DIFFERENTIAL FORM OF GAUSS'S LAW

Lec 5

Gauss's Law in Integral Form

$$\oint_S \overline{D} \bullet d\overline{s} = Q_{enc} = \int_V \rho dv$$

Also called ***Gauss's theorem or Green's theorem.***

$$\oint_S \overline{G} \bullet d\overline{s} = \int_V \nabla \bullet \overline{G} dv$$

Recall from Divergence theorem

$$\oint_S \overline{G} \bullet d\overline{s} = \int_V \nabla \bullet \overline{D} dv = \int_V \rho dv$$

Applying Divergence Theorem to Gauss's Law

$$\nabla \bullet \overline{D} = \rho$$

Because the above must hold for any volume *V, we must have*

Differential form of Gauss's Law

Fields in Materials

- Materials contain charged particles that respond to applied electric and magnetic fields.
- Materials are classified according to the nature of their response to the applied fields.

Classification of Materials

- Conductors
- Semiconductors
- Dielectrics
- Magnetic materials

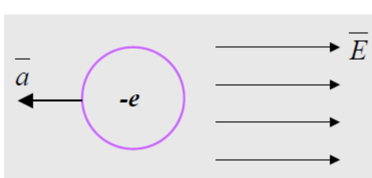
3

Conductors

- A *conductor is a material in which electrons* in the outermost shell of the electron migrate easily from atom to atom.
- Metallic materials are in general good conductors.

Conduction Current

- A constant electric field would cause an electron to move with constant acceleration.



$$\overline{a} = \frac{-e\overline{E}}{m_e}$$

4

Conduction Current (Cont'd)

- In a conductor, electrons are constantly colliding with each other and with the fixed nuclei, and losing momentum.
- The net macroscopic effect is that the electrons move with a (constant) drift velocity ***v_d* which is** proportional to the electric field.

$$\bar{v}_d = -\mu_e \bar{E}$$

Electron mobility

5

Conductor in an Electrostatic Field

- To have an electrostatic field, all charges must have reached their equilibrium positions (i.e., they are stationary).
- Under such static conditions, there must be *zero electric field within the conductor.* (Otherwise charges would continue to flow.)
- If the electric field in which the conductor is immersed suddenly changes, charge flows temporarily until equilibrium is once again reached with the electric field inside the conductor becoming zero.
- In a metallic conductor, the establishment of equilibrium takes place in about 10^{-19} s an extraordinarily short time indeed.

6

Conductor in an Electrostatic Field (Cont'd)

- There are two important consequences to the fact that the electrostatic field inside a metallic conductor is zero:

1- The conductor is an equipotential body.

2- The charge on a conductor must reside entirely on its surface.

- A corollary of the above is that the electric field just outside the conductor must be normal to its surface.

7

ELECTROSTATIC BOUNDARY CONDITIONS

Fundamental Laws of Electrostatics in Integral Form

$$\oint_c \overline{E} \bullet d\,\overline{l} = 0$$

Conservative field

$$\oint_s D \bullet d\,\overline{s} = \int_v \rho dv$$

Gauss's law

$$\overline{D} = \varepsilon\,\overline{E}$$

Electric displacement

8

Fundamental Laws of Electrostatics in Differntial Form

$$\nabla \times \overline{E} = 0$$

Conservative field

$$\nabla \bullet D = \rho$$

Gauss's law

$$\overline{D} = \varepsilon \overline{E}$$

Electric displacement

9

Fundamental Laws of Electrostatics

- The integral forms of the fundamental laws are more general because they apply over regions of space. The differential forms are only valid at a point.
- From the integral forms of the fundamental laws both the differential equations governing the field within a medium and the boundary conditions at the interface between two media can be derived.

10

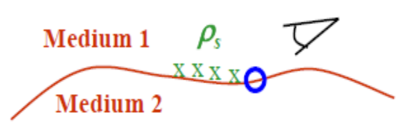
Boundary Conditions

- Within a homogeneous medium, there are no abrupt changes in ***E or D. However, at*** the interface between two different media (having two different values of ε), it is obvious that one or both of these must change abruptly.
- To derive the boundary conditions on the normal and tangential field conditions, we shall apply the integral form of the two fundamental laws to an infinitesimally small region that lies partially in one medium and partially in the other.

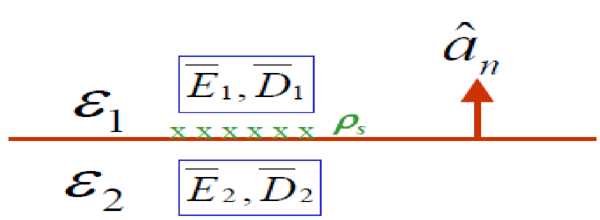
11

Boundary Conditions (Cont'd)

- Consider two semi-infinite media separated by a boundary. A surface charge may exist at the
- interface.



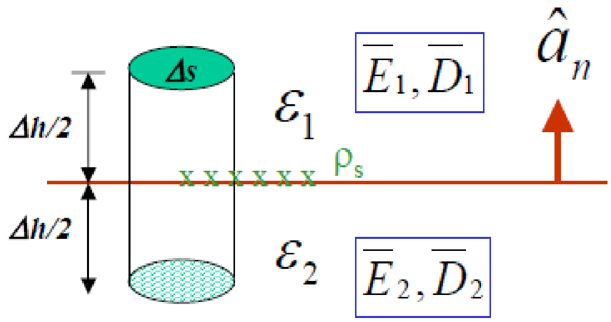
- Locally, the boundary will look planar



12

Boundary Condition on Normal Component of D

- Consider an infinitesimal cylinder (pillbox) with cross-sectional area Δs and height Δh lying half in medium 1 and half in medium 2:



Boundary Condition on Normal Component of D (Cont'd)

- Applying Gauss's law to the pillbox, we have

$$\oint_S \overline{D}\cdot d\overline{s} = \int_V \rho\, dv$$

$$LHS = \int_{top} \overline{D}\cdot d\overline{s} + \int_{bottom} \overline{D}\cdot d\overline{s} + \cancelto{0}{\int_{side} \overline{D}\cdot d\overline{s}}$$

$$= D_{1n}\Delta s - D_{2n}\Delta s$$

$$RHS = \rho_s \Delta s$$

14

Boundary Condition on Normal Component of D (Cont'd)

- The boundary condition is $$D_{1n} - D_{2n} = \rho_s$$

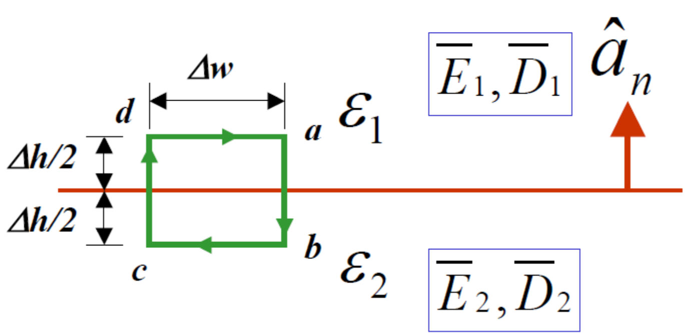
- If there is no surface charge $$D_{1n} = D_{2n}$$

For *non-conducting* materials, ρs *= 0 unless* an *impressed source i*spresent.

15

Boundary Condition on Tangential Component of E

- Consider an infinitesimal path *abcd with width Δw* and height Δ*h lying half in medium 1 and half in* medium 2:



16

Boundary Condition on Tangential Component of E (Cont'd)

- Applying conservative law to the path, we have

$$\oint_C \overline{E} \cdot d\overline{l} = 0$$

$$LHS = \int_a^b \overline{E} \cdot d\overline{l} + \int_b^c \overline{E} \cdot d\overline{l} + \int_c^d \overline{E} \cdot d\overline{l} + \int_d^a \overline{E} \cdot d\overline{l}$$

$$= -E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\Delta w + E_{1n}\frac{\Delta h}{2} + E_{2n}\frac{\Delta h}{2} + E_{1t}\Delta w$$

$$= \left(E_{1t} - E_{2t}\right)\Delta w$$

17

Boundary Condition on Tangential Component of E (Cont'd)

The boundary condition is: $D_{1t} = D_{2t}$

Electrostatic Boundary Conditions - Summary

At any point on the boundary,

– the components of ***E1 and E2 tangential to the*** boundary are equal
– the components of ***D1 and D2 normal to the*** boundary are discontinuous by an amount equal to the surface charge existing at that point

18

Electrostatic Boundary Conditions - Special Cases

- Special Case 1: the interface between two perfect (non-conducting) dielectrics:

– Physical principle: "there can be no free surface charge associated with the surface of a perfect dielectric."

– In practice: unless an impressed surfac charge is explicitly stated, assume it is zero.

19

Electrostatic Boundary Conditions - Special Cases

- Special Case 2: *the interface between a conductor and a perfect dielectric:*

– Physical principle: "there can be no electrostatic field inside of a conductor."

– In practice: a surface charge always exists at the boundary.

$$D_{1n} = \rho_s$$

$$E_{1t} = 0$$

20