## Chapter One

### 1.1. Thermodynamics working fluids

Behavior of the working substance is very essential factor in understanding thermodynamics. In this lecture, focus is given to pure substances such as gases and steam properties and how they are interrelated are important in the design and operation of thermal systems. The ideal gas equation is very well known approximation in relating thermal properties for a state point, or during a process. However, not all gases are perfect, and even the same gas, may behave as an ideal gas under certain circumstances, then changes into non-ideal, or real, under different conditions. There are other equations, or procedures to deal with such conditions. Steam or water vapour is not governed by simple equations but properties of water and steam are found in steam tables or charts.

### 1.2. The Ideal Gas

Ideally, the behavior of air is characterized by its mass, the volume it occupies, its temperature and the pressure condition in which it is kept. An ideal gas is governed by the perfect gas equation of state which relates the state pressure, volume and temperature of a fixed mass ( $m$ is constant) of a given gas ( $R$ is constant ) as:

$$
\begin{equation*}
\frac{P V}{T}=m R \tag{1}
\end{equation*}
$$

Where

$$
\begin{aligned}
& \mathrm{P} \text { - Pressure }(\mathrm{Pa}) \\
& V-\text { Volume }\left(\mathrm{m}^{3}\right) \\
& \mathrm{T}-\text { Absolute } \mathrm{Temperature}(\mathrm{~K}) \\
& \mathrm{T}(\mathrm{~K})=273+\mathrm{t}\left({ }^{\circ} \mathrm{C}\right) \\
& \mathrm{m}-\text { mass }(\mathrm{kg}) \\
& \mathrm{R}-\text { gas constant }(\mathrm{J} / \mathrm{kg} \mathrm{~K})
\end{aligned}
$$

The equation of state can be written in the following forms, depending on what is needed to be calculated

1. In terms of the pressure $P=\frac{m R T}{V}$
2. In terms of the volume $\quad V=\frac{m R T}{P}$
3. In terms of the mass $\quad m=\frac{P V}{R T}$
4. In terms of the temperature $T=\frac{P V}{m R}$
5. In terms of the gas constant $R=\frac{P V}{m T}$
6. In terms of the density

$$
\rho=\frac{m}{V}=\frac{P}{R T}
$$

The specific gas constant $R$, is a property related to the molar mass (M) in $\mathrm{kg} / \mathrm{kmol}$, of the gas and the Universal gas constant Ro as :

$$
\begin{equation*}
R=R_{o} / M \tag{8}
\end{equation*}
$$

where Ro $=8314.3 \mathrm{~J} / \mathrm{kg} \mathrm{K}$
The ideal gas equation can also be written on time basis, relating the mass flow rate ( $\mathrm{kg} / \mathrm{s}$ ) and the volumetric flow rate ( $\mathrm{m}^{3} / \mathrm{s}$ ) as follows:

$$
\begin{equation*}
P V_{t}=m_{t} R T \tag{9}
\end{equation*}
$$

### 1.2.2 Alternative Gas Equation During A Change Of State:

The equation of state can be used to determine the behavior of the gas during a process, i.e. what happens to its temperature, volume and pressure if any one property is changed. This is defined by a simple expression relating the initial and final states such as :

$$
\begin{equation*}
\frac{P_{1} V_{1}}{T_{1}}=\frac{P_{2} V_{2}}{T_{2}} \tag{10}
\end{equation*}
$$

this can be rewritten in terms of the final condition, hence the following equations are geerated:
Final Pressure $\quad P_{2}=P_{1} \times \frac{T_{2}}{T_{1}} \times \frac{V_{1}}{V_{2}}$
Final Temperature $T_{2}=T_{1} \times \frac{P_{2}}{P_{1}} \times \frac{V_{2}}{V_{1}}$
Final Volume $\quad V_{2}=V_{1} \times \frac{P_{1}}{P_{2}} \times \frac{T_{2}}{T_{1}}$

### 1.2.3 Thermodynamic Processes for gases

There are four distinct processes which may be undertaken by a gas (see Figure 1.6):-

a) Constant volume process, known as isochoric process; given by:-

$$
\begin{equation*}
\frac{P_{1}}{T_{1}}=\frac{P_{2}}{T_{2}} \tag{14}
\end{equation*}
$$

b) Constant pressure process; known as isobaric process, given by:-

$$
\begin{equation*}
\frac{V_{1}}{T_{1}}=\frac{V_{2}}{T_{2}} \tag{15}
\end{equation*}
$$

c) Constant temperature process, known as isothermal process, given by:-

$$
\begin{equation*}
P_{1} V_{1}=P_{2} V_{2} \tag{16}
\end{equation*}
$$

d) Polytropic process given by:-

$$
\begin{equation*}
P_{1} V_{1}^{n}=P_{2} V_{2}^{n} \tag{17}
\end{equation*}
$$

Note when $n=C_{p} / C_{v}$, the process is known as adiabatic process. A process is a path in which the state of the system change and some properties vary from their original values.

The important equation for polytropic process :
What equation for an adiabat corresponds to
$P V=m R T_{1}$ for an isotherm?
On raising the gas temperature by $\Delta T$, the change in the internal ener-gy-the sum of molecular kinetic energy, rotational energy and vibrational energy (if any),

$$
\Delta U=m C_{v} \Delta T
$$

This is always true: whether or not the gas is changing volume is irrelevant, all that counts in $\Delta U$ is the sum of the energies of the individual molecules (assuming as we do here that attractive or repulsive forces between molecules are negligible).
In adiabatic compression, all the work done by the external pressure goes into this internal energy,
$-P d V=m C_{v} d T$
Divide the both sides by $m R T$ (since $P V=m R T$, that's OK) to find

$$
\begin{equation*}
-\frac{m R T d V}{V}=m C_{v} d T \quad \text { and } \quad-\frac{d V}{V}=\frac{C_{v} d T}{R T} \tag{a}
\end{equation*}
$$

Recall now that $C_{p}=C_{v}+R$, and $C_{p} / C_{v}=n$. It follows that $\frac{C_{p}-C_{v}}{C_{v}}=\frac{R}{C_{v}}=n-1$
substitute in eq. (a)
$-\frac{d V}{V}=\frac{1}{(n-1)} \frac{d T}{T}$ and $-(n-1) \frac{d V}{V}=\frac{d T}{T}$
$\ln T+(n-1) \ln V=$ const.
from which the equation of an adiabatic is
$T V^{n-1}=$ constant
$\frac{P V}{m R} V^{n-1}=$ constant
$\therefore P V^{n}=$ constant

From equation (11) $\frac{P_{2}}{P_{1}}=\frac{T_{2}}{T_{1}} \times \frac{V_{1}}{V_{2}}----(a)$
and polytropic process $\frac{P_{2}}{P_{1}}=\left(\frac{V_{1}}{V_{2}}\right)^{n}---(b)$
from eq.(a) and eq.(b) we get
$\left(\frac{V_{1}}{V_{2}}\right)^{n}=\frac{T_{2}}{T_{1}} \times \frac{V_{1}}{V_{2}} \quad$ and $\frac{\left(\frac{V_{1}}{V_{2}}\right)^{n}}{\left(\frac{V_{1}}{V_{2}}\right)}=\frac{T_{2}}{T_{1}} \quad$ and $\left(\frac{V_{1}}{V_{2}}\right)^{n-1}=\frac{T_{2}}{T_{1}}$
$\therefore \frac{V_{1}}{V_{2}}=\left(\frac{T_{2}}{T_{1}}\right)^{\frac{1}{n-1}}---(c)$

$$
\begin{aligned}
& \frac{P_{2}}{P_{1}}=\left(\frac{V_{1}}{V_{2}}\right)^{n} \\
& \quad \therefore \frac{V_{1}}{V_{2}}=\left(\frac{P_{2}}{P_{1}}\right)^{\frac{1}{n}}---(d)
\end{aligned}
$$

Substitute eq.(d) in eq. (c) we get
$\left(\frac{P_{2}}{P_{1}}\right)^{\frac{1}{n}}=\left(\frac{T_{2}}{T_{1}}\right)^{\frac{1}{n-1}}$
Or
$\therefore T_{2}=T_{1}\left(\frac{P_{2}}{P_{1}}\right)^{\frac{(n-1)}{n}}$
There are six types of Processes associated with Thermodynamics:
Adiabatic : no heat transfer from or to the fluid Isothermal : no change in temperature of the fluid Isobaric : no change in pressure of the fluid Isochoric: no change in volume of the fluid Isentropic : no change of entropy of the fluid Isenthalpic : no change of enthalpy of the fluid

## 2. Laws of Thermodynamics

There are four laws which relates the thermodynamics of substances.

### 2.1 Zeroth Law of Thermodynamics

If an object with a higher temperature comes in contact with a lower temperature object, it will transfer heat to the lower temperature object. The objects will approach the same temperature, and in the absence of loss to other objects, they will maintain a single constant temperature. Therefore, thermal equilibrium is attained.
The "zeroth law" states that if two systems are at the same time in thermal equilibrium with a third system, they are in thermal equilibrium with each other.


Figure 2.1: Analogy of the Zeroth Law of Thermodynamics.
If $\mathbf{A}$ and $\mathbf{C}$ are in thermal equilibrium with $\mathbf{B}$, then $\mathbf{A}$ is in thermal equilibrium with B. Practically this means that all three are at the same temperature, and it forms the basis for comparison of temperatures. The Zeroth Law states that:-
"two systems which are equal in temperature to a third system are equal in temperature to each other".

### 2.1.1 International Temperature Scale

This scale is used to calibrate temperature measuring devices. It consists of a number of fixed points of known temperature which can be reproduced accurately (Table 2.1).

## Units of Temperature

The Kelvin (SI units) is the fraction(1/273.16) of the thermodynamic temperature of the triple point of water. Generally, conversion of Celsius to Kelvin: $\mathrm{T}(\mathrm{K})=\mathrm{T}\left({ }^{\circ} \mathrm{C}\right)+273.15$

Table 2.1 Example of Thermodynamic fixed temperatures

| Fixed point | Temperature (deg C) |
| :--- | :--- |
| Ice Point | 0.01 |
| Steam Point | 100 |

### 2.2 First Law of Thermodynamics

The first law of thermodynamics is the application of the conservation of energy principle.

### 2.2.1 First Law of Thermodynamics Applied to closed Systems

 consider a closed system where there is no flow into or out of the system, and the fluid mass remains constant. For such system, the first law statement is known as the Non-Flow Energy Equation, or NFEE abbreviated, it can be summarized as follows:$$
\begin{equation*}
\Delta U=Q-W \tag{25}
\end{equation*}
$$

The first law makes use of the key concepts of internal energy $(\Delta U)$, heat (Q), and system work (W).

### 2.2.2 Internal Energy $(\Delta \boldsymbol{U})$

Internal energy is defined as the energy associated with the random, disordered motion of molecules. It is separated in scale from the macroscopic ordered energy associated with moving objects; it refers to the invisible microscopic energy on the atomic and molecular scale. For example, a room temperature glass of water sitting on a table has no apparent energy, either potential or kinetic. But on the microscopic scale it is a seething mass of high speed molecules traveling at hundreds of meters per second. If the water were tossed across the room, this microscopic energy would not necessarily be changed when we superimpose an ordered large scale motion on the water as a whole. During a non flow process the change in internal energy is calculated assuming the closed's system volume remains constant (work $\mathrm{W}=0, \Delta U=Q$ ), the following equation is used

$$
\begin{equation*}
\Delta U=m \cdot C_{v} \cdot \Delta T \tag{26}
\end{equation*}
$$

## Where

$\boldsymbol{C}_{v}$ is the specific heat capacity of the fluid, and
$\Delta \boldsymbol{T}$ is the temperature difference during the process

### 2.2.3 Specific Heat

Heat may be defined as energy in transit from a high temperature object to a lower temperature object. An object does not possess "heat"; the appropriate term for the microscopic energy in an object is internal energy. The internal energy may be increased by transferring energy to the object from a higher temperature (hotter) object - this is properly called heating. In order to heat or cool a given quantity of a gas in a given time, the following equation is used:

## QuantityofHeat

$(Q)=\operatorname{mass}(m) x$ specific heat capacity (C) x temperature difference.
Since this heat exchange may take place
Either at constant pressure: $Q=\dot{m} C_{p}(T 2-T 1)$
Or at constant volume: $\quad Q=\dot{m} C_{v}(T 2-T 1)$
Where:
$\boldsymbol{C}_{\boldsymbol{p}}$ specific heat at constant pressure (kJ/kg K), see Table 2.2
$\boldsymbol{C}_{v}$ specific heat at constant volume (kJ/kg K) , see Table 2.2

$$
\begin{align*}
& \text { Note that fora perfect gas } \\
& \qquad C_{p}=C_{v}+R \text { and } n=C_{p} / C_{v} \tag{29}
\end{align*}
$$

## Specific Heat at Constant Volume $C_{v}$

Consider a closed system of unit mass, the first law of thermodynamics applied to this system is:-

$$
q-w=d u
$$

If the volume is constant, then $w=0$, it follows that $q=d u$
But

$$
q=C_{v} d T
$$

Hence $\quad d u=C_{v} d T$
Or $\quad C_{v}=d u / d T$
This is known as Joule's Law of internal energy which states that "the internal energy of a perfect gas depends upon its temperature only".

## Specific Heat at Constant Pressure $C_{p}$

 Consider a constant pressure non-flow process, the first law:-$$
q-w=d u
$$

For a constant pressure process

$$
W=P\left(V_{2}-V_{1}\right)=\left(P_{2} V_{2}-P_{1} V_{1}\right)
$$

Hence $\quad q-\left(P_{2} V_{2}-P_{1} V_{1}\right)=\mathrm{u}_{2}-\mathrm{u}_{1}$
Or $\quad q=\left(\mathrm{u}_{2}+P_{2} V_{2}\right)-\left(\mathrm{u}_{1}+P_{1} V_{1}\right)=\mathrm{h}_{2}-\mathrm{h}_{1}$
But

$$
q=C_{p} d T
$$

Hence $\quad \mathrm{h}_{2}-\mathrm{h}_{1}=C_{p}\left(T_{2}-T_{1}\right)$
Or $\quad C_{p}=\mathrm{dh} / \mathrm{dT}$
Relationship Between Specific Heats
Since $H=U+P V$

$$
\begin{aligned}
& d H=d U+d(P V) ; \quad d(P V)=d(m R T)=m R d T \\
& m C_{p} d T=m C_{v} d T+m R d T
\end{aligned}
$$

Therefore, $C_{p}=C_{v}+R \quad$ ie $C_{p}>C_{v}$
The ratio $C_{p} / C_{v}=n$ is called the adiabatic index.
The reason for the differences between $\boldsymbol{C}_{\boldsymbol{p}}$ and $\boldsymbol{C}_{\boldsymbol{v}}$ is that in the constant pressure process part of the heat transferred is used in doing work against the moving system boundary and all heat therefore is not available for raising the gas temperature.

Table 2.2 Specific heat capacities for air at standard atmosphere

| Temperature <br> K | $C_{p}$ <br> $\mathrm{~J} / \mathrm{kg} \mathrm{K}$ |  | $C_{v}$ <br> $\mathrm{~J} / \mathrm{kg} \mathrm{K}$ |
| :--- | :--- | :--- | :--- |
| 250 | 1003 | n <br> $C_{p} / C_{v}$ |  |
| 300 | 1005 | 716 | 1.401 |
| 350 | 1008 | 718 | 1.400 |
| 400 | 1013 | 721 | 1.398 |
| 450 | 1020 | 733 | 1.395 |
| 500 | 1029 | 742 | 1.391 |
| 550 | 1040 | 753 | 1.387 |
| 600 | 1051 | 764 | 1.381 |
| 650 | 1063 | 776 | 1.376 |
| 700 | 1055 | 788 | 1.370 |
| 750 | 1087 | 800 | 1.364 |
| 800 | 1099 | 812 | 1.359 |
| 900 | 1121 | 834 | 1.354 |
| 1000 | 1142 | 855 | 1.344 |

### 2.2.4 System Work

Work performed on or by the working fluid within a system's boundary is defined as the summation (or integration) of the product of pressure and volume of the fluid during a process.

$$
\begin{equation*}
W=\int P d V \tag{30}
\end{equation*}
$$

In calculating the process work, it is important to point out that for each process, the work done will be different, since there are four distinctly different processes, in the following sections, an expression for the work done will be evaluated for each process.
a: for a constant pressure process, the work during an isobaric process is simply :

$$
\begin{equation*}
W=P\left(V_{2}-V_{1}\right) \tag{31}
\end{equation*}
$$

b: for a constant volume process,

$$
d v=0 ;
$$

hence the work during an isochoric process is simply :

$$
W=0 \quad \ldots \ldots \ldots(32)
$$

c: for a constant temperature ,isothermal process,

$$
P V=m R T=c
$$

hence $W=\int \frac{c}{\mathrm{v}} d V=c \ln \frac{V_{2}}{V_{1}}=\mathrm{mR} \mathrm{T}_{1} \ln \frac{V_{2}}{V_{1}}$
but $\quad P_{2} V_{2}=P_{1} V_{1} \quad \therefore \frac{\mathrm{~V}_{2}}{\mathrm{~V}_{1}}=\frac{\mathrm{P}_{1}}{\mathrm{P}_{2}}$
$\therefore$ hence the work done can be written in terms of pressure ratio:

$$
\begin{equation*}
\therefore \mathrm{W}=\mathrm{m} \mathrm{R} \mathrm{~T}_{1} \ln \frac{\mathrm{P}_{1}}{\mathrm{P}_{2}} \tag{34}
\end{equation*}
$$

d: for an adiabatic ( polytropic ) process,

$$
P V^{n}=C
$$

integrating between states 1 and 2, the work done is derived:

$$
\begin{align*}
& P V^{n}=\text { constant }=P_{1} V_{1}^{n}=P_{2} V_{2}^{n} \\
& P=\frac{\text { constant }}{V^{n}}=\frac{P_{1} V_{1}^{n}}{V^{n}}=\frac{P_{2} V_{2}^{n}}{V^{n}} \\
& \int_{1}^{2} P d V=\text { constant } \int_{1}^{2} \frac{d V}{V^{n}}=\text { constant }\left(\frac{V^{-n+1}}{-n+1}\right)_{1}^{2} \\
& \int_{1}^{2} P d V=\frac{\text { constant }}{1-n}\left(V_{2}^{1-n}-V_{1}^{1-n}\right)=\frac{P_{2} V_{2}^{n} V_{2}^{1-n}-P_{1} V_{1}^{n} V_{1}^{1-n}}{1-n} \\
& W=\frac{P_{1} V_{1}-P_{2} V_{2}}{n-1} \quad \ldots \ldots \ldots(35) \tag{35}
\end{align*}
$$

and using the ideal gas definition, the work done can be written in terms of initial and final states temperatures:
Since $P_{1} V_{1}^{n}=P_{2} V_{2}^{n}=C$.
For an ideal gas $(P V=m R T)$ it becomes:

$$
\begin{equation*}
W=\frac{m R\left(T_{1}-T_{2}\right)}{n-1} \tag{36}
\end{equation*}
$$

2.2.5 First Law of Thermodynamics Applied to Closed Systems (Cycle)
Since in a cycle the working fluid undergoing changes in its state will retain its initial conditions at any fixed point along the cycle. Hence the energy equation applied to a cycle is:-
$\sum Q-\sum W=0$
where $\Sigma$ means the sum of heat or work around the cycle.
Practical Application of a Closed System (Cycle) - Assume compression and expansion to be adiabatic, from first law :-
$Q_{s}-Q_{r}=W_{e}-W_{c}$

## Where

$\boldsymbol{Q}_{\boldsymbol{s}}$ heat supplied
$\boldsymbol{Q}_{r}$ heat rejected
$\boldsymbol{W}_{\boldsymbol{c}}$ work of compression
$W_{e}$ work of expansion

### 2.2.6 First Law of Thermodynamics Applied to Open Systems

The first law of thermodynamics is based on the conversation of energy within a system.
Open systems are associated with those, which have a steady flow, thus the first law applied to such systems is known as the Steady Flow Energy Equation (SFEE):-

$$
\begin{align*}
& Q-W=(\Delta H+\Delta K E+\Delta P E) \\
& \quad=m\left[\left(h_{2}-h_{1}\right)+\frac{1}{2}\left(V_{2}^{2}-V_{1}^{2}\right)+g\left(Z_{2}-Z_{1}\right)\right] \tag{39}
\end{align*}
$$

| Variable | Symbol | Units |
| :--- | :---: | :---: |
| Heat transfer | Q | W |
| Work transfer | W | W |
| Mass flow rate | m | $\mathrm{kg} / \mathrm{s}$ |
| Specific enthalpy | h | $\mathrm{J} / \mathrm{kg}$ |
| Velocity | V | $\mathrm{m} / \mathrm{s}$ |
| Gravitational acceleration | g | $9.81 \mathrm{~m} / \mathrm{s}^{2}$ |
| Elevation above datum | z | m |

### 2.2.6 Application of SFEE

## a. Turbines or Compressors

if the SFEE is applied to the expansion of a fluid in a turbine as shown


$$
Q-W=(\Delta h+\Delta K e+\Delta P e)
$$

With the following simplifications are made

$$
\begin{align*}
Q & =0 \\
\Delta k e & =0 \\
\Delta P e & =0 \text { are all neglected } . \\
\therefore W & =m(h 1-h 2) \tag{40}
\end{align*}
$$

hence for a turbine, the amount of energy produced "Work " is equal to the enthalpy change between inlet and outlet.

## b. Boilers or Condensers

If the SFEE is applied to the heating or cooling ( evaporation or condensation ) of a fluid in a boiler or condenser


$$
Q-W=(\Delta h+\Delta K e+\Delta P e)
$$

With the following simplifications are made
There is no process work on the fluid $W=0$,
If Kinetic energy and Potential energy changes $\Delta k e=0, \Delta P e=0$ are neglected. Then the SFEE reduces to :

$$
\begin{equation*}
\therefore Q=m\left(h_{2}-h_{1}\right) \tag{41}
\end{equation*}
$$

hence for a boiler or a condenser, the amount of energy supplied or extracted from the fluid "Heat " is equal to the enthalpy change for the fluid between inlet and outlet.

## C. Throttling valve

Consider the flow of fluid through a small valve as shown

if the SFEE is applied between sections 1 and 2 :
$Q-W=m(\Delta h+\Delta k e+\Delta P e)$
$Q=0 \quad$ Assuming adiabatic
$W=0$ No displacement work (no work is inputted or extracted, ie no pump or turbines attached )
$\Delta k e=0$ Assumed (inlet and exit velocities are similar or slow)
$\Delta P e=0$ Assumed ( entry and exit at the same or nearly the same elevation )
Hence, The SFEE, reduces to:
$\therefore m(h 2-h 1)=0$ divide by the mass flow m to get:

$$
\begin{equation*}
\therefore h 2=h 1 \tag{42}
\end{equation*}
$$

hence for a control valve, the enthalpy of the fluid remains constant.

## d. Diffuser or Nozzle

Consider the flow of fluid through a diffuser which is a device used in aircraft to reduce the kinetic energy of exhaust gases, as shown

if the SFEE is applied between sections 1 and 2:
$Q-W=m(\Delta h=\Delta k e+\Delta P e)$
$Q=0 \quad$ Assuming adiabatic
$W=0$ No displacement work ( no work is inputted or extracted, ie no pump or turbine's attached )
$\Delta P e=0$ Assumed ( entry and exit at the same or nearly the same elevation )
Hence, The SFEE, reduces to:

$$
\begin{equation*}
h_{2}-h_{1}=\frac{\left(V_{2}^{2}-V_{1}^{2}\right)}{2} \tag{43}
\end{equation*}
$$

## Example 2.1

A closed rigid container has a volume of $1 \mathrm{~m}^{3}$ and holds air at 345 kPa and $27^{\circ} \mathrm{C}$. Heat is added until the temperature is $327^{\circ} \mathrm{C}$.
Determine the change in Internal Energy:-
a) Using an average value of the specific heat.
b) Taking into account the variation of specific heat with temperature.

Solution:
a)
$\Delta U=m C_{v} \Delta T$
From table (4.2)
$C_{v}=\frac{764+718}{2}=741 \mathrm{~J} / \mathrm{kg} \mathrm{K}$

$$
m=\frac{P V}{R T}=\frac{345 \times 10^{3} \times 1}{287 \times 300}=4.0069 \mathrm{~kg}
$$

Therefore
$\Delta U=4.0069 \times 741(327-27)=890.75 \mathrm{~kJ}$
b)

$$
\begin{aligned}
& \Delta u=\int_{T_{1}}^{T_{2}} C_{v} d T= \\
&=C_{v 2} T_{2}-C_{v 1} T_{1} \\
&=764 \times 600-718 \times 300=243000 \mathrm{~J} / \mathrm{kg} \\
& m=\frac{P V}{R T}=\frac{345 \times 10^{3} \times 1}{287 \times 300}=4.0069 \mathrm{~kg}
\end{aligned}
$$

Therefore
$\Delta U=m \Delta u$
$\Delta U=973.676 K J$

## Example 2.2

Steam at a pressure of 6 MPa and a temperature of $500^{\circ} \mathrm{C}$ enters an adiabatic turbine with a velocity of $20 \mathrm{~m} / \mathrm{s}$ and expands to a pressure of 50 kPa , and a dryness fraction of 0.98 . The steam leaves with a velocity of $200 \mathrm{~m} / \mathrm{s}$. The turbine is required to develop 1 MW .
Determine:
a) the mass flow rate of steam required, when KE is neglected, and
b) What is the effect of KE on the answer?


## Solution:

a)
from table B.1.3 at $\mathrm{P}=6000 \mathrm{Kpa}$ and a temperature of $500^{\circ} \mathrm{C}$

$$
h_{1}=3422.12 \mathrm{KJ} / \mathrm{kg}
$$

from table B.1.2 at $\mathrm{P}=50 \mathrm{Kpa}$

$$
\begin{array}{r}
h_{f 2}=340.47 \mathrm{KJ} / \mathrm{kg} \text { and } h_{g 2}=2645.87 \mathrm{KJ} / \mathrm{kg} \\
h_{2}=340.47+0.98 \times(2645.87-340.47)=2524.3 \mathrm{KJ} / \mathrm{kg}
\end{array}
$$

This situation is governed by the Steady Flow Energy Equation
$Q-W=m\left[\left(h_{2}-h_{1}\right)+\frac{V_{2}^{2}-V_{1}^{2}}{2}+g\left(Z_{2}-Z_{1}\right)\right]$
Neglecting the changes in Kinetic and Potential energies, $\mathrm{Q}=0$ for an adiabatic process

$$
\text { Approximately }-W=m\left[h_{2}-h_{1}\right]
$$

$$
-1 \times 10^{3}=m(2524.3-3422.12) \quad \text { hence } m=1.114 \mathrm{~kg} / \mathrm{s}
$$

(b) taking KE into consideration

$$
\begin{aligned}
& -W=m\left[\left(h_{2}-h_{1}\right)+\frac{V_{2}^{2}-V_{1}^{2}}{2}\right] \\
& -1 \times 10^{3}=m\left[(2524.3-3422.12)+\frac{200^{2}-20^{2}}{2 \times 1000}\right]
\end{aligned}
$$

$$
m=1.1389 \mathrm{~kg} / \mathrm{s} \text { or an error } 2.1 \%
$$

## Example 2.3

Steam at $1.0 \mathrm{MPa}, 0.95 \mathrm{dry}$ is throttled to 10 kPa . What is the quality of the steam after throttling?

## Solution:

## SFEE


$Q-W=m(\Delta h+\Delta k e+\Delta P e)$
The SFEE, reduces to: $\mathrm{h} 2=\mathrm{h} 1$
at 1.0 MPa $=1000$ Kpa from table B.1.2,
$h_{f}=762.79 \mathrm{~kJ} / \mathrm{kg}$ and $h_{g}=2,778 \mathrm{~kJ} / \mathrm{kg}$

$$
\begin{aligned}
h 1 & =762.81+0.95 \times(2778-762.79) \\
& =2677.24 \frac{\mathrm{~kJ}}{\mathrm{~kg}}
\end{aligned}
$$

at 10 kPa from table B. 1.2 with $h_{g}=2584.63 \mathrm{~kJ} / \mathrm{kg}$

$$
h_{g}<h_{2}
$$

since $h_{2}=h_{1}=2677.24 \mathrm{~kJ} / \mathrm{kg}$, which is higher than $h_{g}$ at 10 kPa , the steam will be in a superheated condition.

## Example 2.4

An insulated, constant-volume system containing 1.36 kg of air receives 53 kJ of paddle work. The initial temperature is $27^{\circ} \mathrm{C}$. Determine
a) the change of internal energy.
b) the final temperature.

Assume a mean value $C_{v}=0.718 \mathrm{~kJ} / \mathrm{kgK}$.

## Solution:

a)
$Q-W=\Delta U$
$Q=0$ (insulated system)
$W=-53 k J$ ( externally inputted work) see figure (3.1)
The change in internal energy $\Delta U$ is
$\Delta U=-W=+53 k J \quad$ Since $Q=0$
b)

$$
\begin{aligned}
& \Delta U=m C_{v} \Delta T \\
& \therefore 53=1.36 \times 0.718\left(T_{2}-27\right) \\
& T_{2}=27+\frac{53}{1.36 \times 0.718}=81.3^{\circ} \mathrm{C}
\end{aligned}
$$

## Example 2.5

A boiler receives feed water at $40^{\circ} \mathrm{C}$ and delivers steam at 2 MPa and $500^{\circ} \mathrm{C}$.
If the furnace is oil fired, the calorific value of oil being $42000 \mathrm{~kJ} / \mathrm{kg}$ and 4000 kg oil are burned while 45000 kg of steam are produced, determine :
a) the heat supplied in the boiler.
b) the efficiency of the boiler.

Assume the values of enthalpies at the two state points as :
$\mathrm{h} 1=\mathrm{hf} @ 40^{\circ} \mathrm{C}=169.33 \mathrm{~kJ} / \mathrm{kg} \quad$ at 2 MPa , $500^{\circ} \mathrm{C}, \mathrm{h} 2=3467.6 \mathrm{~kJ} / \mathrm{kg}$


## Solution:

a) Constant pressure process.

$$
h 1=h f @ 40^{\circ} \mathrm{C}=169.33 \mathrm{~kJ} / \mathrm{kg} \quad \mathrm{~h} 2=3467.6 \mathrm{~kJ} / \mathrm{kg}
$$

SFEE ignoring $\mathrm{W}, \Delta \mathrm{ke}$ and $\Delta \mathrm{Pe}$ :

$$
\begin{aligned}
Q s & =m_{s}(h 2-h 1) \\
& =45000(3467.6-169.33)=1.484 \times 10^{8} \mathrm{~kJ}
\end{aligned}
$$

b) The heat generated by burning oil in the furnace is

$$
=\text { mass of oil burned } x \text { calorific value }
$$

$$
=4000 \times 42000=1.68 \times 10^{8} \mathrm{~kJ}
$$

$\therefore$ Boiler efficiency $=\frac{\text { Energy output }}{\text { Energy input }}=\frac{1.484 \times 10^{8}}{1.68 \times 10^{8}}=88 \%$

## Example 2.6

Air at $27^{\circ} \mathrm{C}$ receives heat at constant volume until its temperature reaches $727^{\circ} \mathrm{C}$. Determine the heat added per kilogram?
Assume for air $C_{v}=0.7865 \mathrm{~kJ} / \mathrm{kgK}$.

## Solution:

Closed system for which the first law of Thermodynamics applies,

$$
Q-W=\Delta U
$$

$W=0 \quad$ No work transfer at constant volume process.
$\therefore Q=\Delta U$
$\Delta U=Q=m C_{v}\left(T_{2}-T_{1}\right)$

$$
=1 \times 0.7865(727-27)=550.55 \mathrm{~kJ} / \mathrm{kg}
$$

Hence

$$
Q=\Delta U=550.55 \mathrm{~kJ} / \mathrm{kg} .
$$

Note that the $C_{v}$ used is an average value.
$C_{v}=\frac{718+855}{2}=0.7865 \mathrm{~kJ} / \mathrm{kgK}$ from table 4.2

## Example 2.7

Air, which may be considered a perfect gas, enters an adiabatic nozzle with negligible velocity. The entry pressure is 6 bar and the exit pressure is 1 bar; the entry temperature is 760 K . The flow throughout the nozzle is reversible and the mass flow rate is $2 \mathrm{~kg} / \mathrm{s}$.
Calculate the exit velocity.
Take $\mathrm{Cp}=1004.5 \mathrm{~J} / \mathrm{kg} \mathrm{K}$ and $\mathrm{n}=1.4$

## Solution:

This situation is an open system for which the SFEE applies

$$
Q-W=m\left[\left(h_{2}-h_{1}\right)+\frac{V_{2}^{2}-V_{1}^{2}}{2}+g\left(Z_{2}-Z_{1}\right)\right]
$$

$\mathrm{Q}=0$ adiabatic
W = 0 no work transfer in the system
For a perfect gas
$h_{2}-h_{1}=C_{P}\left(T_{2}-T_{1}\right)$
$g\left(Z_{2}-Z_{1}\right)=0$ negligable

$$
T_{2}=T_{1}\left(\frac{P_{2}}{P_{1}}\right)^{\frac{n-1}{n}}=760\left(\frac{1}{6}\right)^{\frac{0.4}{1.4}}=455.5 \mathrm{~K}
$$

The SFEE reduces to (dividing by the mass)

$$
\begin{aligned}
\therefore & 0=m\left[C_{P}\left(T_{2}-T_{1}\right)+\frac{V_{2}^{2}-V_{1}^{2}}{2}\right] \\
& 0=1004.5(455.5-760))+\frac{V_{2}^{2}}{2} \\
\therefore & V_{2}=782 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

