Chapter Two

Microwave transmission lines:

Transmission lines are used to confine electromagnetic waves so that they do not spread out and dissipate. Energy is transmitted through a transmission line by the propagating transverse electromagnetic (TEM) waves.

Types of transmission lines:

1. Parallel plate transmission line:

This type of transmission line consists of two parallel conducting plates separated by a dielectric slab of a uniform thickness. At microwave frequencies parallel plate transmission lines can be fabricated on a dielectric using printed circuit technology.

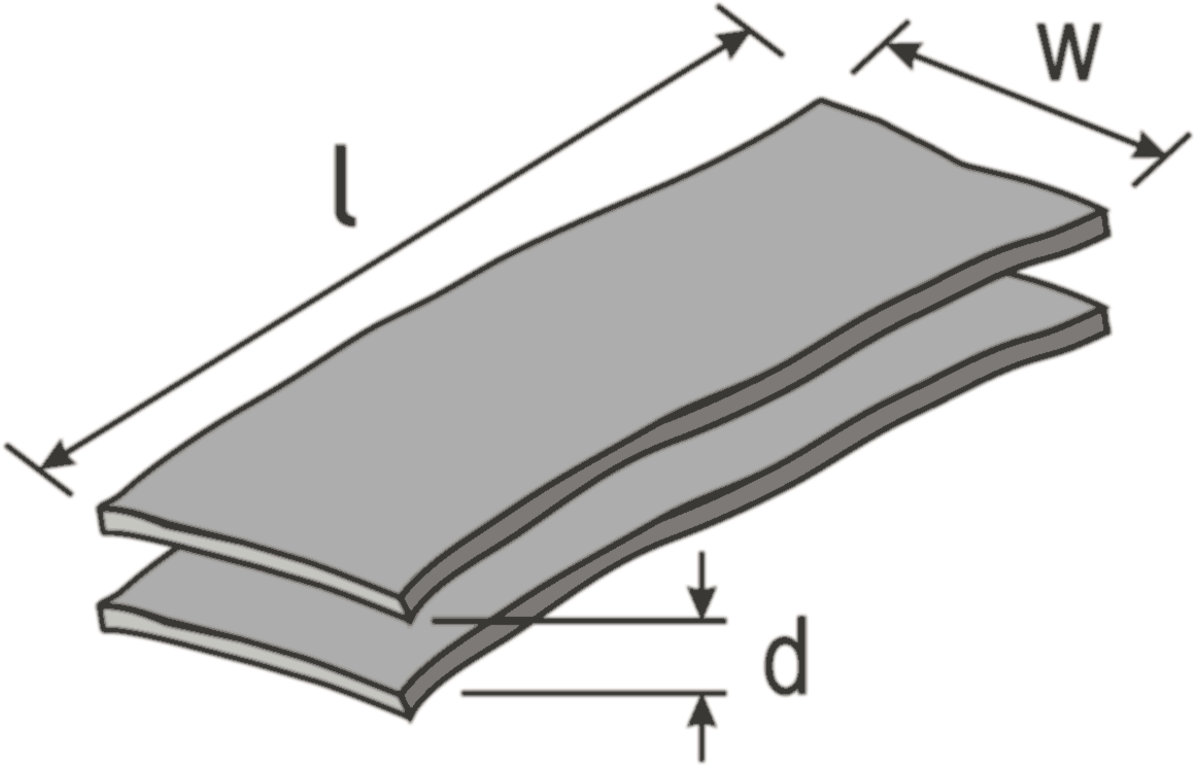


Figure (2.1) Parallel plate transmission line

2. Two wire transmission line:

This transmission line consists of a pair of parallel conducting wires separated by a uniform distance. Example, telephone wires, the flat lead-in-lines from antenna to a television receiver.

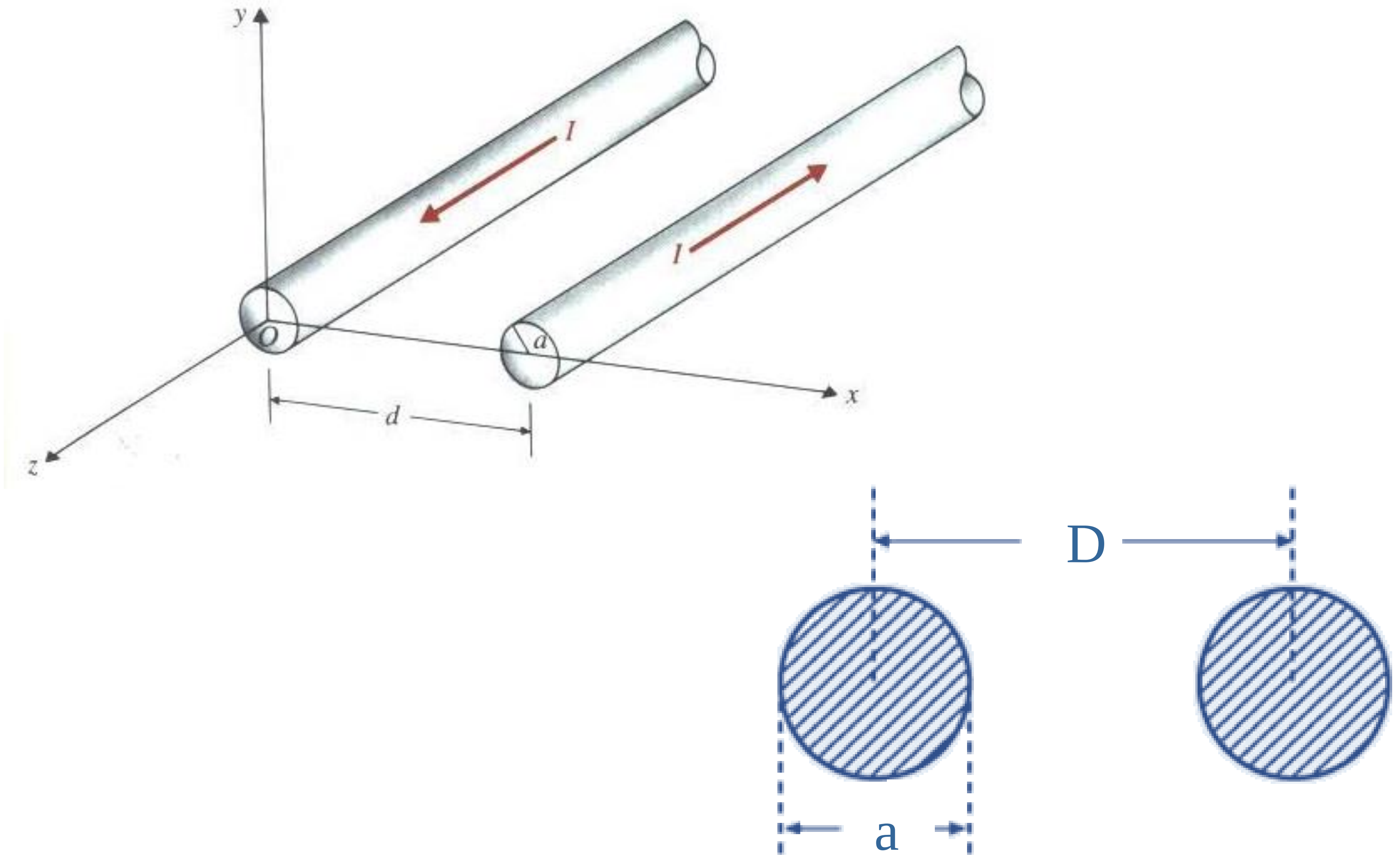
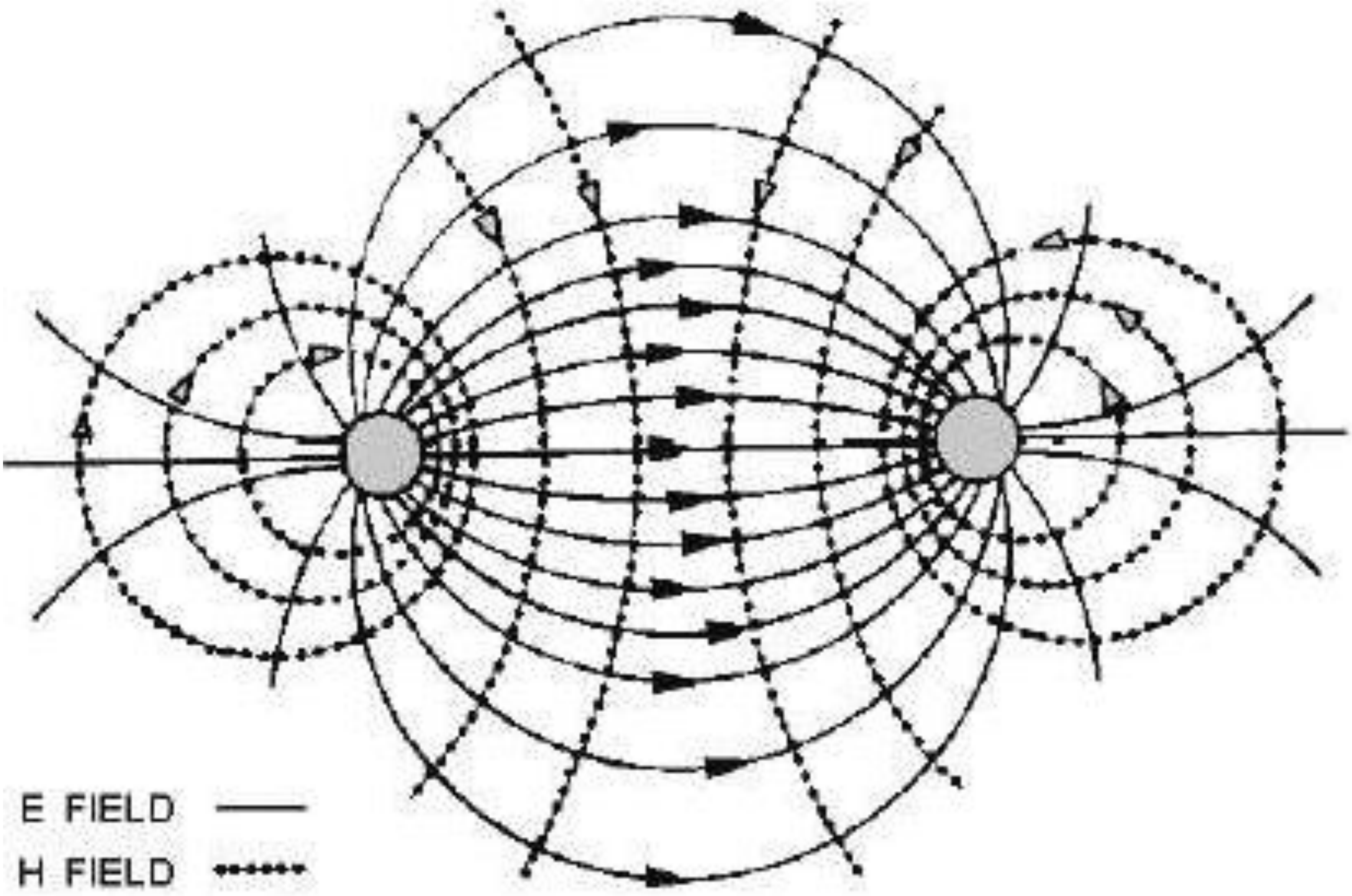


Figure (2.2) Two wire transmission line



Electric and magnetic field on a transmission line

3. Coaxial cable transmission line:

This consists of an inner conductor and a coaxial outer conducting sheet separated by a dielectric medium. This structure has the advantage of confining the electric and magnetic fields entirely within the dielectric region.

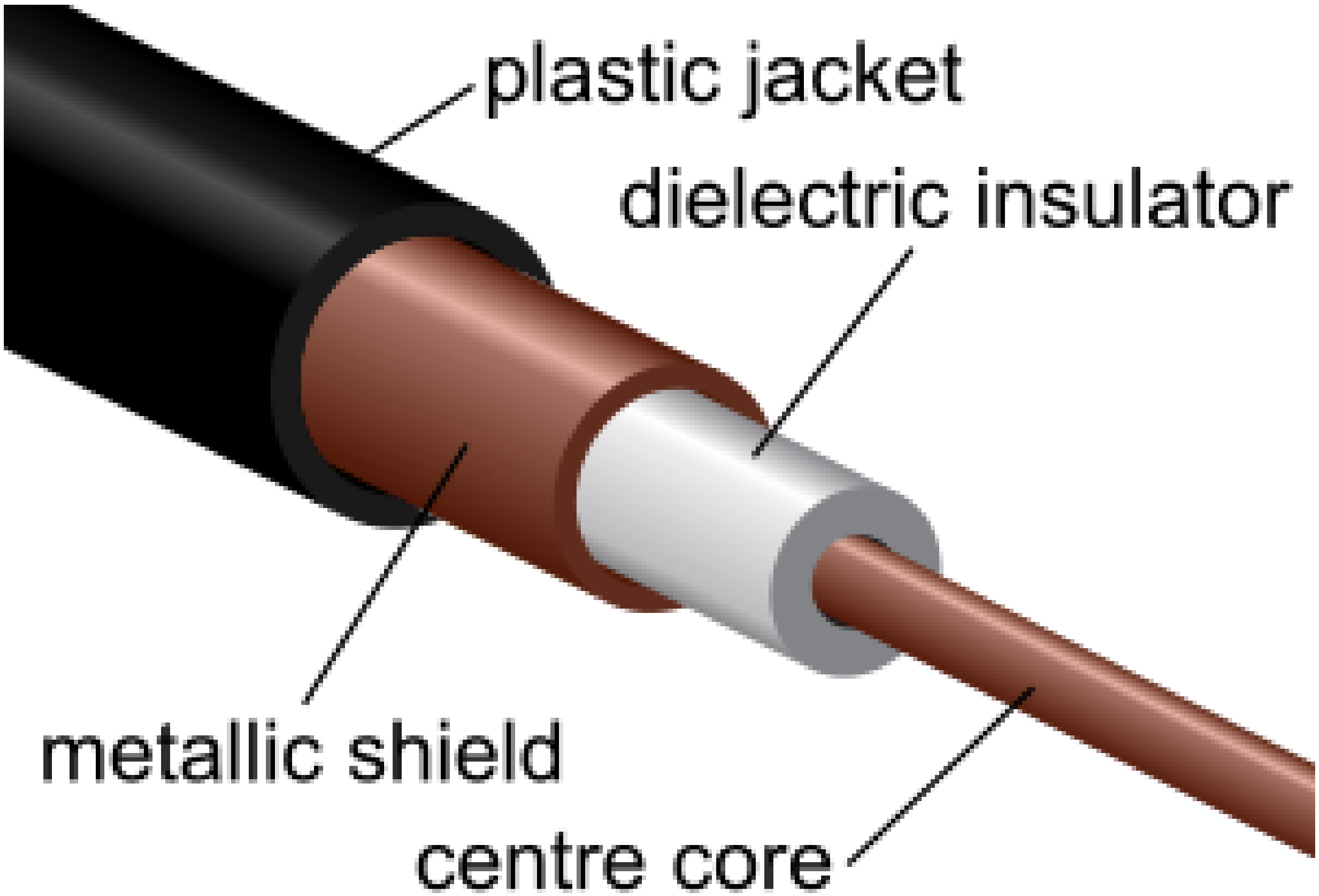


Figure (2.3) Coaxial cable transmission line

Transverse electromagnetic wave along a parallel plate transmission line:

Let us consider a y-polarized TEM wave propagating in the Z- direction along a uniform parallel plate transmission line as shown below:

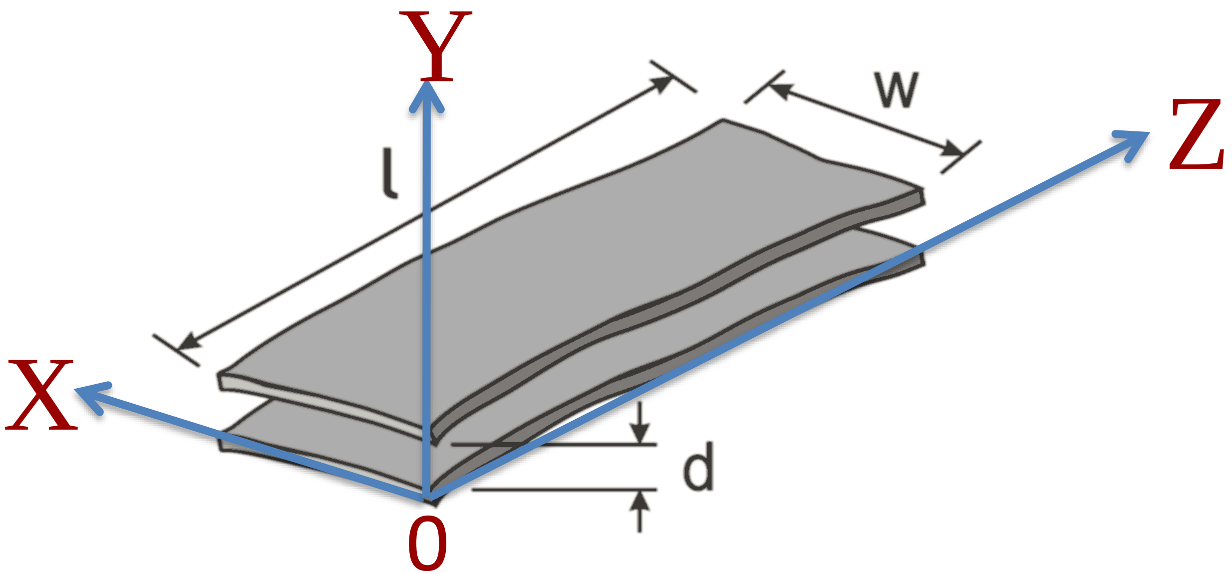


Figure (2.4) Parallel plate transmission line

$$E_y = E e^{-\gamma z}{}_{ay}$$

ay : *is a unit vector*

$$H_x = H e^{-\gamma z}{}_{ax}$$

Where γ is the propagation constant.
Assuming perfectly conducting plates and a lossless dielectric, we have:

$$\gamma = j\beta = j\omega\sqrt{\mu\varepsilon}$$

$$and \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$L = \mu\frac{d}{w} \qquad (\frac{H}{m})$$

Is the inductance per unit length of the parallel plate transmission line

d: *is the spacing between the plates*

w: *is the width of the plates*

$$C = \varepsilon\frac{w}{d} \qquad (\frac{F}{m})$$

Is the capacitance per unit length of the parallel plate transmission line

$$\beta = \omega\sqrt{LC} \qquad (\frac{rad}{m})$$

Is the phase constant.

$$Z_o = \frac{d}{w}\sqrt{\frac{\mu}{\varepsilon}} = \frac{d}{w}\eta \qquad (\Omega)$$

Is the intrinsic impedance of the line.

The velocity of propagation along the line is:

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}} \qquad (\frac{m}{s})$$

Lossy parallel plat transmission line:

The conductance between two parallel plates separated by a dielectric medium having a permittivity ε and a conductivity σ is:

$$G = \sigma\frac{w}{d} \qquad \left({}^{S}/_{m}\right)$$

The surface impedance Z_s is defined as the ratio of the tangential component of the electric field to the surface current density of the conductor surface.

$$Z_s = \frac{E_t}{J_s} \qquad (\Omega)$$

For the upper plate,

$$Z_s = \frac{E_z}{H_x} = \eta_c$$

Where η_c is the intrinsic impedance of the plate conductor.

The intrinsic impedance is:

$$Z_s = R_s + JX_s = (1 + j)\sqrt{\frac{\pi f \mu_c}{\sigma_c}} \qquad (\Omega)$$

σ_c and μ_c are for conductor.

The ohmic power dissipated in a unit length of the plate:

$$P = \frac{1}{2} I^2 \left(\frac{R_s}{w}\right) \qquad \left({}^{W}/_{m}\right)$$

The effective series resistance per unit length for both plates of a parallel plate transmission line of width w is:

$$R = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} \qquad \left(\Omega/m\right)$$

Parameter	Formula	Unit
R	$\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$	Ω/m
L	$\mu \frac{d}{w}$	H/m
G	$\sigma \frac{w}{d}$	S/m
C	$\varepsilon \frac{w}{d}$	***F/m***

Table (2.1) Parallel plate transmission line parameters (width = w, separation = d)

Microstrip Transmission Lines

As circuits have been reduced in size with integrated semiconductor electron devices, a transmission structure was required that was compatible with circuit construction techniques to provide guided waves over limited distances

Microstrip is a type of electrical transmission line which can be fabricated using printed circuit board [PCB] technology, and is used to convey microwave-frequency signals. It consists of a conducting strip separated from a ground plane by a dielectric layer known as the substrate

The disadvantages of microstrip compared with waveguide are the generally lower power handling capacity, and higher losses. Also, unlike waveguide, microstrip is not enclosed, and is therefore susceptible to cross-talk and unintentional radiation.

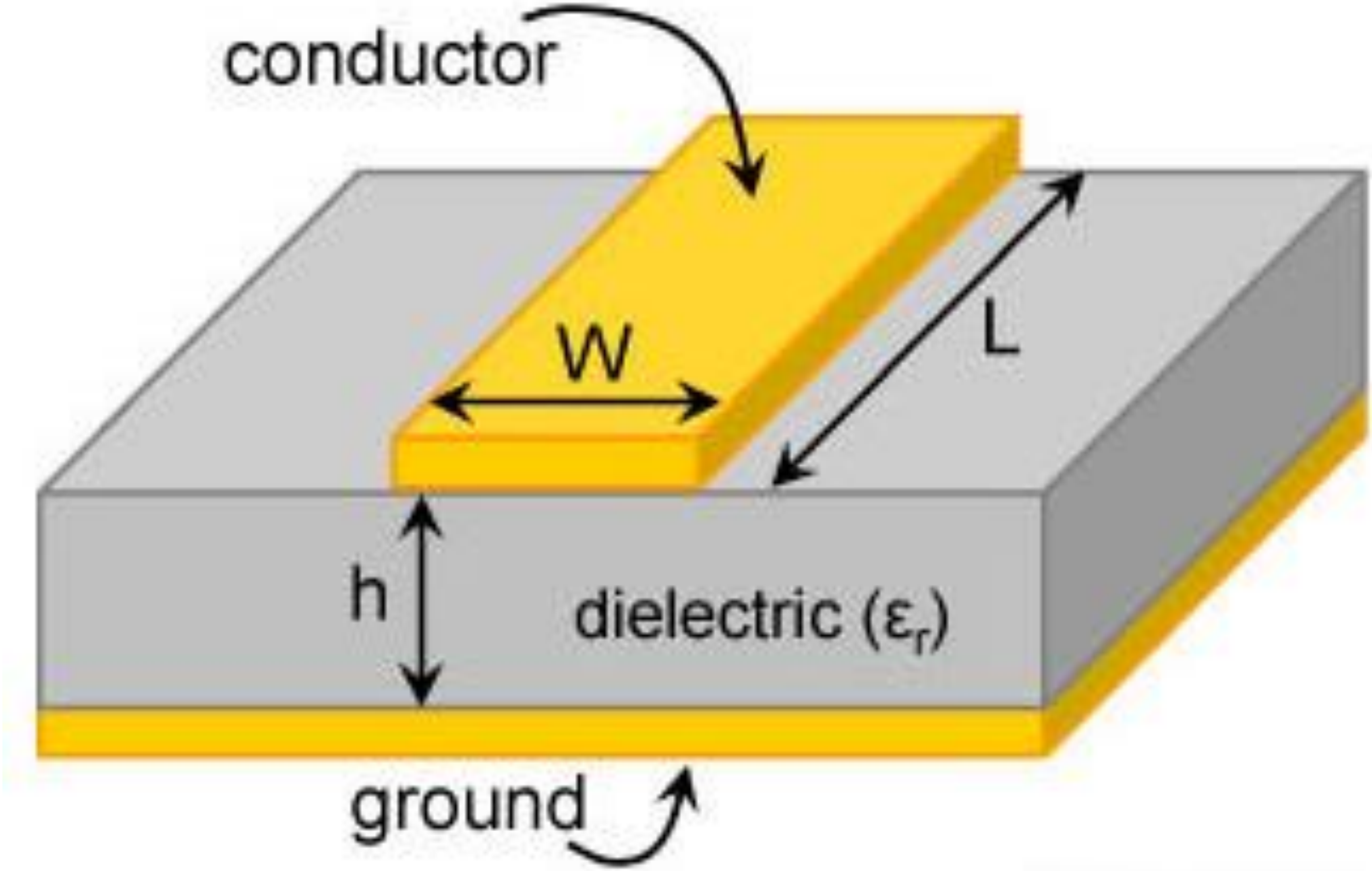


Figure (2.5) Microstrip transmission line

General Transmission Line Equation:

Consider a differential length ΔZ of a transmission line which is described by the following four parameters:

R, resistance per unit length (both conductors), in (Ω/m)

L, inductor per unit length (both conductors), in (H/m)

G, conductance per unit length, in (S/m)

C, capacitance per unit length, in (F/m)

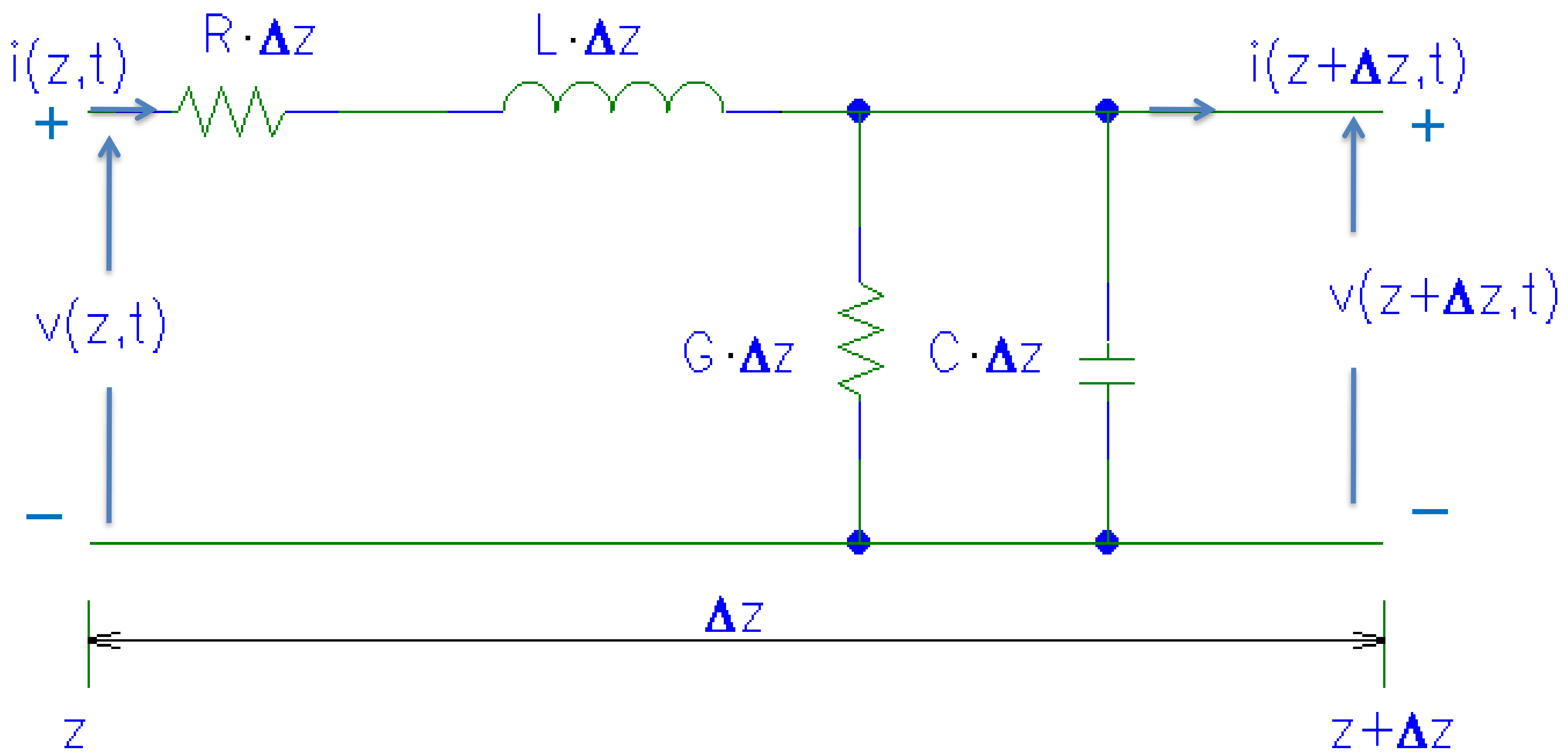


Figure (2.6) equivalent circuit of a two conductor transmission line

R and L are series elements, G and C are shunt elements. The quantities v(z,t) and v(z+Δz,t) are the instantaneous voltage at Z and (z+Δz) respectively. Similarly, i(z,t) and i(z+Δz,t) are the instantaneous currents at z and (z+Δz).
Applying Kirchhoff's voltage law:

$$v(z,t) - R\Delta z i(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$$

Which leads to:

$$-\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z} = Ri(z,t) + L\frac{\partial i(z,t)}{\partial t}$$

On the limit as $\Delta z \rightarrow 0$,the above equation becomes:

$$-\frac{\partial v(z,t)}{\partial z} = Ri(z,t) + L\frac{\partial i(z,t)}{\partial t} \qquad (2.1)$$

Similarly, applying Kirchhoff's current law, we get:

$$i(z,t) - G\Delta z v(z+\Delta z,t) - C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

On dividing by Δz and letting Δz approaches zero, we get:

$$-\frac{\partial i(z,t)}{\partial z} = Gv(z,t) + C\frac{\partial v(z,t)}{\partial t} \qquad (2.2)$$

Equation (1) and (2) are the general transmission line equations

Wave characteristics on an infinite transmission line:

The propagation constant is:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \qquad m^{-1}$$

Whose real and imaginary parts, α and β are the attenuation constant (Np/m) and phase constant (rad/m) of the line respectively.

The ratio of the voltage and current at any -z for the line is called the characteristic impedance of the line.

$$Z_o = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad (\Omega)$$

Note, that γ and Z_o are characteristic properties af a transmission line whether or not the line is infinitely long. They depend on R, L, G, C and ω not the length of the line.

The following three limiting cases have special significance.

1. Lossless line (R=0, G=0)

a. Propagation constant:

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}$$

$$\alpha = 0$$

$$\beta = \omega\sqrt{LC} \quad (linear\ function\ of\ \omega)$$

b. Phase velocity:

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

c. Characteristic impedance:

$$Z_o = R_o + jx_o = \sqrt{\frac{L}{C}}$$

2. Low-loss line $(R \ll \omega L), G \ll \omega C)$

a. Propagation constant:

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}\left(1 + \frac{R}{j\omega L}\right)^{1/2}\left(1 + \frac{G}{j\omega C}\right)^{1/2}$$

$$\cong j\omega\sqrt{LC}\left[1+\frac{1}{2j\omega}\left(\frac{R}{L}+\frac{G}{C}\right)\right]$$

$$\alpha \cong \frac{1}{2}\left(R\sqrt{\frac{C}{L}}+G\sqrt{\frac{L}{C}}\right)$$

$$\beta = \omega\sqrt{LC}$$

b. Phase velocity:

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

c. Characteristic impedance:

$$Z_o = R_o + jx_o \cong \sqrt{\frac{L}{C}}\left[1+\frac{1}{2j\omega}\left(\frac{R}{L}-\frac{G}{C}\right)\right]$$

$$R_o \cong \sqrt{\frac{L}{C}}$$

.

$$x_o \cong -\sqrt{\frac{L}{C}}\frac{1}{2\omega}\left(\frac{R}{L}-\frac{G}{C}\right)$$

3. Distortionless line $\left(\frac{R}{L}=\frac{G}{C}\right)$

a. Propagation constant:

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)\left(\frac{RC}{L}+j\omega C\right)} \cong \sqrt{\frac{C}{L}}(R+j\omega L)$$

$$\alpha = R\sqrt{\frac{C}{L}} \qquad \beta = \omega\sqrt{LC}$$

b. Phase velocity:

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

c. Characteristic impedance:

$$Z_o = R_o + jx_o \cong \sqrt{\frac{R + j\omega L}{\left(\frac{RC}{L}\right) + j\omega C}} = \sqrt{\frac{L}{C}}$$

$$R_o = \sqrt{\frac{L}{C}}$$

$$x_o = 0$$

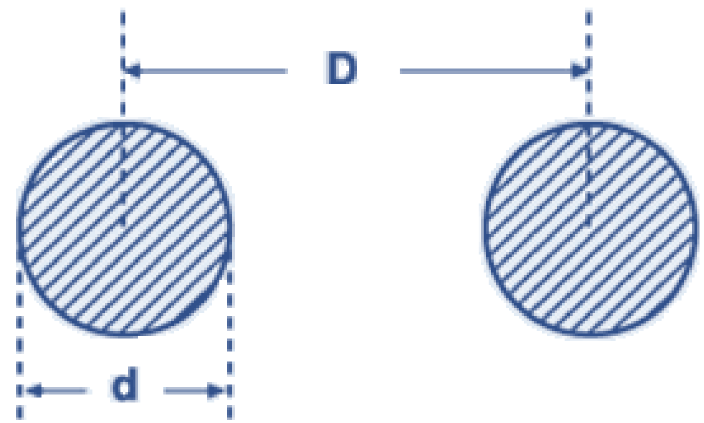
Transmission line parameters:

The electrical properties of a transmission line at a given frequency are completely characterized by its four distributed parameters R, L, G and C. These parameters have been obtained for parallel plate transmission line. We will obtain them for two pair wire and coaxial cable transmission line.

1. Two wire transmission line:

The capacitance per unit length of a two wire transmission line, whose wires have radius a and are separated by a distance D is:

$$C = \frac{\pi\varepsilon}{\cosh^{-1}\left(\frac{D}{2a}\right)} \quad \left(F/m\right)$$



The inductance is:

$$L = \frac{\mu}{\pi}\cosh^{-1}\left(\frac{D}{2a}\right) \quad \left(H/m\right)$$

And the conductance is:

$$G = \frac{\pi\sigma}{\cosh^{-1}\left(\frac{D}{2a}\right)} \qquad \left({}^{S}\!/_{m}\right)$$

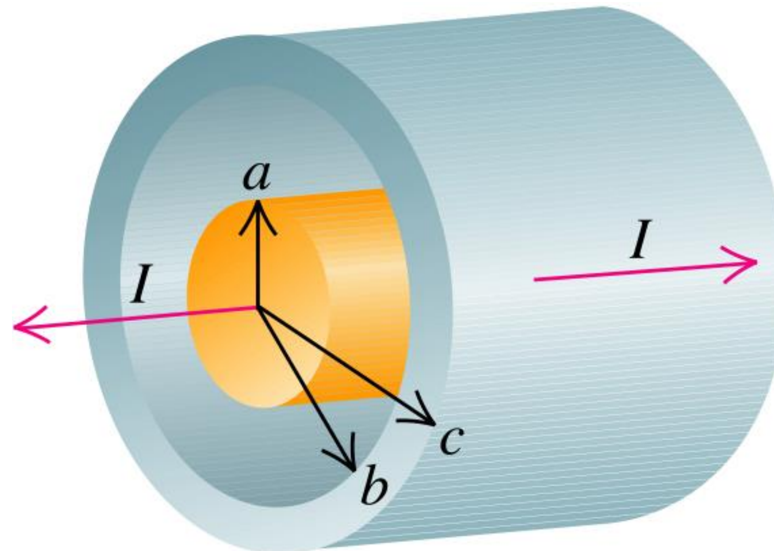
The series resistance per unit length is:

$$R = 2\left(\frac{R_s}{2\pi a}\right) = \frac{1}{\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} \qquad \left({}^{\Omega}\!/_{m}\right)$$

2. Coaxial transmission line:

The external inductance per unit length of a coaxial transmission line with a center conductor of radius a and an outer conductor of radius b is:

$$L = \frac{\mu}{2\pi} \ln\frac{b}{a} \qquad \left({}^{H}\!/_{m}\right)$$



The capacitance is:

$$C = \frac{2\pi\varepsilon}{ln\left(\frac{b}{a}\right)} \qquad \left({}^{F}/_{m}\right)$$

The conductance is:

$$G = \frac{2\pi\sigma}{ln\left(\frac{b}{a}\right)} \qquad \left({}^{S}/_{m}\right)$$

The resistance per unit length is:

$$R = \frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{1}{2\pi}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}\left(\frac{1}{a}+\frac{1}{b}\right) \qquad \left({}^{\Omega}/_{m}\right)$$

Parameter	Two wire line	Coaxial line	Unit
R	$\frac{R_s}{\pi a}$	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	Ω/m
L	$\frac{\mu}{\pi} cosh^{-1}\left(\frac{D}{2a}\right)$	$\frac{\mu}{2\pi} ln \frac{b}{a}$	H/m
G	$\frac{\pi\sigma}{cosh^{-1}\left(\frac{D}{2a}\right)}$	$\frac{2\pi\sigma}{ln\left(\frac{b}{a}\right)}$	S/m
C	$\frac{\pi\varepsilon}{cosh^{-1}\left(\frac{D}{2a}\right)}$	$\frac{2\pi\varepsilon}{ln\left(\frac{b}{a}\right)}$	F/m

Table (2.2) Distributed parameters of two wire and coaxial transmission lines

<u>Mismatched transmission line:</u>

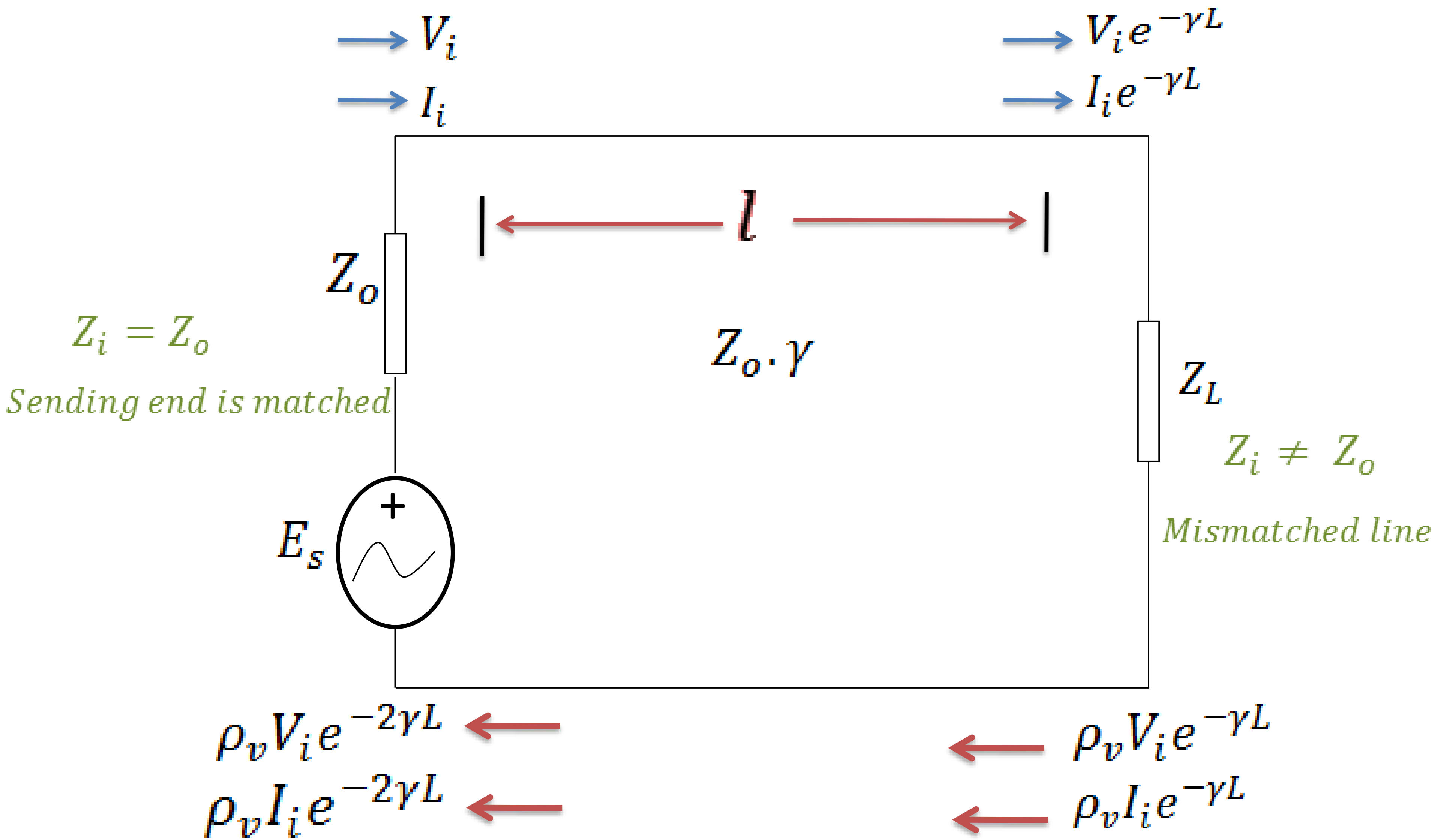


Figure (2.7) Currents and voltages on a mismatched transmission line

At a distance x from the sending end of the line:

$$V_x = V_i e^{-\gamma x} + V_r e^{\gamma x} = \rho_v V_i e^{-\gamma(2\gamma - x)} \qquad (2.1)$$

V_r is the reflected voltage at the receiving end of the line.

$$V_r = \rho_v V_i e^{-\gamma l}$$

Similarly the current at a distance x is:

$$I_x = I_i e^{-\gamma x} + I_r e^{\gamma x}$$

$$Or \;\; I_x = \frac{V_i e^{-\gamma x}}{Z_o} - \frac{V_r e^{\gamma x}}{Z_o}$$

The minus sign is because the reflected current is always out of phase with the reflected voltage.

$$V_s = V_i + V_r \qquad and \;\; I_s = \frac{V_i}{Z_o} - \frac{V_r}{Z_o}$$

Adding these two equations gives:

$$V_i = \frac{(V_s + I_s Z_o)}{2}$$

And subtracting the two equations gives:

$$V_r = \frac{(V_s - I_s Z_o)}{2}$$

Equation (2.1) becomes:

$$V_x = \left[\frac{V_s + I_s Z_o}{2}\right] e^{-\gamma x} + \left[\frac{V_s - I_s Z_o}{2}\right] e^{\gamma x}$$

$$= V_s \left[\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right] - I_s Z_o \left[\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right]$$

$$Or \quad V_x = V_s cosh\gamma x - I_s Z_o sinh\gamma x$$

$$Similarly, \quad I_x = I_s cosh\gamma x - \frac{V_s}{Z_o} sinh\gamma x$$

Voltage and current reflection coefficients:

$$Z_L = \frac{V_L}{I_L} = \frac{V_I + V_r}{\frac{V_I}{Z_o} - \frac{V_r}{Z_o}}$$

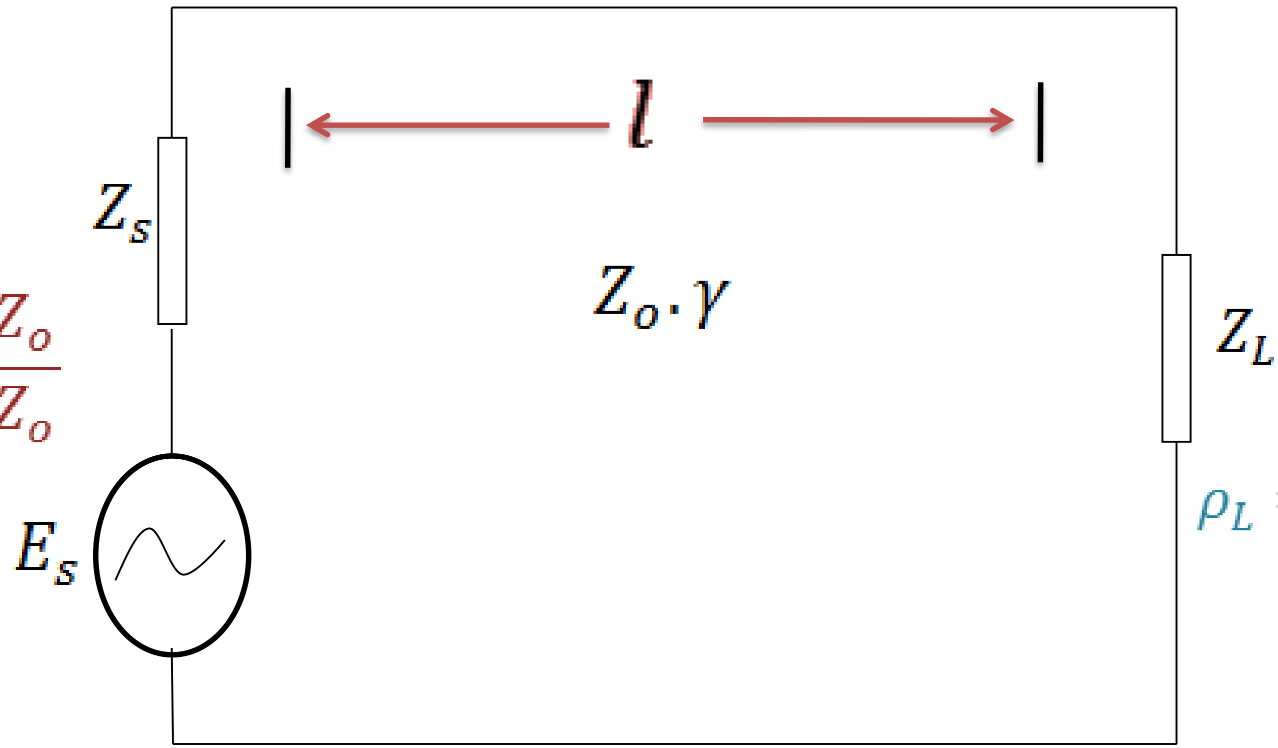
V_I: Incident voltage at load

$$\rho_v(Voltage\,reflection\,coefficient) = \frac{Z_L - Z_o}{Z_L + Z_o}$$

Since the reflected current is always out of phase with the reflected voltage:

$$\therefore \quad \rho_i = -\rho_v = \frac{Z_o - Z_L}{Z_L + Z_o}$$

Line mismatched at both ends



$$\rho_s = \frac{Z_s - Z_o}{Z_s + Z_o}$$

$$\rho_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$V_L = \frac{V_s e^{-\gamma x}(1 + \rho_L)}{1 - \rho_L \rho_s e^{-2\gamma l}}$$

$$Z_{in} = Z_o \left[\frac{Z_L + jZ_o tan\beta l}{Z_o + jZ_L tan\beta l}\right]$$

Standing waves and voltage standing wave ratio:

In case of a transmission line not terminated to the characteristic impedance of a line, some power is absorbed and the rest is reflected, the reflected waves travelling towards the generator. The incident waves and reflected waves combine together to form an interference pattern known as standing waves having maxima and minima. The maximum voltage (V_{max}) occurs whenever the incident and reflected waves are in-phase with one another.

$$V_{max} = V_i + V_r = V_i(1 + |\rho_v|)$$

The minimum voltage (V_{min}) occurs at those points on the line where the incident and reflected waves are out of phase with one another.

$$V_{min} = V_i - V_r = V_i(1 - |\rho_v|)$$

The voltage standing ratio S is the ratio of:

$$S = \frac{V_{max}}{V_{min}} = \frac{V_i(1 + |\rho_v|)}{V_i(1 - |\rho_v|)} = \frac{1 + |\rho_v|}{1 - |\rho_v|}$$