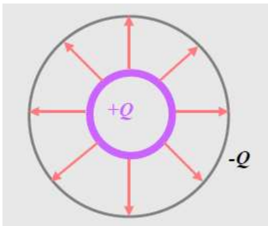
Gausses law, Electric potential

Lecture 3

Electric Displacement (Electric Flux)

- The density of electric displacement is the *electric (displacement) flux density,* ***D.***
- In free space the relationship between *flux density* and electric field is

$$\overline{D} = \varepsilon_0 \overline{E}$$



- *The electric (displacement) flux density for a point charge centered at the origin is*

$$\overline{D} = \hat{a}_R \frac{Q}{4\pi R^2}$$

Gauss's Law

- Gauss's law states that "the net electric flux emanating from a close surface *S is equal to* the total charge contained within the volume *V bounded by that surface."*

$$\oint_S \overline{D} \bullet d\overline{s} = Q_{encl}$$

3

Gausses law in differential form

Simple application of the divergence theorem:

$$\begin{cases} \oint_S \vec{E} \cdot d\vec{A} = \int_V \nabla \cdot \vec{E} dV \\ \oint_S \vec{E} \cdot d\vec{A} = 4\pi Q = 4\pi \int_V \rho dV \end{cases} \rightarrow \int_V (\nabla \cdot \vec{E} - 4\pi\rho) dV = 0$$

This is valid for any surface V:

$$\nabla \cdot \vec{E} = 4\pi\rho$$

Comments:

- First Maxwell's equations

4

Applications of Gauss's Law

- Gauss's law is an *integral equation for the* unknown electric flux density resulting from a given charge distribution.

$$\oint_S \overline{D} \bullet d\overline{s} = Q_{encl}$$

Known

Unknown

5

Applications of Gauss's Law (Cont'd)

- In general, solutions to *integral equations* must be obtained using numerical techniques.
- However, for certain symmetric charge distributions closed form solutions to Gauss's law can be obtained.
- Closed form solution to Gauss's law relies on our ability to construct a suitable family of *Gaussian surfaces.*
- A *Gaussian surface is a surface to which* the electric flux density is normal and over which equal to a constant value.

6

Electric Flux Density of a Point Charge Using Gauss's Law

- Consider a point charge at the origin:

1- Construct a family of Gaussian surfaces, Spheres of radius R where

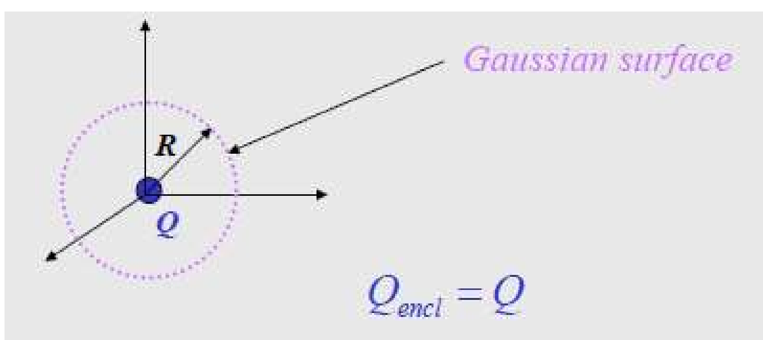
$$0 \leq R \leq \infty$$

2- Evaluate the total charge within the volume enclosed by each Gaussian surface

$$Qencl = \int_v \rho dv$$

7

Electric Flux Density of a Point Charge Using Gauss's Law (Cont'd)



8

Electric Flux Density of a Point Charge Using Gauss's Law (Cont'd)

3- For each Gaussian surface, evaluate the integral

$$\oint_s D \bullet ds = DS$$

Surface area of Gaussian surface

Magnitude of D on Gaussian surface

9

Electric Flux Density of a Point Charge Using Gauss's Law (Cont'd)

- Solve for ***D on each Gaussian surface***

$$D = \frac{Q_{encl}}{S}$$

$$\overline{D} = \hat{a}_R \frac{Q}{4\pi R^2} \Rightarrow \overline{E} = \frac{\overline{D}}{\varepsilon_0} = \hat{a}_R \frac{Q}{4\pi\varepsilon_0 R^2}$$

10

MORE APPLICATIONS OF GAUSS'S LAW

- ***Electric Flux Density of an Infinite Line Charge Using Gauss's Law***
- ***Electric Flux Density of an surface Charge Using Gauss's Law***

11

ELECTRIC POTENTIAL

- An electric field is a *force field.*
- If a body being acted on by a force is

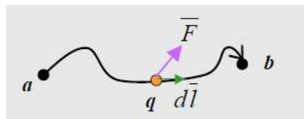
moved from one point to another, then *work* is done.

- The concept of *scalar electric potential* provides a measure of the work done in moving charged bodies in an electrostatic field.

12

Electrostatic Potential (Cont'd)

- The work done in moving a test charge from one point to another in a region of electric field:



$$W_{a\to b} = -\int_a^b \overline{F}.d\overline{l} = -q\int_a^b \overline{E}.d\overline{l}$$

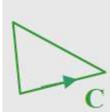
13

Electrostatic Potential (Cont'd)

- The electrostatic field is *conservative:*

– The value of the line integral depends only on the end points and is independent of the path taken.

– The value of the line integral around any closed path is zero.



$$\oint_C \overline{E}\cdot d\overline{l} = 0$$

14

Electrostatic Potential (Cont'd)

- The work done per unit charge in moving a test charge from point *a to point b is the electrostatic potential difference between the two points:*

$$V_{ab} = \frac{W_{a\to b}}{q} = -\int_a^b \overline{E} \bullet d\overline{l}$$

Electrostatic potential difference

15

Electrostatic Potential (Cont'd)

- Since the electrostatic field is conservative we can write

$$V_{ab} = -\int_a^b \overline{E} \bullet d\overline{l} = -\int_a^{p_0} \overline{E} \bullet d\overline{l} - \int_{p_0}^b \overline{E} \bullet d\overline{l}$$

$$= -\int_{p_0}^b \overline{E} \bullet d\overline{l} - (-\int_{p_0}^a \overline{E} \bullet d\overline{l}) = V(b) - V(a)$$

16

Electrostatic Potential (Cont'd)

- Thus the *electrostatic potential V is a scalar* field that is defined at every point in space.
- In particular the value of the *electrostatic potential at any point P is given by*

$$V(P) = -\int_{p_0}^{p} \overline{E} \bullet d\bar{l}$$

Reference Point

The ***reference point (P0) is where the potential is*** zero (analogous to ***ground in circuit theory).***
• Often the reference is taken to be at infinity so that the potential of a point in space is defined as

$$V(P) = -\int_{\infty}^{p} \overline{E} \bullet d\bar{l}$$

17

Electrostatic Potential and Electric Field

The work done in moving from point *a to* point *b can be written as*

$$W_{a \to b} = qV_{ab} = q\{V_{(b)} - V_{(a)}\} = -q\int_{a}^{b} \overline{E} \bullet d\bar{l}$$

Along a short path of length Δ***l we have***

$$\Delta W = q \Delta V = -q \overline{E} \bullet dl$$

$$\Delta V = -\overline{E} \bullet \Delta \bar{l}$$

Along an incremental path of length ***dl we*** have $dV = -\overline{E} \bullet d\bar{l}$

Recall from the definition of *directional derivative:* $dV = \nabla V \bullet d\bar{l}$

Thus:

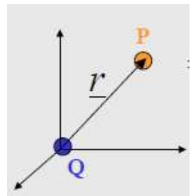
$$\overline{E} = -\nabla V$$

18

Electrostatic Potential of a Point Charge at the Origin

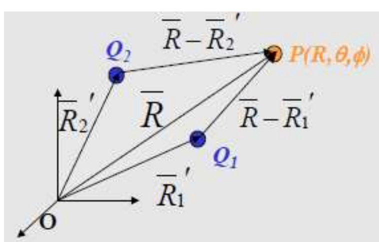
$$V(R) = -\int_{\infty}^{R} \overline{E} \bullet d\overline{l} = -\int_{\infty}^{R} \hat{a}_R \frac{Q}{4\pi\varepsilon_0 (R'^2)} \cdot \hat{a}_R \, dR$$

$$= \frac{Q}{4\pi\varepsilon_0} \int_{R}^{\infty} \frac{dR}{(R)^2} = \frac{Q}{4\pi\varepsilon_0 R}$$



19

Electrostatic Potential Resulting from Multiple Point Charges



$$V(\overline{R}) = \sum_{k=1}^{n} \frac{Q_k}{4\pi\varepsilon_0 \left| \overline{R} - \overline{R}'_k \right|}$$

20

Electrostatic Potential Resulting from Continuous Charge Distributions

$$V(\overline{R}) = \frac{1}{4\pi\varepsilon_0}\int_{L'} \frac{\rho_l(\overline{R}')\,dl'}{\left|\overline{R} - \overline{R}'\right|}$$ ⇐ **line charge**

$$V(\overline{R}) = \frac{1}{4\pi\varepsilon_0}\int_{s'} \frac{\rho_s(\overline{R}')\,ds'}{\left|\overline{R} - \overline{R}'\right|}$$ ⇐ **surface charge**

$$V(\overline{R}) = \frac{1}{4\pi\varepsilon_0}\int_{v'} \frac{\rho_v(\overline{R}')\,dv'}{\left|\overline{R} - \overline{R}'\right|}$$ ⇐ **volume charge**

21