Chapter One

Maxwell equation and its application in EM waves

Points of review

Del (∇) operator:

$$\nabla = a_x \frac{\partial}{\partial x} + a_y \frac{\partial}{\partial y} + a_z \frac{\partial}{\partial z} \qquad \text{1/m}$$

Gradient of a scalar V ($= \nabla V$):

$$\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z$$

Gradient of a scalar field is a vector that represents both the magnitude and the direction of the maximum space rate of increase of a scalar field.
Imagine height being a scalar, and then the gradient of the height would be a vector pointing "uphill", the length of the vector is proportional to the steepness of the slope.

Divergence of a vector A (=$\nabla . A$):

$$\nabla . A = divA = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

The divergence indicates the spreading or diverging of a quantity from a point.
It is positive if the net flow is outward, and it is negative if the net flow is inward.
Example, leaking air from balloon yields positive divergence.
Imagine that the vector field **F** below gives the velocity of some fluid flow.
It appears that the fluid is exploding outward from the origin.

Divergence of a vector field is associated with conserved quantities, if the divergence is zero there are no "sources" or "sinks".

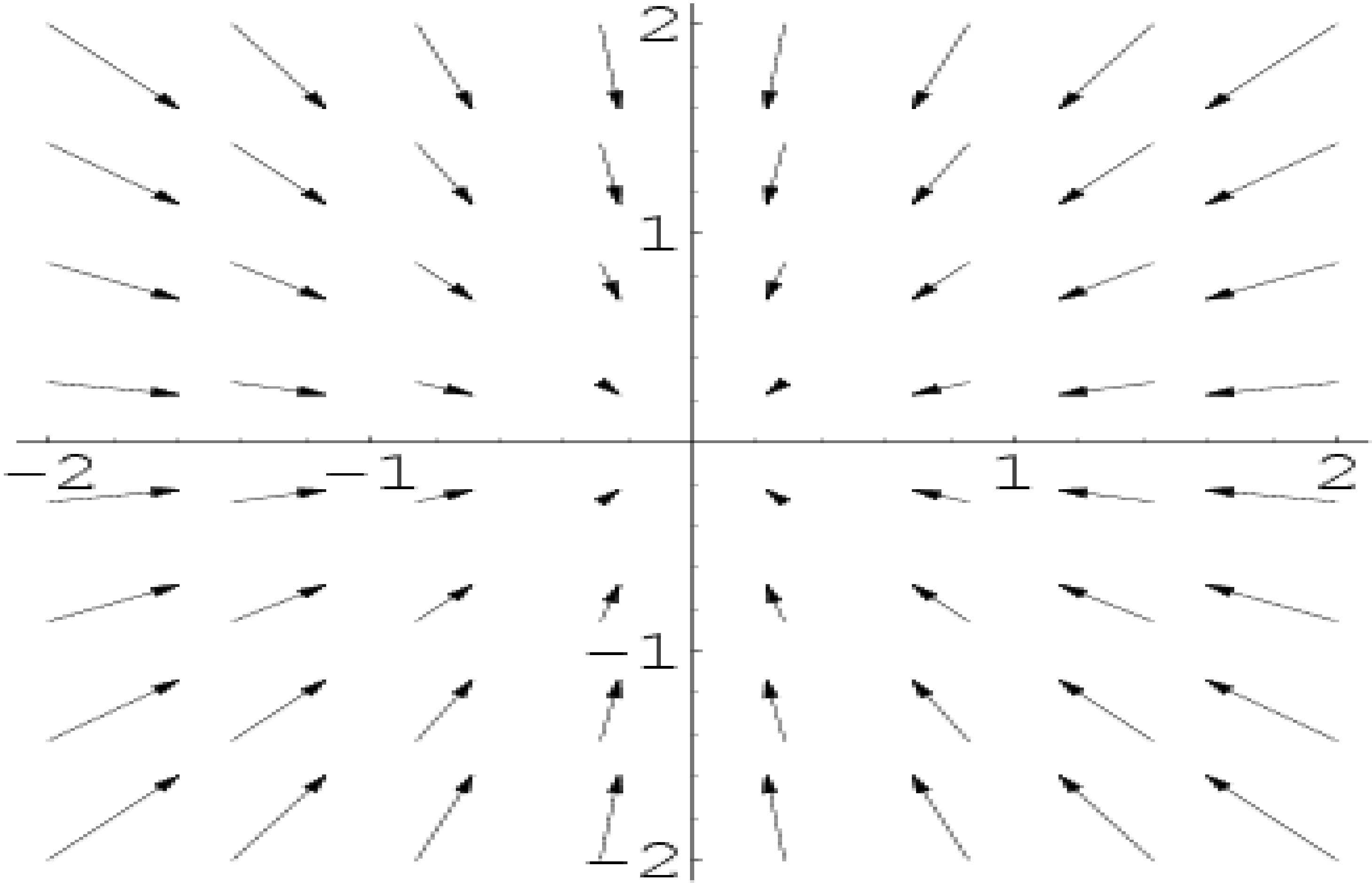


Figure (1.1) Negative divergence

In contrast, the below vector field represents fluid flowing so that it compresses as it moves toward the origin. Since this compression of fluid is the opposite of expansion, the divergence of this vector field is negative.

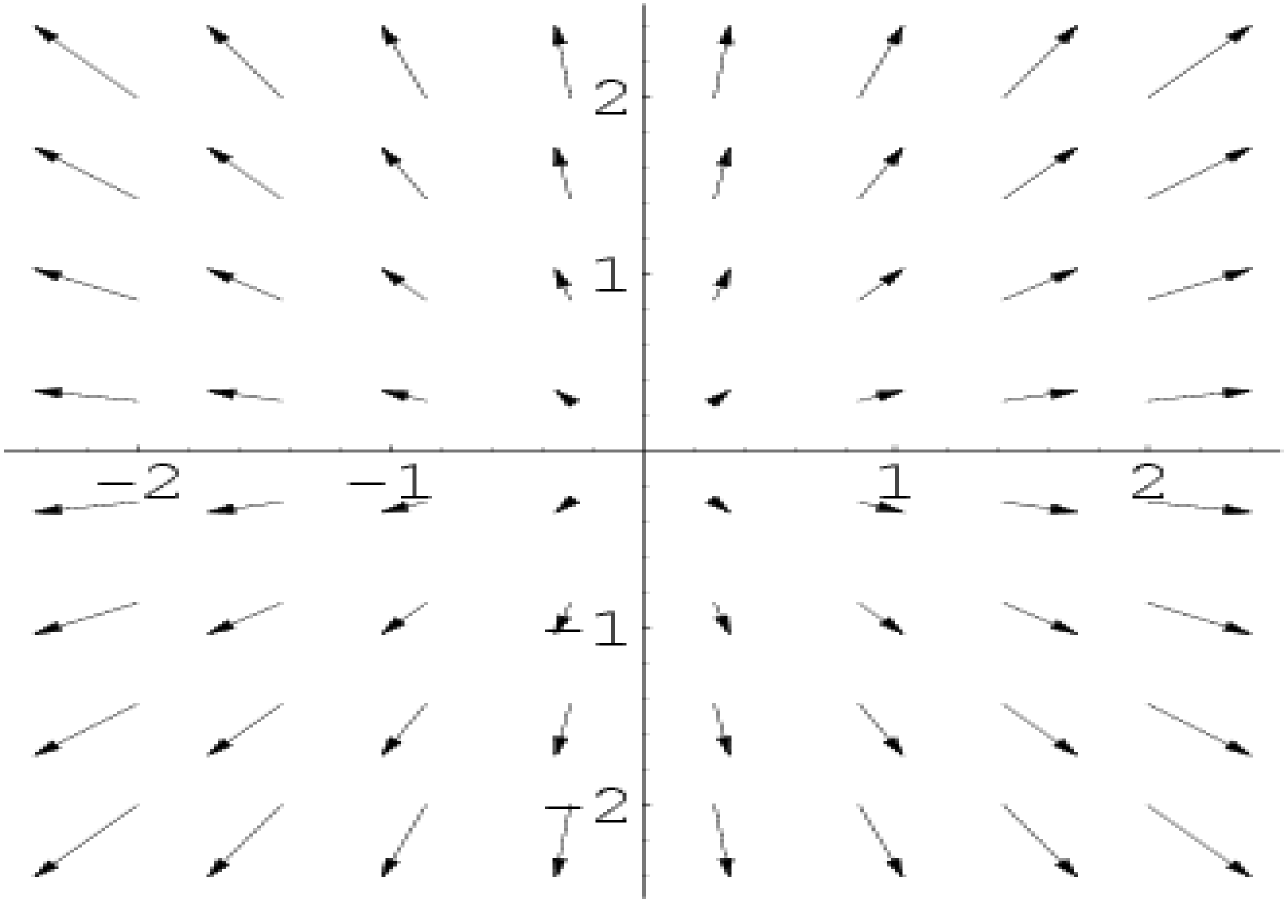


Figure (1.2) Positive divergence

Curl of a vector $(= \nabla X A)$

$$Curl\ A = \nabla X A = \begin{vmatrix} a_x & a_y & a_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

A curl of a vector represents rotation, it is also written as:

$$Curl\ A = rot A = \nabla X A \qquad div(curl) = 0$$

The curl of a vector field is slightly more complicated than the divergence. It captures the idea of how a fluid may rotate. Imagine that the below vector field F represents fluid flow. It appears that fluid is circulating around a bit

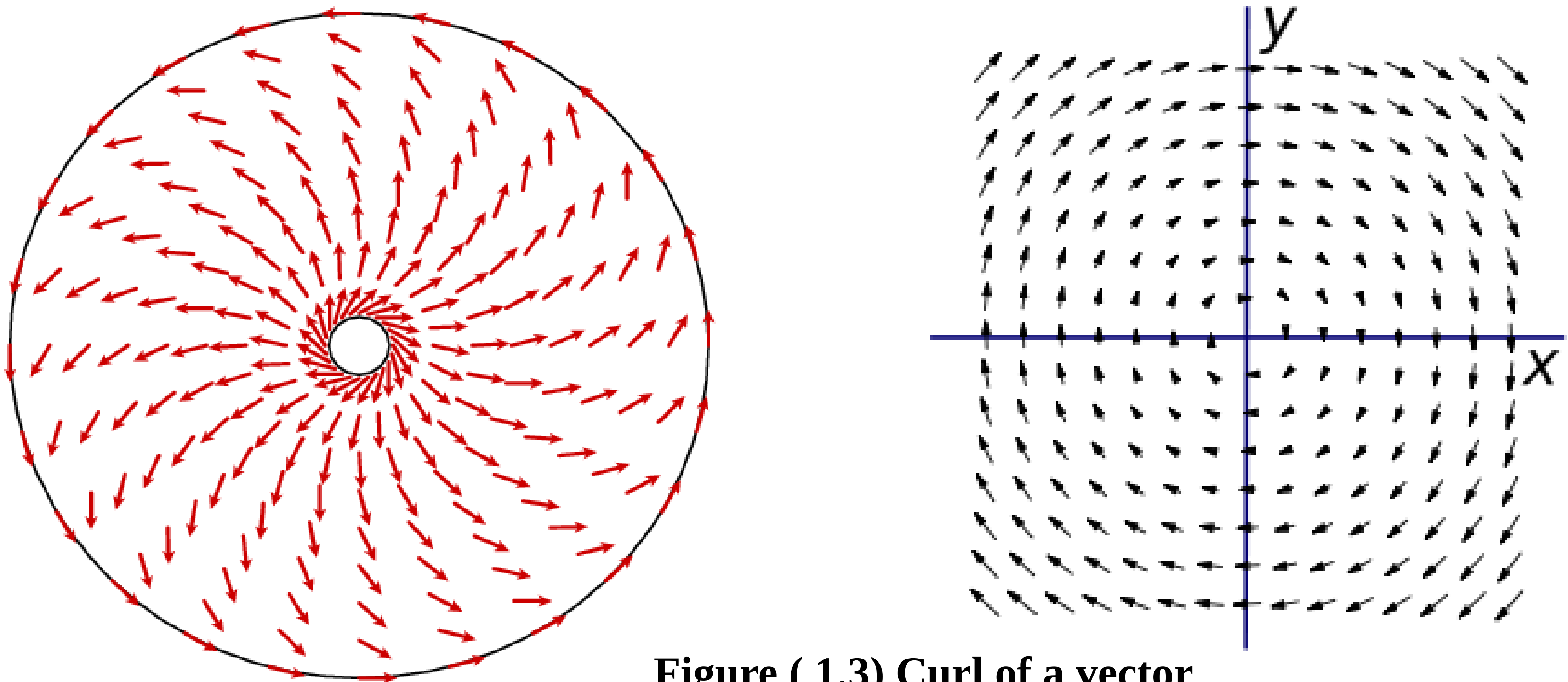


Figure (1.3) Curl of a vector

Curl of a vector field is associated with it's rotation, if the curl is zero the field is irrotational.

Maxwell equations for time varying fields

Maxwell equations in differential form

1. Ampere's law $$\nabla \, x \; H = J + \frac{\partial D}{\partial t} \qquad \textbf{(1.1)}$$

2. Faraday's law of induction $$\nabla x \, E = -\frac{\partial B}{\partial t} \qquad \textbf{(1.2)}$$

3. Gauss' law for electricity $$\nabla \cdot D = \rho \qquad \textbf{(1.3)}$$

4. Gauss' law for magnetism $$\nabla \cdot B = 0 \qquad \textbf{(1.4)}$$

H=magnetic field strength (A/M)

D=Electric flux density (C/m^2)

$\frac{\partial D}{\partial t}$ =Displacement current density (A/m^2)

J=Conduction current density (A/m^2)

E=Electric field (V/m)

B=Magnetic flux density (wb/m^2 or Tesla)

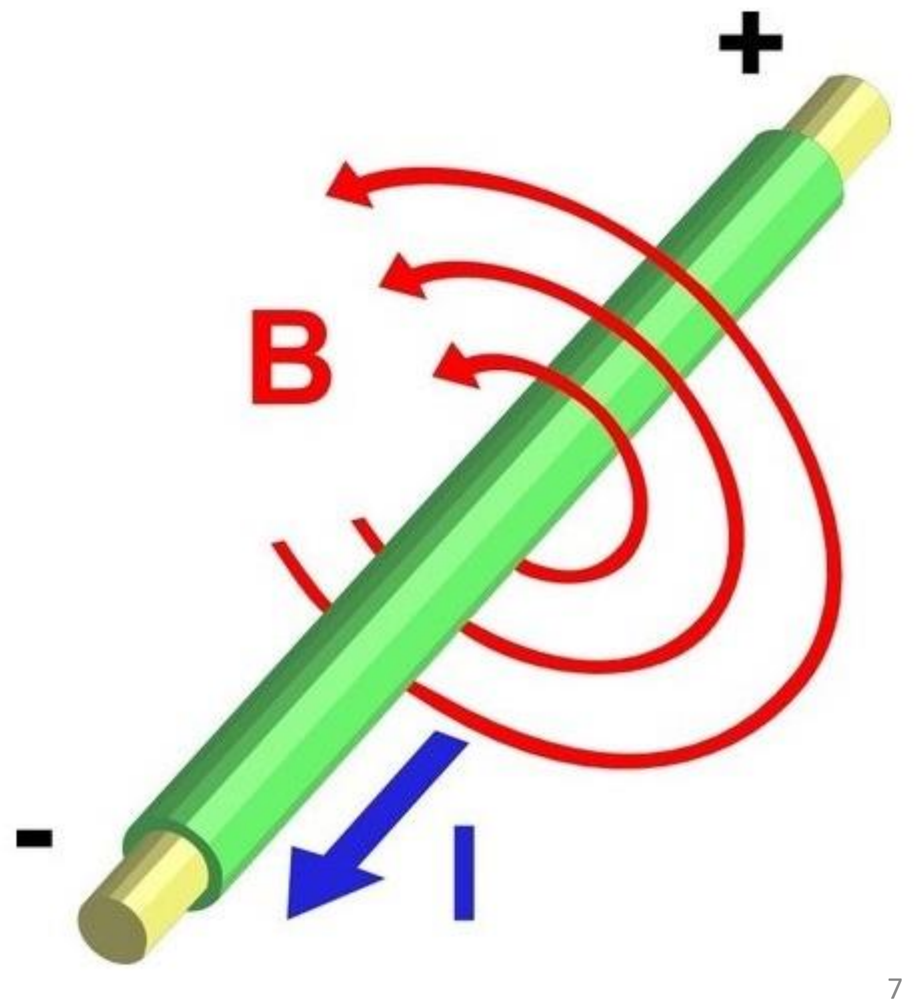
$\frac{\partial B}{\partial t}$ =Magnetic current density (V/m^2)

ρ_v =Volume charge density (C/m^3)

Ampère-Maxwell law

The magnetic field induced around a closed loop is proportional to the electric current plus displacement current (rate of change of electric field) that the loop encloses.

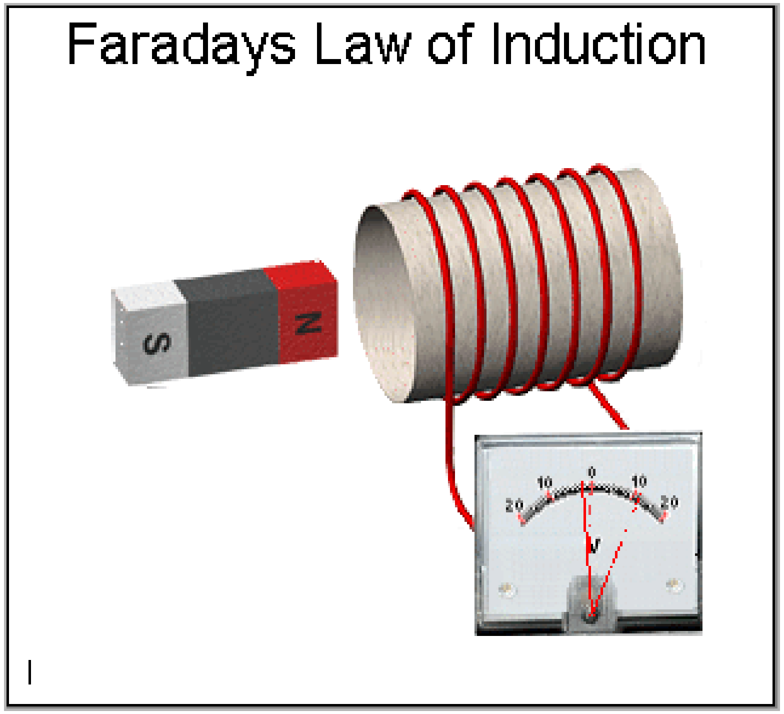
$$\nabla \times B = \mu_0 \left(J + \varepsilon_0 \frac{\partial E}{\partial t} \right)$$



Maxwell–Faraday equation

The voltage induced in a closed loop is proportional to the rate of change of the magnetic flux that the loop encloses, i.e., *every time the magnetic field change there is the creation of a electric field.*

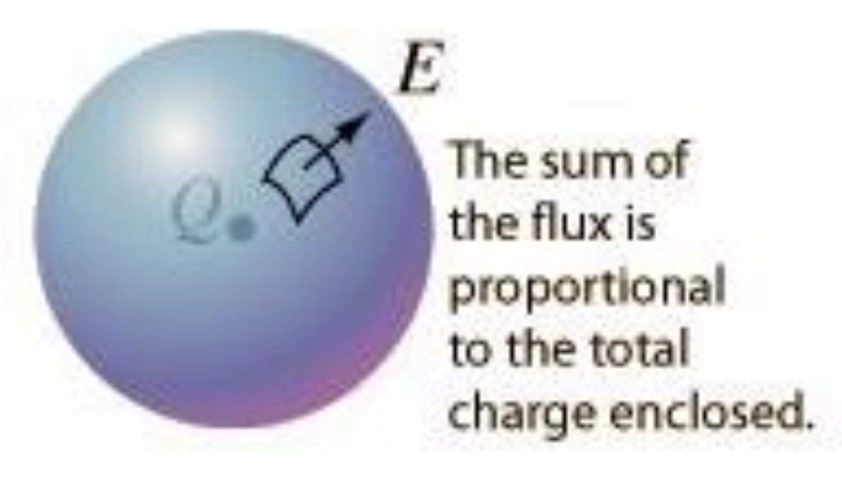
$$\nabla \times \mathbf{E} = -\frac{\partial B}{\partial t}$$



Gauss Law

Essentially it's stating that the net quantity of the electric flux leaving a sample volume is proportional to the charge inside the volume.

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$



Gauss Law for Magentism

Essentially it is stating the impossibility of creating a magnetic monopole; the total magnetic flux through a closed surface is zero.

$$\nabla \cdot B = 0$$

Influence of medium on the fields

When the sources of electric and magnetic fields exist in a medium, the medium has the influence on the characteristics of the fields. The characteristics are described as follow:

$$\left.\begin{array}{l}\mathbf{D}=\varepsilon\mathbf{E}\\ \mathbf{B}=\mu\mathbf{H}\\ \mathbf{J}=\sigma\mathbf{E}\end{array}\right\}$$

$\varepsilon = \varepsilon_r \varepsilon_o$ = permittivity (F/m)

$\mu = \mu_r \mu_o$ = permeability (H/m)

σ = conductivity of the medium (mho/m)

$$\varepsilon_o = 8.854 * 10^{-12} \quad (F/m)$$

$$\mu_o = 4\pi * 10^{-7} \quad (H/m)$$

Characteristics of free space

Free space is characterized by the following parameters:

Relative permittivity, $\varepsilon_r=1$

Relative permeability, $\mu_r=1$

Conductivity, $\sigma = 0$

Conduction current density, J=0

Volume charge density, $\rho_v=0$

Intrinsic impedance or characteristic impedance, $\eta=120\pi$ or 377Ω

Mediums can be divided into five types

1. Homogeneous medium
2. Non- homogeneous medium
3. Isotropic medium
4. Anisotropic medium
5. Source free region

Homogeneous medium

It is a medium for which ε, μ and σ are constant throughout the medium. An example is free space

Non- homogeneous medium

It is a medium for which ε, μ and σ are not constant and they are different from point to point in the medium.

An example is the human body

Isotropic medium

It is a medium for which ε is a scalar constant

An example is free space

Source free region

It is a medium in which there is no field sources.

Broadcast Frequencies:

Long wave AM Radio = 148.5 - 283.5 kHz (LF)
Medium wave AM Radio = 530 kHz - 1710 kHz (MF)
Shortwave AM Radio = 3 MHz - 30 MHz (HF)
TV Band I (Channels 2 - 6) = 54 MHz - 88 MHz (VHF)
FM Radio Band II = 88 MHz - 108 MHz (VHF)
TV Band III (Channels 7 - 13) = 174 MHz - 216 MHz (VHF)
TV Bands IV & V (Channels 14 - 69) = 470 MHz - 806 MHz (UHF)

Application of Maxwell's equations: EM waves and propagation of energy.

Wave equations in free space:

Wave equations in free space is given by :

$$\nabla^2 \boldsymbol{E} = \mu_o \varepsilon_o \frac{\partial^2 \boldsymbol{E}}{\partial t^2} \qquad (1.5)$$

$$\nabla^2 \boldsymbol{H} = \mu_o \varepsilon_o \frac{\partial^2 \boldsymbol{H}}{\partial t^2} \qquad (1.6)$$

$\varepsilon_r = 1 \qquad \varepsilon = \varepsilon_o$

$\mu_r = 1 \qquad \mu = \mu_o$

$B = \mu_o H$

$\sigma = 0$

$J = 0$

$\rho_v = 0$

$\varepsilon_o = 8.854 * 10^{-12} \qquad F/M$

$\mu_O = 4\pi * 10^{-7} \qquad H/M$

To proof it:

$$\nabla X\boldsymbol{E} = -\frac{\partial B}{\partial t} \qquad B = \mu_o H$$

$$\nabla X\boldsymbol{E} = -\mu_o \frac{\partial H}{\partial t}$$

Taking the curl of both sides we get:

$$\nabla X\nabla X\boldsymbol{E} = -\mu_o \nabla X(\frac{\partial \boldsymbol{H}}{\partial t})$$

$$\nabla X\nabla X\boldsymbol{E} = \nabla(\nabla.\boldsymbol{E}) - \nabla^2\boldsymbol{E}$$

$$\nabla(\nabla.\boldsymbol{E}) = 0$$

$$\therefore \quad -\nabla^2\boldsymbol{E} = -\mu_o \varepsilon_o \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

$$Where \quad \nabla X\boldsymbol{H} = \frac{\partial D}{\partial t} + J \qquad \text{and} \qquad D = \varepsilon_o \boldsymbol{E}$$

$$\therefore \quad \nabla^2\boldsymbol{E} = \mu_o \varepsilon_o \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

The wave will experience no loss as it propagating in free space

Wave propagations for a conducting medium:

Waves which propagate in a medium other than vacuum will experience some loss due to absorption in the medium.

$$\nabla^2 \boldsymbol{E} = \mu\varepsilon \frac{\partial^2 \boldsymbol{E}}{\partial t^2} + \mu\sigma \frac{\partial \boldsymbol{E}}{\partial t} \qquad (1.7)$$

$$\nabla^2 \boldsymbol{H} = \mu\varepsilon \frac{\partial^2 \boldsymbol{H}}{\partial t^2} + \mu\sigma \frac{\partial \boldsymbol{H}}{\partial t} \qquad (1.8)$$

The effect of the new loss term ($\mu\sigma \frac{\partial \boldsymbol{E}}{\partial t}$ and $\mu\sigma \frac{\partial \boldsymbol{H}}{\partial t}$) is to attenuate the propagating wave.

Plane waves:

Plane waves are waves that vary only in the direction of propagation and are uniform in planes normal to the direction of propagation.

Uniform plane waves:

An EM wave propagating in x-direction is said to be a uniform plane wave if its field is independent of y and z directions.

The plane wave equation is: $$\frac{\partial^2 \boldsymbol{E}}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

Or

$$\frac{\partial^2 \boldsymbol{Ex}}{\partial x^2} ax + \frac{\partial^2 \boldsymbol{Ey}}{\partial x^2} ay + \frac{\partial^2 \boldsymbol{Ez}}{\partial x^2} az = \mu_o \varepsilon_o \left[\frac{\partial^2 \boldsymbol{Ex}}{\partial t^2} ax + \frac{\partial^2 \boldsymbol{Ey}}{\partial t^2} ay + \frac{\partial^2 \boldsymbol{Ez}}{\partial t^2} az \right]$$

Then we get:

$$\frac{\partial^2 \boldsymbol{E}x}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 \boldsymbol{E}x}{\partial t^2} \quad , \quad \frac{\partial^2 \boldsymbol{E}y}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 \boldsymbol{E}y}{\partial t^2} \quad , \quad \frac{\partial^2 \boldsymbol{E}z}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 \boldsymbol{E}z}{\partial t^2}$$

In the region which there is no charge density

$$\Delta . \boldsymbol{E} = \frac{1}{\varepsilon} \nabla . D = 0, \qquad D = \varepsilon \boldsymbol{E}$$

That is,

$$\frac{\partial \boldsymbol{E}x}{\partial x} + \frac{\partial \boldsymbol{E}y}{\partial y} + \frac{\partial \boldsymbol{E}z}{\partial z} = 0$$

For a uniform plane wave in which **E** is independent of y and z, the last two terms in the above equation are equal to zero.

$$\frac{\partial \boldsymbol{E}x}{\partial x} = 0$$

Therefore there is no variation of **E**x in the x-direction

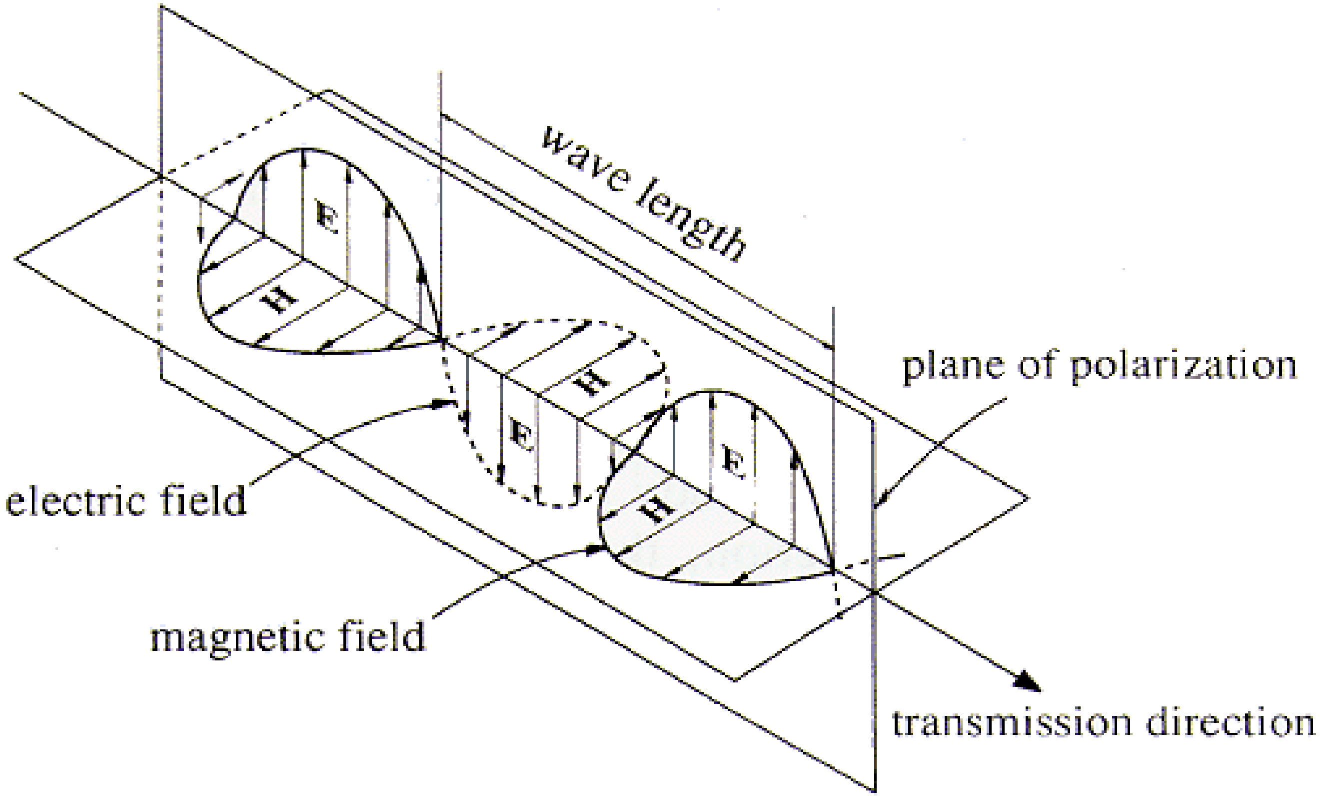


Figure (1.4) Electromagnetic radiation

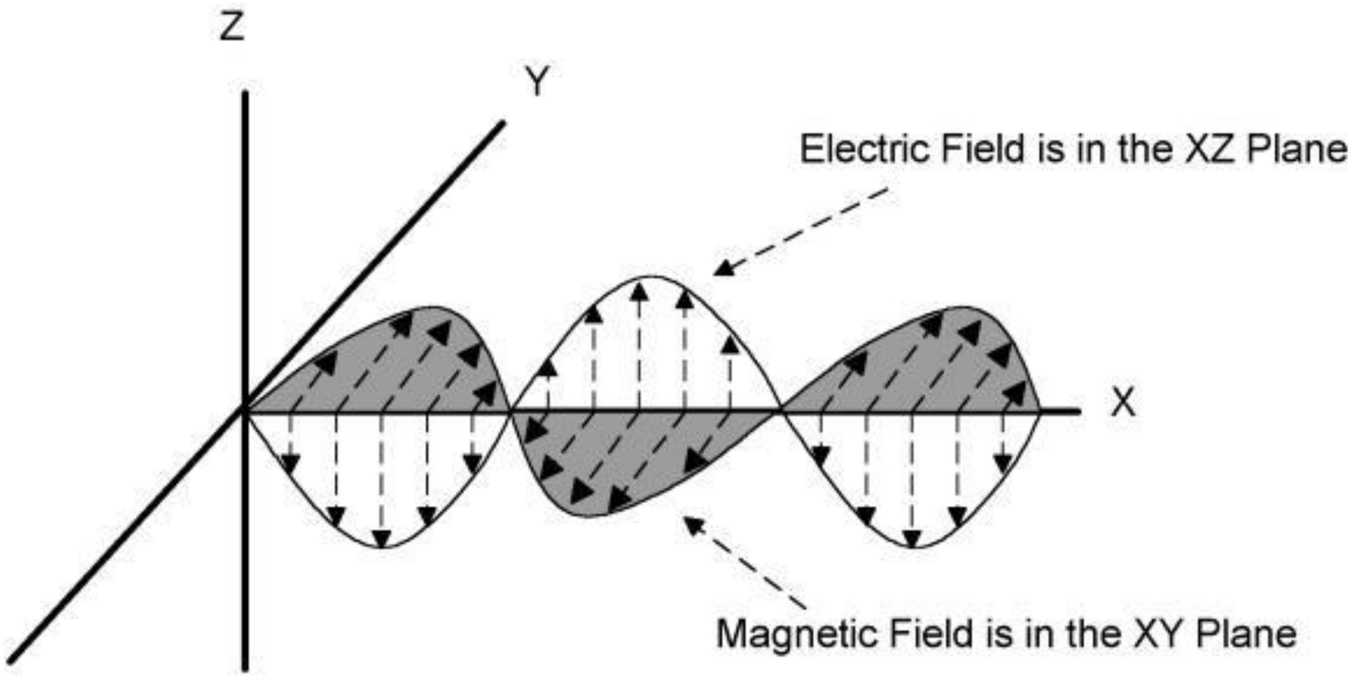


Figure (1.5) plane wave

Relation between **E** and **H**:

The relation is: $$\left|\frac{\boldsymbol{E}}{\boldsymbol{H}}\right| = \frac{\boldsymbol{E}}{\boldsymbol{H}} = 120\pi = 377\Omega$$

Wave propagation in lossless mediums:

The wave equation is: $$\nabla^2 \boldsymbol{E} = -\omega^2 \mu\varepsilon \boldsymbol{E} \qquad Where, \qquad E = E e^{j\omega t}$$

That is,

$$\frac{\partial^2 \boldsymbol{E}}{\partial x^2} = \omega^2 \mu\varepsilon \boldsymbol{E} = -\beta^2 \boldsymbol{E} \qquad where,\ \ \beta = \omega\sqrt{\mu\varepsilon}$$

The y-component of E may be written as:

$$\boldsymbol{E}_y = \underbrace{Ae^{-j\beta x}}_{\text{Forward wave}} + \underbrace{Be^{j\beta x}}_{\text{Reflected wave}}$$

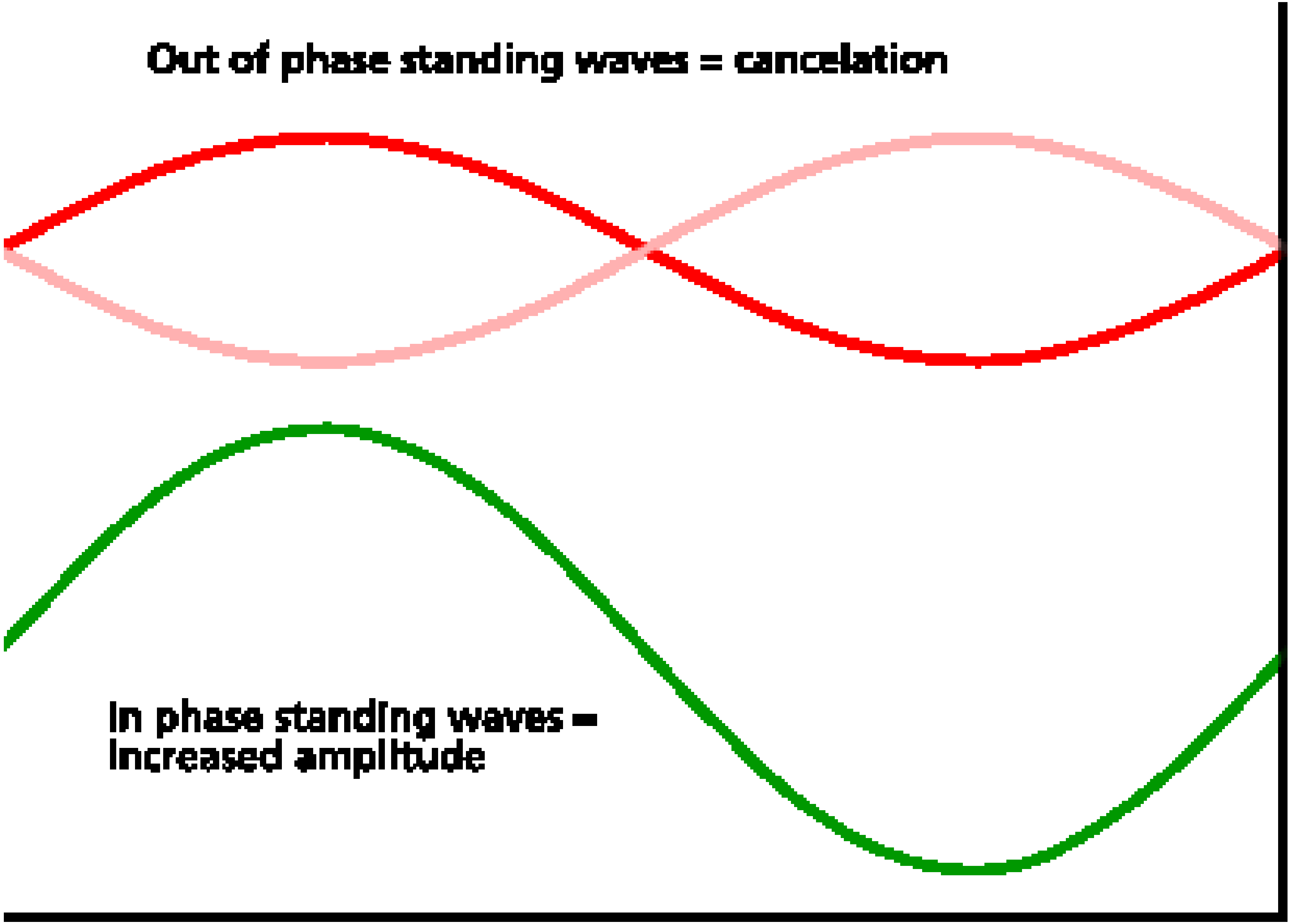
If A and B are arbitrary complex constants. Then,

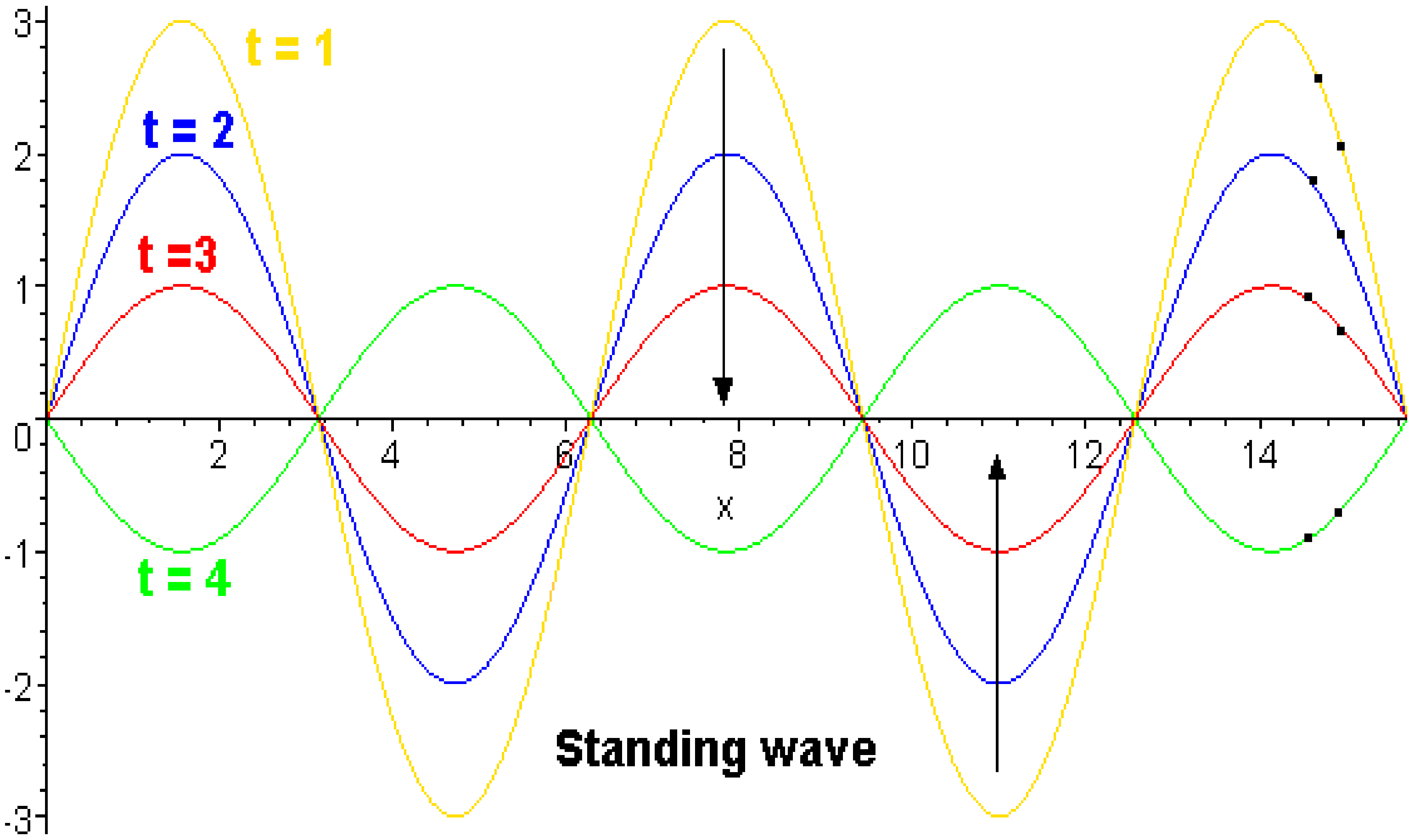
$$\boldsymbol{E}_y(x,t) = Re\left[\boldsymbol{E}_y(x)e^{j\omega t}\right] = Re\left[Ae^{j(\omega t-\beta x)} + Be^{j(\omega t+\beta x)}\right] \qquad (1.9)$$

If A and B are real, equation (1.9) becomes:

$$\boldsymbol{E}_y(x,t) = Acos(\omega t - \beta x) + Bcos(\omega t + \beta x) \qquad (1.10)$$

Equation (1.10) is the sum of two waves. They travel in opposite directions. If A= B, the waves combine together and form a standing wave. Such waves do not progress.


Out of phase standing waves = cancelation
In phase standing waves =
increased amplitude


t = 1
t = 2
t =3
t = 4
Standing wave
x
3
2
1
0
-1
-2
-3
2
4
6
8
10
12
14

The wave velocity:

It is defined as the velocity of propagation of the wave.

$$v = \frac{\omega}{\beta}$$

Where

$$\omega = 2\pi f = angular\ frequency$$

$$\beta = phase\ constant\ (phase\ number), \qquad radians/m$$

Phase constant (β):

Is defined as a measure of the phase shift in radians per unit length. Phase constant is also called wave number.

Intrinsic (or characteristic) impedance:

The intrinsic impedance is a property of a medium - an area of space. For a vacuum (outer space) or for wave propagation through the air around earth (often called 'free space'), the intrinsic impedance is given by:

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi 10^{-7}}{8.854e-12}} \approx 120\pi \approx 377 \text{ Ohms}$$

This parameter is the ratio of the magnitude of the E-field to the magnitude of the H-field for a plane wave.

$$Z = \frac{|\mathbf{E}|}{|\mathbf{H}|}$$

For a general medium with permittivity and permeability given by $(\varepsilon, \mu) = (\varepsilon_r \varepsilon_0, \mu_r \mu_0)$, the intrinsic impedance is given by:

$$Z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r \mu_0}{\varepsilon_r \varepsilon_0}} = \sqrt{\frac{\mu_r}{\varepsilon_r}} Z_0$$

For a medium with a conductivity σ associated with it, the intrinsic impedance is given by:

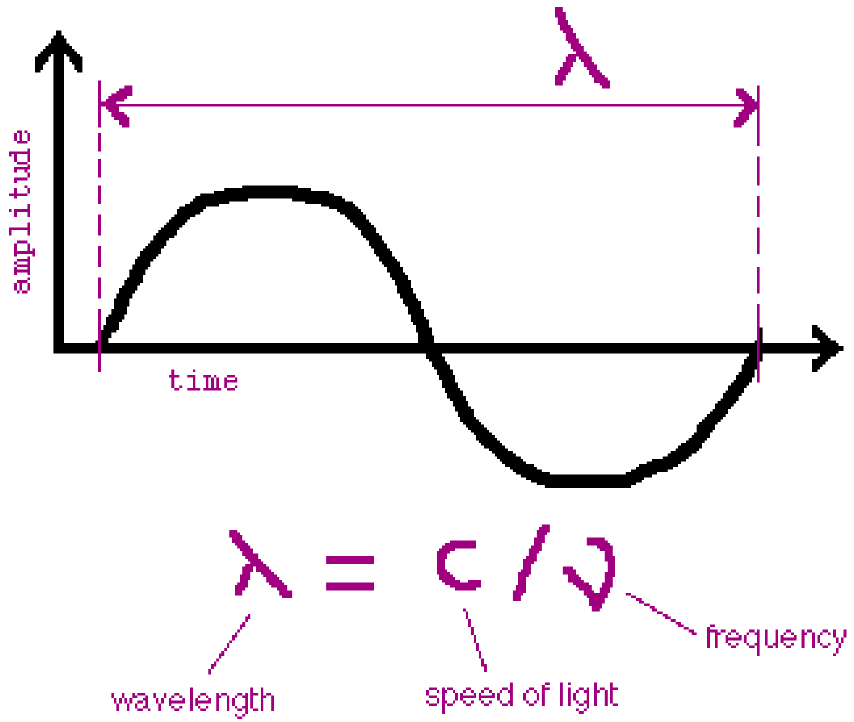
$$Z = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

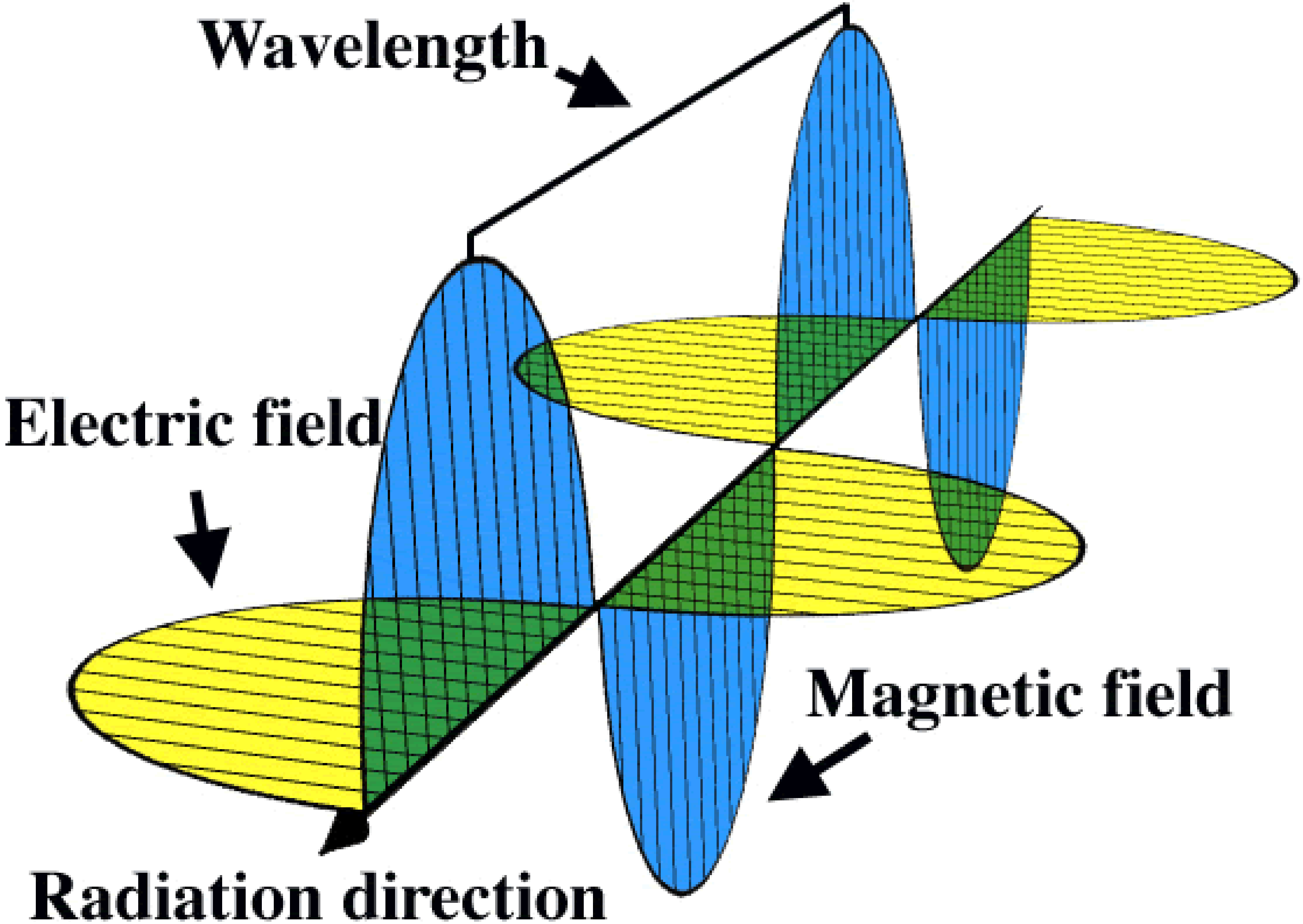
When the conductivity is non-zero, the above intrinsic impedance is a complex number, indicating that the electric and magnetic fields are not in-phase.

Wave length of the wave:

λ is defined as the distance through which the sinusoidal wave passes through a full cycle of 2π radians.

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f}$$




Wavelength
Electric field
Magnetic field
Radiation direction

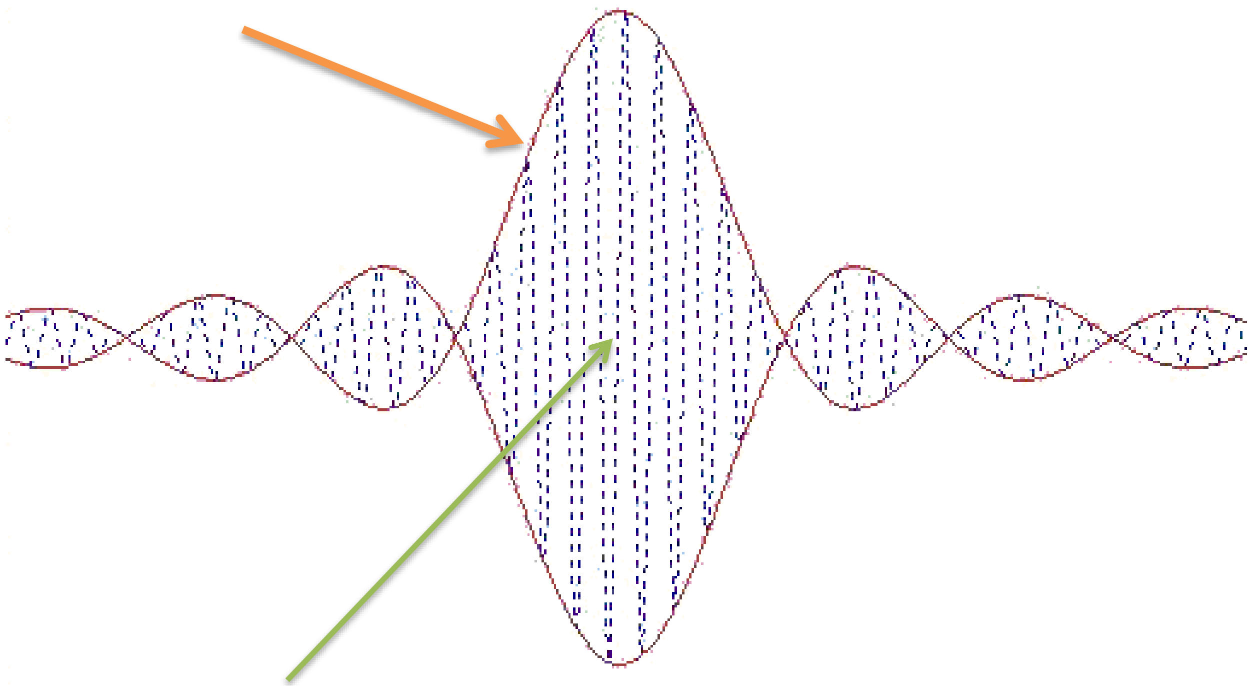
Phase velocity (v_p):

Is defined as the velocity of some point in the sinusoidal waveform, or the velocity with which the phase is progressing.

Group velocity (v_g):

Group velocity is the velocity with which the energy propagates.

Envelope velocity (group velocity v_g)



Wave velocity (phase velocity v_p)

Figure (1.6) Phase and Group velocities

Conductors and Dielectrics

The materials can be classified as dielectrics and conductors.
The displacement current density,

$$J_d = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} = j\omega\varepsilon E$$

And conduction current density,

$$J_c = \sigma E$$

Therefore,

$$\frac{J_c}{J_d} = \frac{\sigma}{\omega\varepsilon}$$

Thus for dielectrics, $$\frac{\sigma}{\omega\varepsilon} < 0.01$$

And for conductors, $$\frac{\sigma}{\omega\varepsilon} > 100$$

Materials in the range $0.01 < \frac{\sigma}{\omega\varepsilon} > 100$ can be called quasi-conductors. σ is the conductivity of the medium in mhos/meter, ε is the permittivity of the medium and ω=2πf. It is seen that the frequency is an important factor to determine whether a material acts as a conductor or a dielectric. Earth behaves like a conductor at1 kHz and as a dielectric at frequencies of 30 GHz. At 10 MHz it behaves as a quasi conductor.

Propagation characteristics of EM wave in free space:

$$\nabla^2 = -\omega^2 \mu_o \varepsilon_0 \boldsymbol{E} = \gamma^2 \boldsymbol{E}$$

$$\gamma = \sqrt{-\omega^2 \mu_o \varepsilon_o} = j\omega\sqrt{\mu_o \varepsilon_o}$$
$$= j\beta, \quad (m^{-1}) \qquad (Propagation\ constant)$$

$$Phase\ shift\ constant,\ \beta = \omega\sqrt{\mu_o \varepsilon_o}\,, \qquad (^{rad}/_{m})$$

$$Velocity\ of\ propagation, \qquad v_o = \frac{1}{\sqrt{\mu_o \varepsilon_o}}\,, \qquad (^{m}/_{s})$$

The velocity of propagation of EM wave is the same as phase velocity.
That is:

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu_o \varepsilon_o}} = \frac{1}{\sqrt{\mu_o \varepsilon_o}}\,, \qquad (^{m}/_{s})$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu_o \varepsilon_o}}, \qquad (m)$$

$$\eta = \frac{E}{H} = 120\pi = \sqrt{\frac{\mu_o}{\varepsilon_o}}, \qquad (\Omega)$$

$$Attenuation\ constant,\ \ \alpha = 0$$

Maxwell's equations in a conducting medium:

$$\nabla X H = \sigma E + \varepsilon \frac{\partial E}{\partial t}$$

$$\nabla X E = -\mu \frac{\partial H}{\partial t}$$

The wave equation for the electric field E in a conducting medium is given by:

$$\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

The wave equation for the magnetic field H in a conducting medium is given by:

$$\nabla^2 H = \mu\sigma \frac{\partial H}{\partial t} + \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

Assuming sinusoidal time variation, $(E = Ee^{j\omega t}$),

The equation reduces to a more general form as:

$$\nabla^2 E - j\omega\mu\sigma E + \omega^2 \mu\varepsilon E = 0 \qquad (1.11)$$

Or

$$\nabla^2 E - j\omega\mu(\sigma + j\omega\varepsilon)E = 0 \qquad (1.12)$$

Or

$$\nabla^2 E - \gamma^2 E = 0 \qquad (1.13)$$

Where

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

Similarly, the wave equation for the magnetic field H is given by:

$$\nabla^2 H - \gamma^2 H = 0 \qquad (1.14)$$

γ is called the propagation constant and is a complex number having the real and imaginary. That is

$$\gamma = \alpha + j\beta \qquad (\frac{1}{m})$$

α is called the attenuation constant, (dB/m)

β is phase constant, (rad/m)

Wave propagation in conductors:

A good conductor is one which the conductivity σ is very large $\sigma \gg \omega\varepsilon$

Under such assumption, the propagation constant γ may be written as:

$$\gamma = \sqrt{j\omega\mu\sigma(1 + \frac{j\omega\varepsilon}{\sigma}}$$

Since $\frac{\sigma}{\omega\varepsilon} \gg 1$

The expression for γ reduces to:

$$\gamma = \sqrt{j\omega\mu\sigma}$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^o$$

$$v = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}$$

Wave propagation in dielectrics:

A good dielectric is one which $\dfrac{\sigma}{\omega\varepsilon} \ll 1$

Under such approximation the expression for α (attenuation constant) is:

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}} - 1\right)}$$

Or can be written as: $\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$ dB/m

In the same way, $\beta = \omega\sqrt{\mu\varepsilon}\left(1 + \frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$ rad/m

$$v = \frac{1}{\sqrt{\mu\varepsilon}}\left(1 - \frac{\sigma^2}{8\omega^2\varepsilon^2}\right) \quad \text{m/s}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\left(1 + j\frac{\sigma}{2\omega\varepsilon}\right) \quad \Omega$$

Depth of penetration (δ):

A wave proceeding along a conducting medium is seen to be attenuated greatly. This attenuation increases greatly as the frequency increases.

Depth of penetration is defined as that depth in which the wave attenuates to $\frac{1}{e}$ or approximately 37 percent of the original amplitude. Depth of penetration is also called as skin depth. It is a measure of depth to which an EM wave can penetrate the medium.

$$Depth\,of\,penetration, \qquad \delta = \frac{1}{\alpha}$$

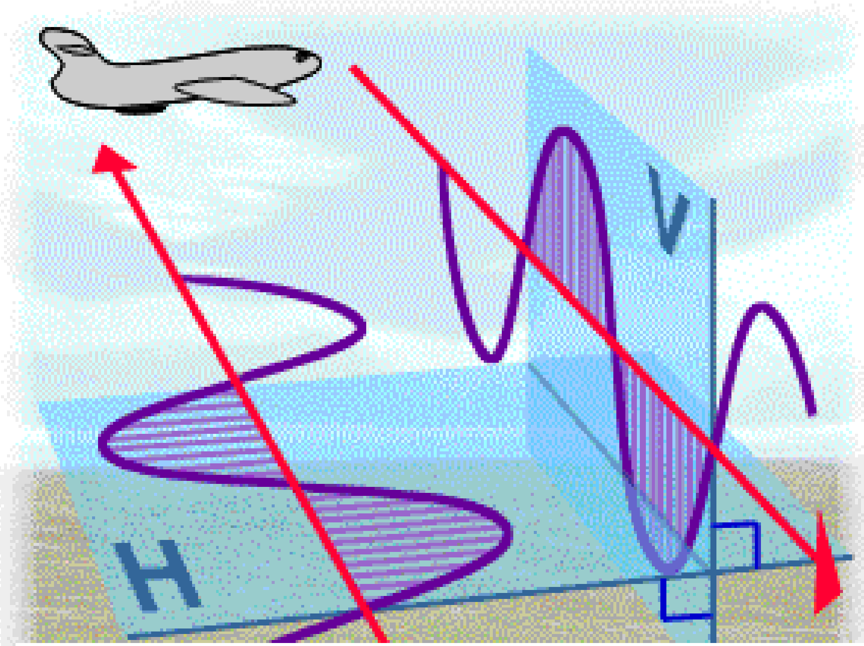
$$For\,good\,conductor, \qquad \delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \frac{1}{\sqrt{f\pi\mu\sigma}}$$

Polarization of the wave:
Is defined as the direction of the electric field at a given point as a function of time.

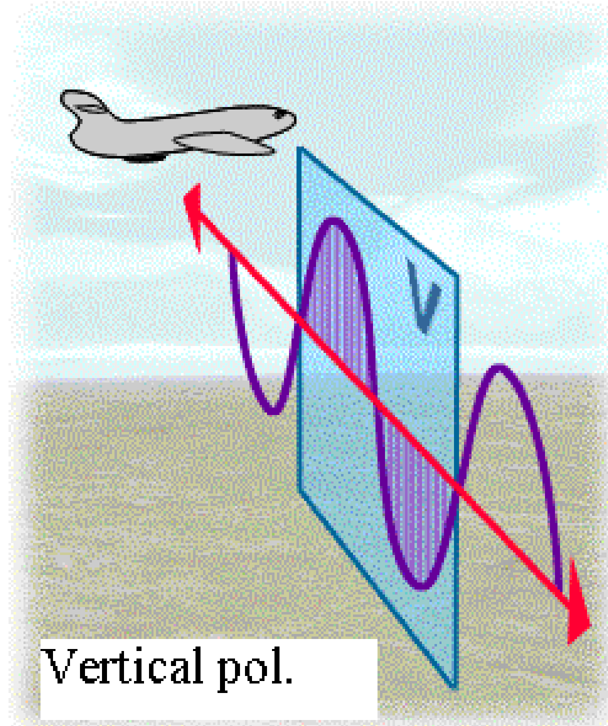
Types of polarization:
1.Linear polarization
2.Circular polarization
3.Elliptical polarization

1. Linear polarization: A wave is said to be linearly polarized if the electric field as a function of time remains along a straight line at some point in the medium.
Linear polarization has (Vertical polarization) and (Horizontal polarization)

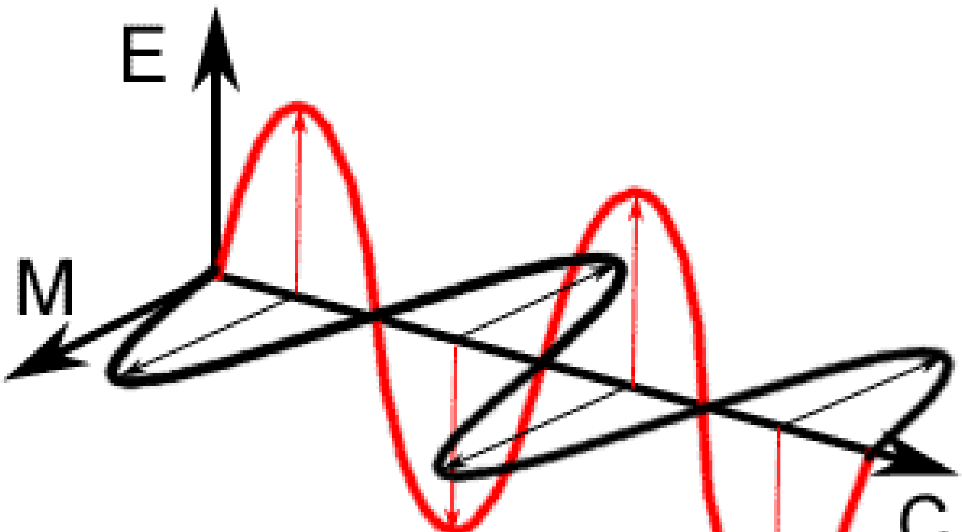
2. Circular polarization: A wave is said to be circularly polarized when the electric field traces a circle.



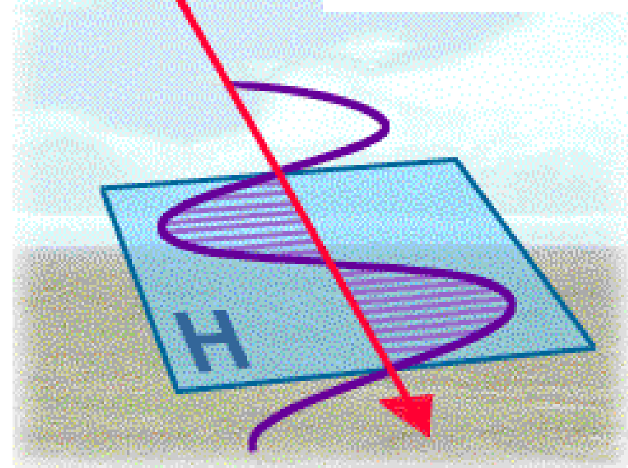
Vertical and horizontal polarization



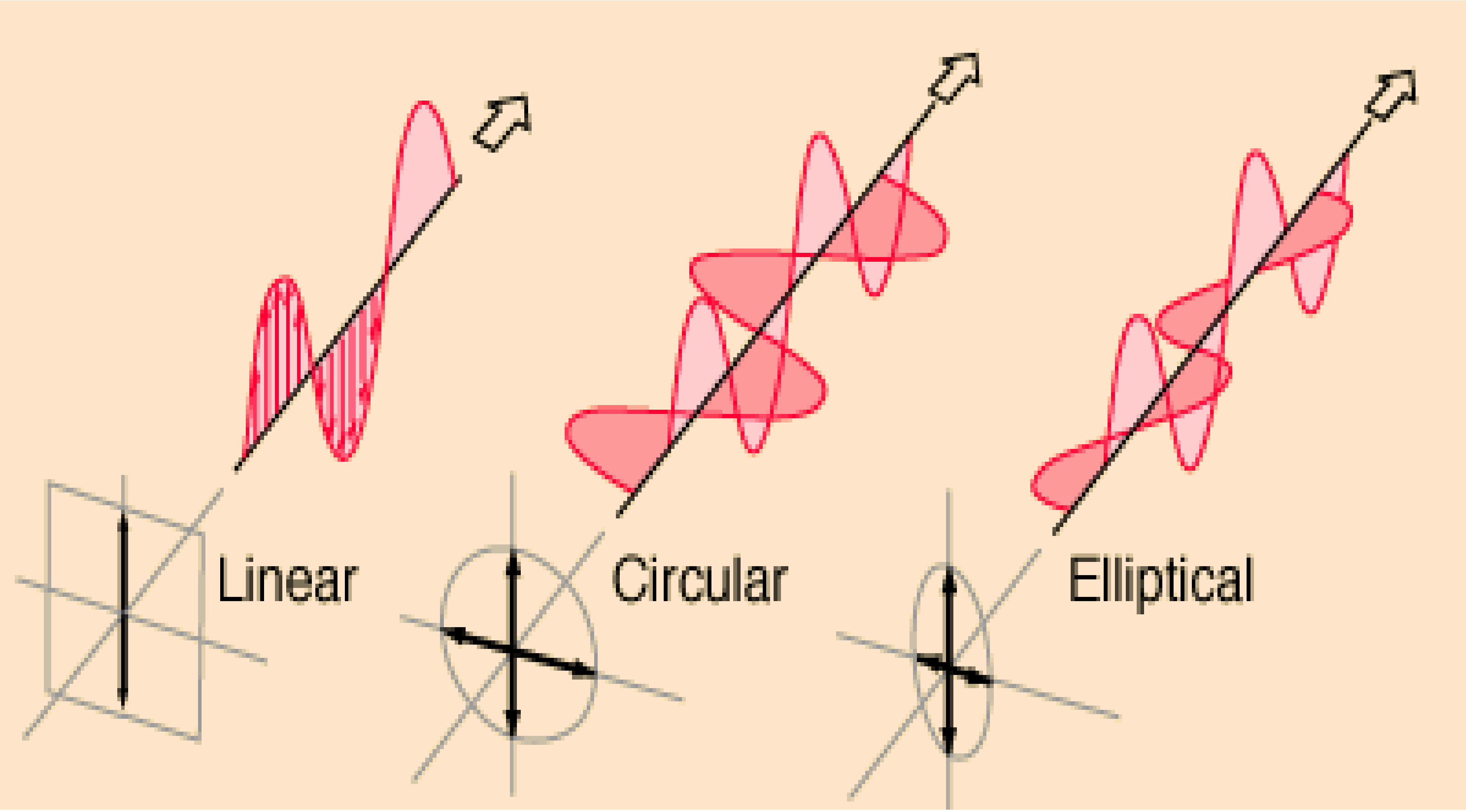
Vertical pol.



Vertical polarization



Horizontal polarization



Linear, Circular and elliptical polarization

Sources of different polarized EM waves:

1.Horizontal dipole produces horizontally polarized waves.
2.Vertical dipole produces vertically polarized waves
3. Circular slots produces circularly polarized waves
4. Elliptical slots produces elliptically polarized waves

Reflection of plane waves:

Reflection by a perfect conductor normal incidence:

When a wave in air is incident on a perfect conductor normally, it is entirely reflected. When an EM wave travelling in one medium is incident upon a second medium with a different dielectric constant, permeability and conductivity, it is partially reflected and partially transmitted.
If the electric field in the incident wave is expressed by the equation:

$$E_{yi} = E_i e^{j(\omega t+\beta z)} \qquad 1.15$$

for a wave travelling in –z direction, then the reflected wave can be expressed by the equation:

$$E_{yr} = E_r e^{j(\omega t-\beta z)} \qquad 1.16$$

The resultant electric field is:

$$E_{resultant} = E_{yi} + E_{yr} = E_i e^{j(\omega t+\beta z)} + E_{yr} = E_r e^{j(\omega t-\beta z)}$$
$$= 0$$

$$E_i + E_r = 0 \quad \Longrightarrow \quad E_i = -E_r$$

$reflection\ coefficient,$

$$\rho_r = \frac{E_r}{E_i} = 1\ \ (completely\ reflection)$$

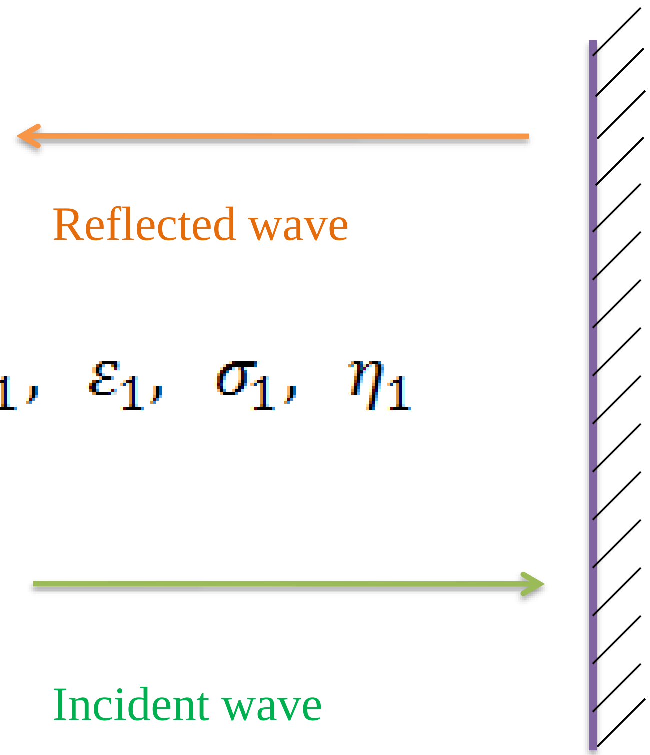
$transmission\ coefficient,$

$$\rho_t = \frac{E_t}{E_i} = 1\ \ (no\ transmission)$$

$$Standing\ wave\ ration, (SWR) = \frac{1+|\rho_r|}{1-|\rho_r|}$$
$$= \infty\ (completely\ reflection)$$

Medium 1

Medium 2 (perfect conductor)



$\mu_1, \; \varepsilon_1, \; \sigma_1, \; \eta_1$

$\mu_2, \; \varepsilon_2, \; \sigma_2, \; \eta_2$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^o$$

Reflection by a perfect conductor normal incidence

Since there are points at which the two waves are in phase and out of phase, the standing waves ratio is defined as the ratio of the amplitude of the standing wave at a maximum to that at a minimum.

$$VSWR = \frac{|V_{max}|}{|V_{min}|}$$

If both magnetic and electric field strengths were reversed there would be no reversal of direction of energy propagation.

Reflection by a perfect dielectric normal incident:

When an EM wave is incident normally on the surface of a perfect dielectric, part of the energy is transmitted and part of it is reflected.

$$Let\ \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}\,,\quad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

$$E_i = \eta_1 H_i \quad , \quad E_r = -\eta_1 H_r \quad , \quad E_t = \eta_2 H_t$$

$$E_i + E_r = E_t \quad , \quad H_i + H_r = H_t$$

Subtracting these equations:

$$-\begin{array}{l} E_i = \eta_1 H_i \\ E_r = -\eta_1 H_r \end{array} \quad \Rightarrow \quad \eta_1 (H_i + H_r) = E_i - E_r = \eta_1 H_t = \frac{\eta_1}{\eta_2} E_t = \frac{\eta_1}{\eta_2}(E_i + E_r)$$

So,

$$\eta_2 (E_i - E_r) = \eta_1 (E_i + E_r)$$

$$E_i (\eta_2 - \eta_1) = E_r (\eta_1 + \eta_2)$$

$$\therefore \quad \frac{E_r}{E_i} = \rho_r = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} \qquad (1.17)$$

$$\rho_t = \frac{E_t}{E_i} = \frac{E_i + E_r}{E_i} = 1 + \frac{E_r}{E_i} = 1 + \rho_r$$

$$= \frac{2\eta_2}{\eta_2 + \eta_1} \qquad (1.18)$$

$$\rho_t = 1 + \rho_r$$

$$\frac{H_r}{H_i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} \qquad (1.19)$$

$$Similarly, \quad \frac{H_t}{H_i} = \frac{2\eta_1}{\eta_2 + \eta_1} \qquad (1.20)$$

Reflection by a perfect conductor – Oblique incidence:

In order to find out the reflection coefficient when a wave incident on a good conductor obliquely, two type of polarization are considered:

1. Parallel polarization: is defined as the polarization in which electric field is parallel to the plane of incidence. The plane of incidence is the plane which contains the incident ray. It is also called vertical polarization.

2. Perpendicular polarization: is defined as the polarization in which electric field is perpendicular to the plane of incidence. It is also called horizontal polarization.

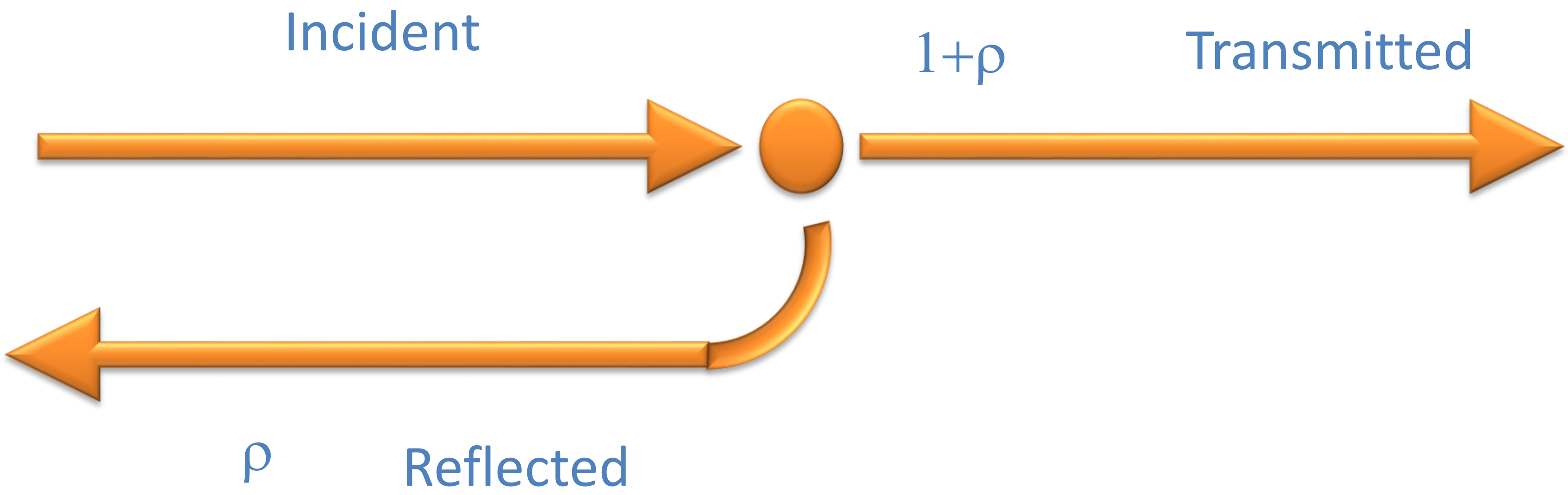
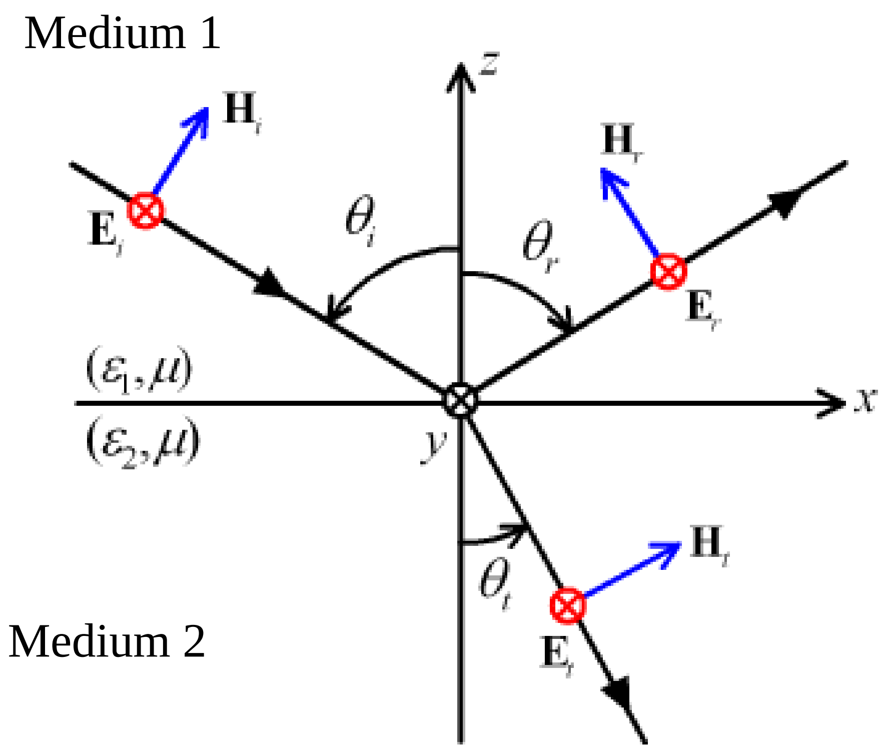


Figure (1.7) Incident, reflected and transmitted waves



Plane wave incident obliquely on a conducting plane (perpendicular polarization)

θ_i: *angle of incidence*

θ_r: *angle of reflection*

θ_t: *angle of transmission*

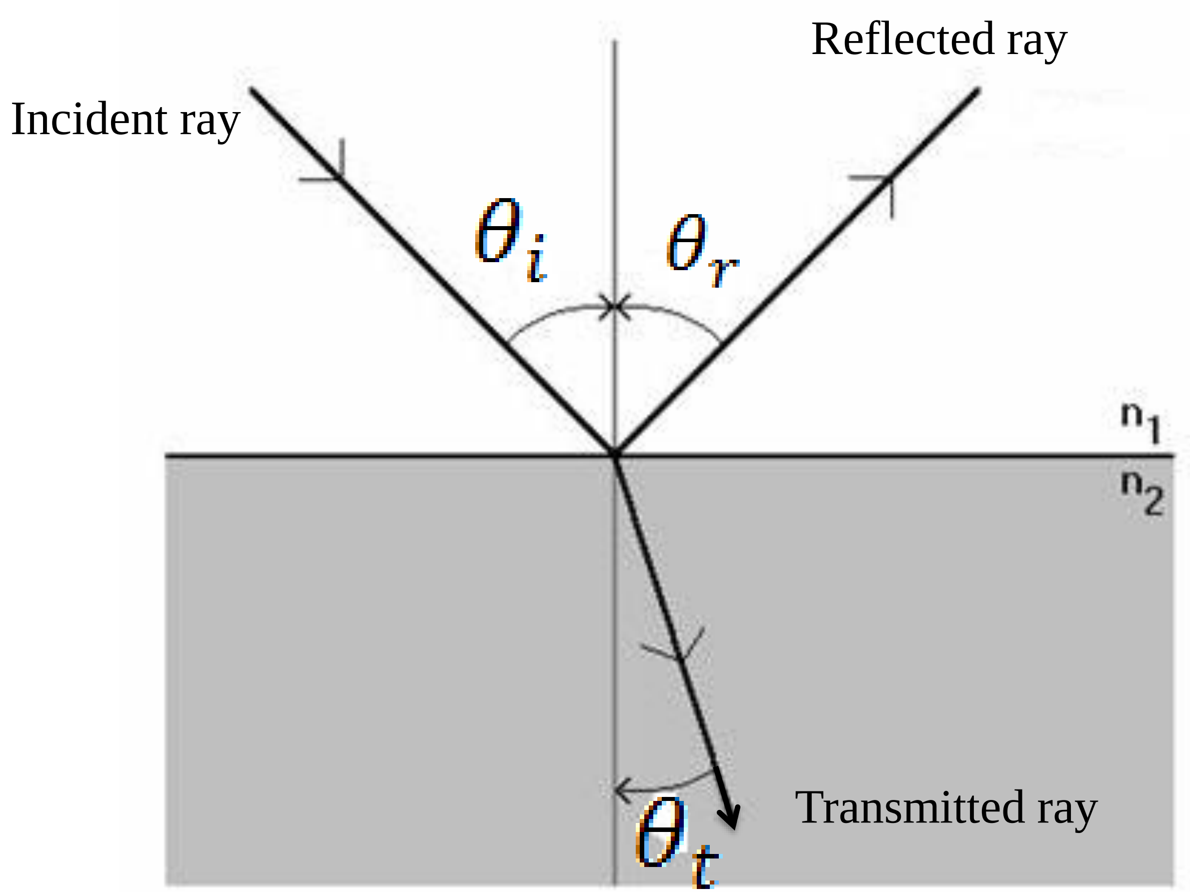


Figure (1.8) Reflection and transmission of wave

According to Snell’s law, angle of incidence and angle of reflection are related by:

$$\frac{sin\theta_i}{sin\theta_t} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \qquad \theta_i = \theta_r$$

The reflection coefficient for perpendicular polarization is given by:

$$\rho_r = \frac{E_r}{E_i} = \frac{cos\theta_i - \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - sin^2\theta_i}}{cos\theta_i + \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - sin^2\theta_i}} \qquad (1.21)$$

The power transmitted per square meter is:

$$P = \frac{E^2}{\eta}$$

The reflection coefficient for parallel polarization is given by:

$$\frac{E_r}{E_i} = \frac{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) cos\theta_i - \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - sin^2\theta_i}}{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) cos\theta_i + \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - sin^2\theta_i}} \qquad (1.22)$$

For no reflection, $\rho_r = 0$

Equation (1.21) becomes:

$$\cos\theta_i - \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_i} = 0 \implies \left(\frac{\varepsilon_2}{\varepsilon_1}\right) = \sin^2\theta_i + \cos^2\theta_i$$

$$\therefore \quad \varepsilon_2 = \varepsilon_1$$

It means, there will be always reflection.

Brewster angle: is defined as the angle of incidence at which there is no reflection. This occurs when the numerator of equation (1.22) is zero.

$$\sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_i} = \left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i \implies \frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i = \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 \cos^2\theta_i$$

$$\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i = \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 - \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 \sin^2\theta_i$$

$$\sin^2\theta_i \left(1 - \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2\right) = \frac{\varepsilon_2}{\varepsilon_1} - \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2$$

$$sin^2\theta_i\left(\frac{{\varepsilon_1}^2-{\varepsilon_2}^2}{{\varepsilon_1}^2}\right)=\frac{\varepsilon_1\varepsilon_2-{\varepsilon_2}^2}{{\varepsilon_1}^2} \Longrightarrow ({\varepsilon_1}^2-{\varepsilon_2}^2)sin^2\theta_i$$
$$=\varepsilon_2(\varepsilon_1-\varepsilon_2)$$

$$sin^2\theta_i=\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2} \quad , \quad cos^2\theta_i=\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2}$$

$$\theta_b=tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

This angle is called Brewster angle at which there is no reflection.

Pointing theorem: The electromagnetic wave is seen to be radiated outward from the source of time varying current to distance points. It is of interest to measure the energy transferred or propagated and the rate of this energy transfer. The relation between the rate of energy flow and the strength of electric and magnetic fields can be obtained by using Maxwell's equations.

$$\nabla XH=\varepsilon\frac{\partial E}{\partial t}+J$$

Thus, the pointing theorem states that the cross product of E and H at any point is a measure of the rate of energy flow per unit area at that point, that is:

$$P = E X H \text{ watts/m}^2$$