lateral dimension. Thus,

$$Lateral\ Strain\ = \frac{Change\ in\ Lateral\ Dimension}{Origenal\ Lateral\ Dimension} = \frac{b' - b}{b} = \frac{\delta b}{b}$$

Tensile stresses and strain are considered positive increase in length. **Compressive stresses and strain** are considered negative producing a decrease in length.

1.4 Types of Materials

Materials may be classified into:-

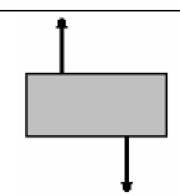
1) Elastic material which undergoes a deformation when subjected to an external loading such that, the deformation disappears on the removed of the loading, (Rubber).

2) A plastic material undergoes a continuous deformation during the period of loading and the deformation is permanent and the material does not regain its original dimensions on the removal of the loading, (Aluminum).

3) A rigid material does not undergo any deformation when subjected to an external loading, (Glass and Cast iron).

1.5 Types of Loads

1- Dead loads: static in nature such as the self weight of the roof.
2- Live loads: fluctuating in nature, does not remain constant such as a weight of a vehicle moving on a bridge.
3- Tensile loads.
4- Compressive loads.
5- Shearing loads.

Load	*Stress*	*Strain*	
Tensile	Tensile	Tensile	
Compressive	Compressive	Compressive	
Shearing	Shear	Shear	

1.6 Stress-Strain relationships

Hooke's law and Young's modulus

The actual values of modulus of elasticity *E* and maximum stress ***σ*** *ultimate* are determined by carrying out a standard tensile test on a specimen of the material. The bar is subjected to a gradually increasing tensile load until failure occurs. Measurements of the change in length of a selected gauge length of the bar are recorded throughout the loading operation by means of extensometers. And a graph of load against extension or stress against strain is produced as shown in Fig.1.3.

1. Form ***OA*** Hooke's low is obeyed, i.e. the material behaves **elastically** and stress is proportional to strain, giving the straight line graph indicated. For elastic materials, stress is proportional to strain:

$$E = \frac{Stress}{Strain} = \frac{\sigma}{\varepsilon} = \frac{P/A}{\boldsymbol{\delta L}/L}$$ and the unit of *E* is N/m^2 $(GN/m)^2$.

2. After ***A*** the linear nature of the graph disappears and this point is termed the limit of proportionality.
3. **Elastic limit (B)**, i.e. the deformation are completely recovered when the load is removed (i.e. strain returns to zero), but Hook's law does not apply.
4. ***Upper Yield Point (C):*** *This is the stress at which, the load starts reducing and the extension increases.* This phenomenon is called yielding of material
5. ***Lower Yield Point (D):*** *At this stage the stress remains same but strain increases for some time.*

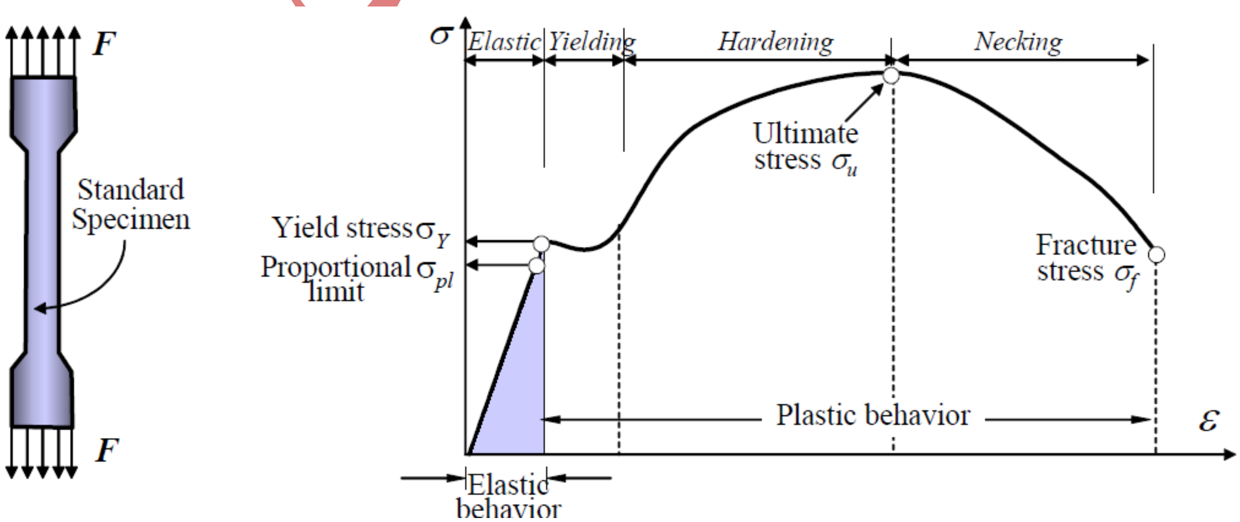


Fig.1.3 Material test and *Stress-Strain Diagram*

6. Beyond the yield point some increase in load is required to take the strain to point ***E***, between ***D*** and ***E*** the material is said to be in the **elastic-plastic state.**

7. Beyond ***E*** the cross-sectional area of the bar begins to reduce rapidly over a relatively small length of the bar and the bar is said to neck. This necking takes place whilst the load reduces, and **fracture** of the bar finally occurs at point ***F***.

8. Stress at failure, termed the **maximum or ultimate tensile stress** is given by the load at ***E*** divided by the original cross-sectional area of the bar. (Tensile Strength)

1.7 Proof stress

It should be noted that certain materials, such as aluminum alloy and high tensile steel, do not exhibit a definite yield point, such as that shown in Figure 1.4. For such cases, a 0.1%or 0.2% proof stress is used instead of a yield stress, as shown in Figure 1.4. To determine the 0.1% proof stress, a strain of 0.1% is set off along the horizontal axis of Figure 1.4, and a straight line is drawn from this point, parallel to the bottom section of the straight line part of the stress-strain relationship.
The 0.1% proof stress is measured where the straight line intersects the stress-strain curve, as shown in Figure 1.4. A similar process is used to determine the 0.2% proof stress.

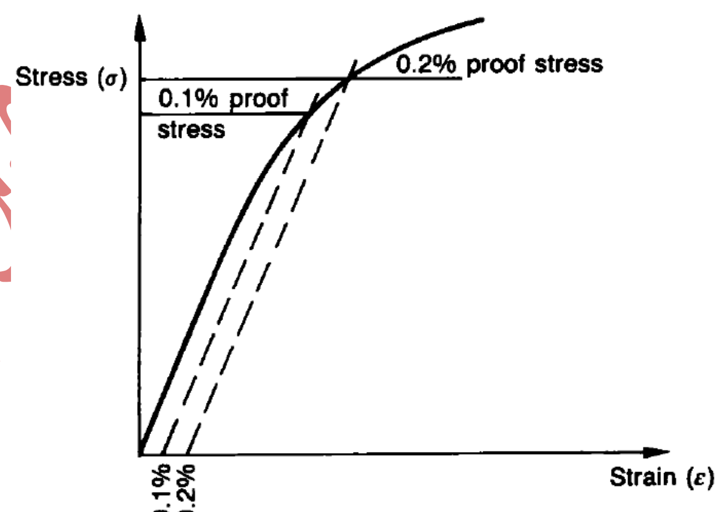


***Figure* 1.4** Stress-strain curve for aluminum alloy or a typical high tensile steel

1.8 EXTENSION / SHORTENING OF A BAR

Consider the bars shown in Fig. 1.5

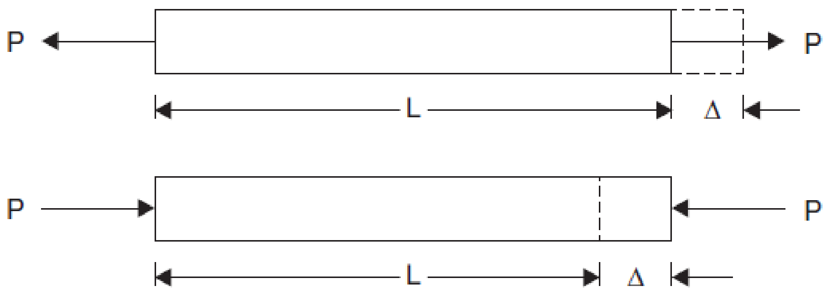


Fig. 1.5

From equation Stress is $\sigma = \frac{P}{A}$

From equation Strain is $\varepsilon = \frac{\Delta L}{L}$

From Hooke's Law we have, $E = \frac{Stress}{Strain}$

$$E = \frac{\frac{P}{A}}{\frac{\Delta L}{L}} = \frac{PL}{A\Delta L} \rightarrow \Delta L = \frac{PL}{AE}$$

1.9 Toughness

The area under the entire stress-strain curve from zero to rupture gives the property known as the **modulus of toughness** (O-F) [The energy per unit volume necessary to rupture the material].

1.10 Modulus of Resilience

The area the stress-strain curve and if evaluated from zero to the elastic limit(O-B) it is defined as the maximum strain energy per unit volume that a material will absorb without permanent deformation.

1.11 Ductile Materials

The capacity of a material to allow large extensions, i.e. the ability to be drawn out plastically is termed its ductility.
A quantitative value of the ductility is obtained by measurement of the percentage elongation or percentage reduction in area

Percentage Elongation $= \frac{\text{increase in gauge length to fracture}}{\text{Original gauge length}} \times 100 = \frac{L'-L}{L} \times 100$

Reduction in area$= \frac{\text{Reduction in cross} - \text{sectional area of necked portion}}{\text{Original area}} = \frac{A'-A}{A} \times 100$

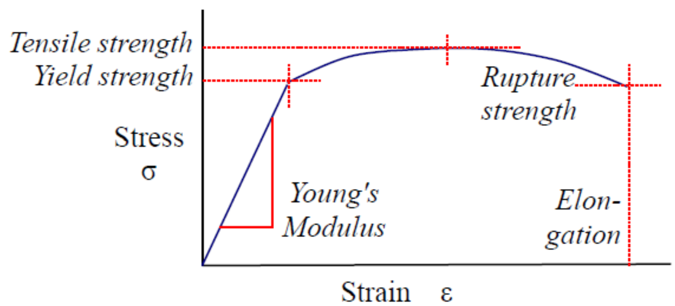
Where
L – original length, *L′*– length at rupture
A–original cross-sectional area, *A′*–minimum cross-sectional area

1.12 Malleability

Materials ability to be hammered out into thin sheets such as lead is called malleability.

1.13 Brittle Materials

They exhibit relatively small extensions to fracture such as glass and cast iron. There is little or no necking at fracture for brittle materials.



Example 1.1

A rod 150 cm long and of diameter 2 cm is subjected to an axial pull of 20 KN. If the modulus of elasticity of the material of the rod is $2 \times 10^5\ N/mm^2$; determine : i)the stress ;ii)the strain and iii) the elongation of the rod.

Example 1.2

Find the minimum diameter of a steel wire, which is used to raise a load of 4000N if the stress in the rod is not to exceed 95 MN/m^2.

Example 1.3

Find the Yong's Modulus of a brass rod of diameter 25 mm and of length 250 mm which is subjected to a tensile load of 50 KN when the extension of the rod is equal to 0.3 mm.

Example 1.4

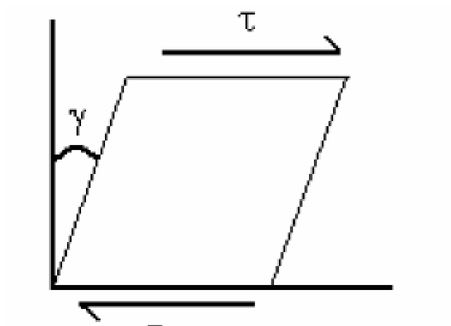
A 70 kN compressive load is applied to a 5 cm diameter, 3 cm tall, steel cylinder. Calculate stress, strain, and deflection.

1.14 Shear Stress

Material is subjected to a set of equal opposite forces. There is a tendency for one layer of the material to slide over another to produce the form of failure, if this failure is restricted then shear stress (τ) is set up.

$$\tau = \frac{\text{Shear load}}{\text{Area resisting shear}} = \frac{P}{A}$$

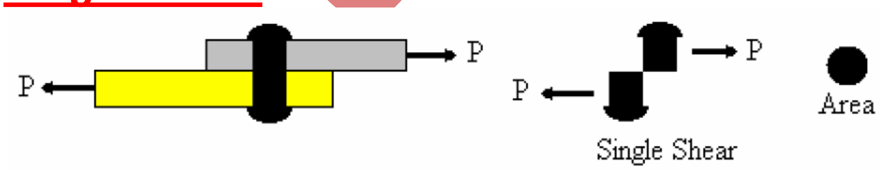
The shear stress is tangential to the area and has units similar to normal stress, ***Pa*** or ***N/m*²**.
Shear strain (γ) is measured in radians (non dimensional) has no units.



Within the elastic range shear strain is proportional to shear stress.

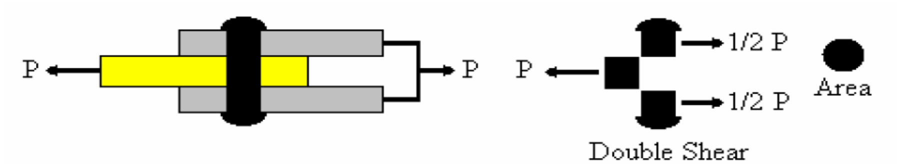
Modulus of rigidity (G) $= \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\gamma}$

Single Shear



$$\tau = \frac{P}{A}$$

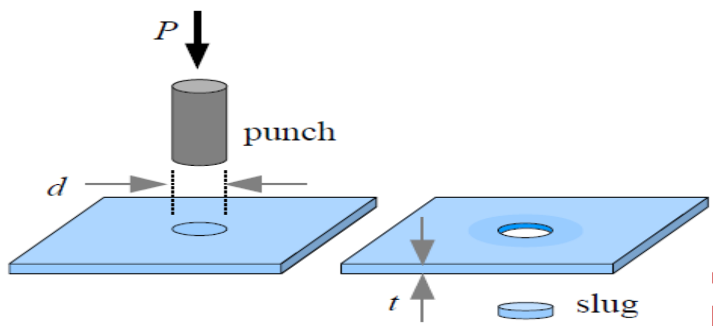
Double Shear



$$\tau = \frac{P}{2A}$$

Example 1.5

A 3 mm thick aluminum sheet is cut with a 4 cm diameter round punch. If the punch exerts a force of 6 kN, what is the shear stress in the sheet?



1.15 Factor of Safety

Factor of Safety $= \dfrac{\text{MaximumStress}}{\text{Allowable working Stress}}$ modified to

Factor of Safety$= \dfrac{\text{Yield Stress (or Proof Stress)}}{\text{Allowable working Stress}}$

Typical values range from 2.5 (for relatively low consequence, static load) to 10 (for shock load and high Safety risk applications).

The actual strength of a structure must exceed its required strength.

$$\text{Factor of safety n } = \frac{\text{Actual Strength}}{\text{Required Strength}}$$

n > 1 to avoid failure.
Usually *n* is chosen to be between 1 and 10.
Commercial aircraft: $1.2 < n < 1.5$
Military aircraft : $n < 1.1$ (but the crews wear parachutes!)
Missiles : $n = 1$ (not expected to return!)
For aircraft, small factors of safety are necessary to keep weight low and are justified by sophisticated modeling, testing of the actual materials used, extensive testing of prototype designs, and rigourous in-service inspections.

1.16 Allowable Stresses and Loads

To avoid permanent deformation of a structure when the loads are removed, we try to load the structure only in the elastic region. Hence, we can calculate an allowable stress based on the yield stress.

$$Allowable\ stress\ (or\ working\ stress) = \frac{\text{YieldStrength}}{\text{Factor of safety}} = \frac{\sigma_y}{n}$$

For prismatic bars in direct tension or compression (no buckling), allowable loads or required areas can be found once allowable stresses are calculated.

$$P_{allow} = \sigma_{allow}\ A_o$$

Example 1.6

A steel bar with diameter 30 mm functions in tension as part of a truss. We do not want the bar to yield. An experienced design engineer recommends a safety factor of 2.5 for this application. What is the allowable load?

1.17 BARS WITH CROSS-SECTIONS VARYING IN STEPS

A typical bar with cross-sections varying in steps and subjected to axial load is as shown in Fig.1.6(*a*). Let the length of three portions be L_1, L_2 and L_3 and the respective cross-sectional areas of the portion be A_1, A_2, A_3 and E be the Young's modulus of the material and P be the applied axial load. Figure 1.6(*b*) shows the forces acting on the cross-sections of the three portions. It is obvious that to maintain equilibrium the load acting on each portion is *P* only. Hence stress, strain and extension of each of these portions are as listed below:

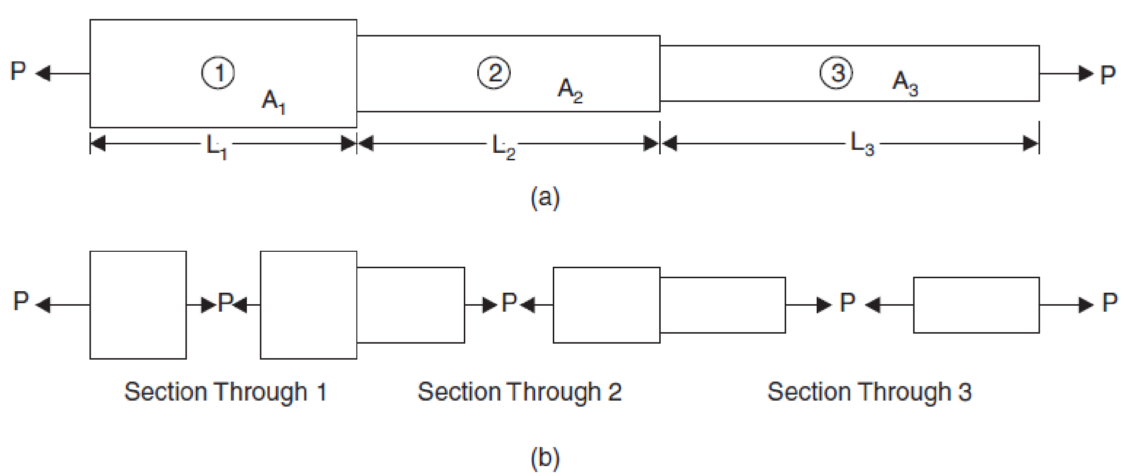


Fig. 1.6. Typical Bar with Cross-section Varying in Step

Hence total change in length of the bar

$$\Delta = \Delta_1 + \Delta_2 + \Delta_3 = \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E} = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]$$

The stress for the each section

$$\sigma_1 = \frac{P}{A_1}\ ,\ \sigma_2 = \frac{P}{A_2} \quad \text{and} \quad \sigma_3 = \frac{P}{A_3}$$

Similarly the strain for the each section

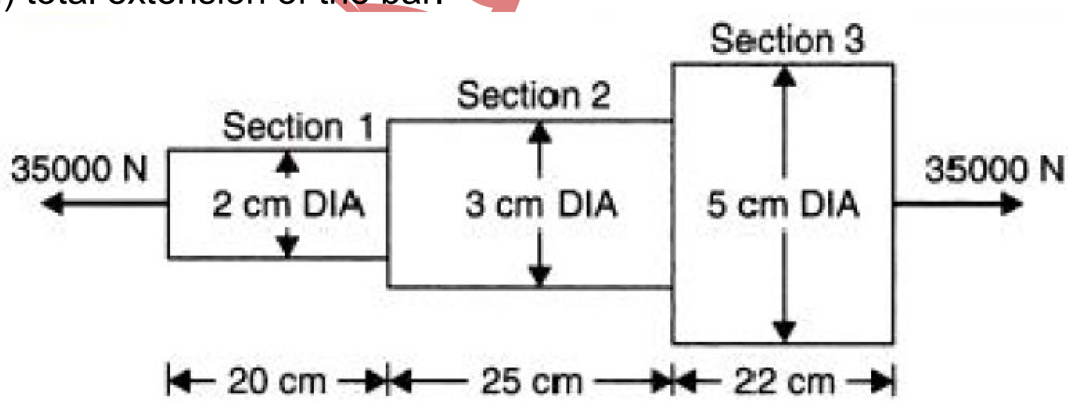
$$\varepsilon_1 = \frac{\sigma_1}{E} = \frac{P}{A_1E}\ ,\ \varepsilon_2 = \frac{\sigma_2}{E} = \frac{P}{A_2E} \quad \text{and} \quad \varepsilon_3 = \frac{\sigma_3}{E} = \frac{P}{A_3E}$$

EXAMPLE 1.7

An axial pull of 35000N is acting on a bar consisting of three lengths as shown in fig. below . If the Young's modulus $= 2.1 \times 10^5 N/mm^2$, determine:

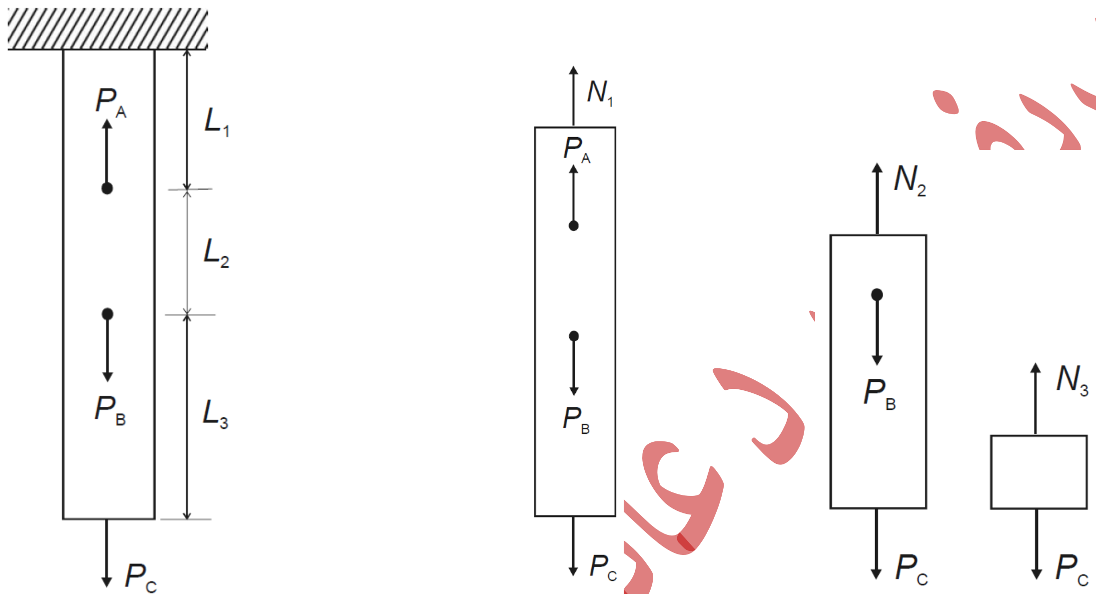
i) stresses in each section and

ii) total extension of the bar.



1.18 Bars with intermediate axial loads

The change in length of the bar δ can be determined by adding algebraically the elongations and shortenings of the individual segments.
Identify segments of the bar (*i*).
Draw free-body diagrams and determine internal axial forces N_i.



Note that *Ni* is always drawn as if the bar were in **tension**.

Note that we have not needed to include the reaction force.

$$\sum F_1 = N_1 + P_A - P_B - P_C = 0 \quad \rightarrow \quad N_1 = P_B + P_C - P_A$$
$$\sum F_2 = N_2 - P_B - P_C = 0 \quad \rightarrow \quad N_2 = P_B + P_C$$
$$\sum F_3 = N_3 - P_C = 0 \quad \rightarrow \quad N_3 = P_C$$

Determine change in length δ_i for each segment.

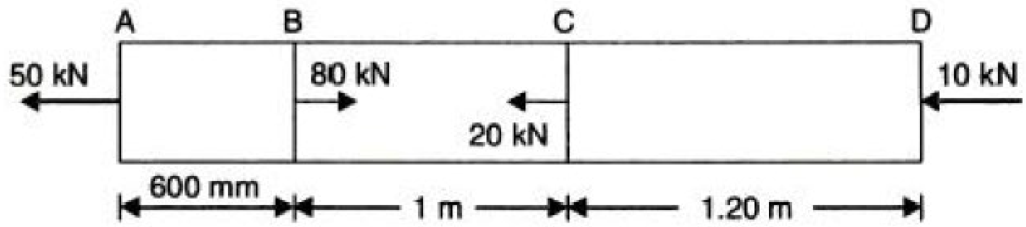
$$\delta_1 = \frac{N_1 L_1}{EA} \qquad \delta_2 = \frac{N_2 L_2}{EA} \qquad AND \qquad \delta_3 = \frac{N_3 L_3}{EA}$$

Add the δ_i's algebraically to determine the overall change in length.

$$\delta = \delta_1 + \delta_2 + \delta_3$$

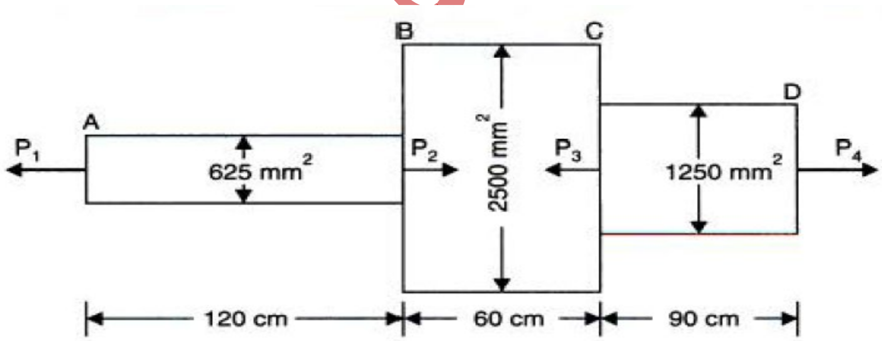
EXAMPLE 1.8

A brass bar, having cross sectional area of 1000 mm^2, is subjected to axial forces as shown in fig below.
Find the total elongation of the bar. Take $E = 1.05 \times 10^5 N/mm^2$



EXAMPLE 1.9

A member ABCD is subjected to point loads P_1, P_2, P_3 and P_4 as shown in Fig. below. Calculate the force P_2 necessary for equilibrium.
If $P_1 = 45\ KN, P_3 = 450\ KN$ and $P_4 = 130\ KN$. Dtermine the total elongation of the member , assuming the modulus of elasticity to be $2.1 \times 10^5 N/mm^2$



1.19 BARS WITH CONTINUOUSLY VARYING CROSS-SECTIONS

When the cross-section varies continuously, an elemental length of the bar should be considered and general expression for elongation of the elemental length derived. Then the general expression should be integrated over entire length to get total extension. The procedure is illustrated with Examples 1.10 and 1.12.

Example 1.10.

A bar of uniform thickness **'t'** tapers uniformly from a width of $\mathbf{b_1}$ at one end to $\mathbf{b_2}$ at other end in a length **'L'** as shown in Fig. 1.7. Find the expression for the change in length of the bar when subjected to an axial force **P**.

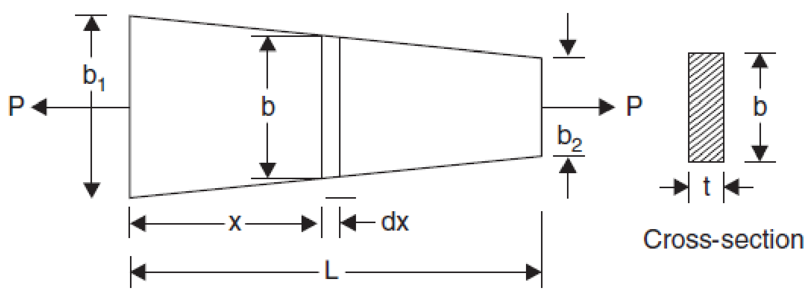


Fig.1.7

Solution:

Consider an elemental length dx at a distance *x* from larger end.

Rate of change of breadth is $\frac{b_1 - b_2}{L}$

$$\frac{b_1 - b}{x} = \frac{b_1 - b_2}{L} \quad \rightarrow b_1 - b = \frac{b_1 - b_2}{L} x \ \rightarrow b = b_1 - \frac{b_1 - b_2}{L} x$$

Hence , width at section x is $b = b_1 - \frac{b_1 - b_2}{L} x = b_1 - kx$

where

$$k = \frac{b_1 - b_2}{L}$$

Cross-section area of the element = $A = t(b_1 - kx)$

Since force acting at all sections is ***P*** only,

Extension of element $= \frac{Pdx}{AE}$ [where length = *dx*]

$$= \frac{Pdx}{(b_1 - kx)tE}$$

Total extension of the bar $\Delta = \int_0^L \frac{P\,dx}{(b_1 - kx)tE} = \frac{P}{tE}\int_0^L \frac{dx}{(b_1 - kx)}$

$$= \frac{P}{tE}\left(\frac{\mathbf{1}}{-\boldsymbol{k}}\right)[\ln(b_1 - kx)]_0^L$$

$$= \frac{-P}{tEk}\left[\ln(b_1 - \frac{b_1 - b_2}{L}x)\right]_0^L$$

$$= \frac{-P}{tEk}\left[(\ln b_1 - \ln(b_1 - b_2)\frac{L}{L} - (\ln b_1)\right]$$

$$= \frac{-P}{tEk}[(\ln b_1 - \ln(b_1 - b_2) - (\ln b_1)] = \frac{-P}{tEk}[-\ln(b_1 - b_2)]$$

$$= \frac{P}{tEk}[\ln(b_1 - b_2)]$$

$$= \frac{P}{tEk}\ln\frac{b_1}{b_2}$$

$$\therefore \Delta = \frac{PL}{tE(b_1 - b_2)}\ln\frac{b_1}{b_2}$$

Example 1.11:
A rectangular bar made of steel is 2.8 m long and 15mm thick. The rod is subjected to an axial tensile load of 40 KN. The width of the rod varies from 75mm at one end to 30mm at the other.
Find the extension of the rod if $E = 2 \times 10^5\ N/mm^2$

Solution:

$$\therefore \Delta = \frac{PL}{tE(b_1 - b_2)}\ln\frac{b_1}{b_2} = \frac{40000 \times 2800}{15 \times 2 \times 10^5\,(75 - 30)}\ln\frac{75}{30}$$

$$\therefore \Delta = \mathbf{0.8296 \times 0.9163 = 0.76}\,\boldsymbol{mm}$$

Example 1.12.
*A tapering rod has diameter $\boldsymbol{d_1}$ at one end and it tapers uniformly to a diameter $\boldsymbol{d_2}$ at the other end in a length **L** as shown in Fig. 1.8. If modulus of elasticity of the material is E, find its change in length when subjected to an axial force **P**.*

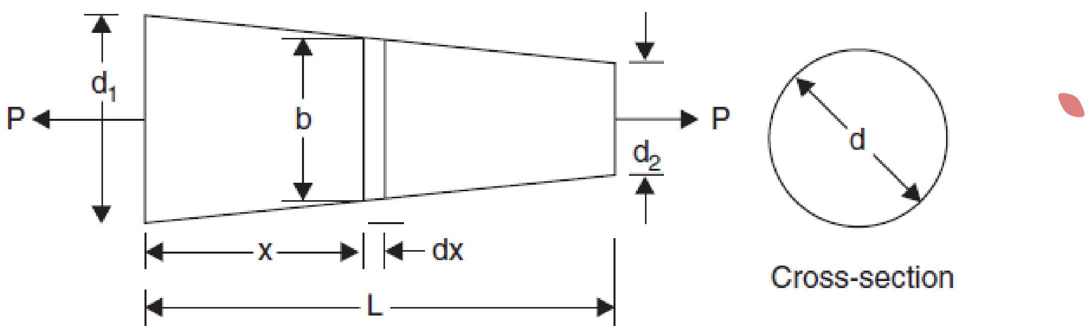


Fig. 1.8

***Solution*:**

Change in diameter in length *L* is $d_1 - d_2$ $\left[Hence\ \frac{d_1-d}{x} = \frac{d_1-d_2}{L}\right]$

Rate of change of diameter, $k = \frac{d_1-d_2}{L}$

Consider an elemental length of bar *dx* at a distance *x* from larger end.
The diameter of the bar at this section is $d = d_1 - kx$

Cross-sectional area $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(d_1 - kx)^2$

Extension of the element $= \frac{Pdx}{\frac{\pi}{4}(d_1-kx)^2 E}$

Extension of the entire bar

$$\Delta = \int_0^L \frac{Pdx}{\frac{\pi}{4}(d_1-kx)^2 E} = \frac{4P}{\pi E}\int_0^L \frac{dx}{(d_1-kx)^2}$$

$$\Delta = \frac{4P}{\pi E}\int_0^L (d_1 - kx)^{-2}dx$$

$$\Delta = \frac{4P}{(-1)\pi E(-k)}\left(\frac{1}{d_1 - kx}\right)_0^L = \frac{4P}{\pi Ek}\left(\frac{1}{d_1 - kx}\right)_0^L$$

$$\Delta = \frac{4P}{\pi E \frac{(d_1 - d_2)}{L}} \left(\frac{1}{d_1 - \frac{d_1 - d_2}{L} x} \right)_0^L = \frac{4P}{\pi E \frac{(d_1 - d_2)}{L}} \left\{ \left(\frac{1}{d_1 - \frac{d_1 - d_2}{L} L} \right) - \frac{1}{d_1} \right\}$$

$$= \frac{4PL}{\pi E (d_1 - d_2)} \left(\frac{1}{d_2} - \frac{1}{d_1} \right)$$ **Since** $d_2 = d_1 - kL$

$$\therefore \Delta = \frac{\mathbf{4PL}}{\boldsymbol{\pi E}(d_1 - d_2)} \left(\frac{(d_1 - d_2)}{d_1 d_2} \right) = \frac{\mathbf{4PL}}{\boldsymbol{\pi E} d_1 d_2}$$

Example 1.13.

A rod , which tapers uniformly from 40mm diameter to 20 mm diameter in a length of 400mm is subjected to an axial load of 5000N.
If the $E = 2.1 \times 10^5 \, N/mm^2$ .Find the extension of the rod

Solution:

$$\therefore \Delta = \frac{4PL}{\pi E d_1 d_2} = \frac{4 \times 5000 \times 400}{\pi \times 2.1 \times 10^5 \times 40 \times 20} = 0.01515mm$$