RELATIVISTIC PROBLEM OF TWO BODIES

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ABSTRACT

The paper presents a developed theory of gravitational interaction of two-point bodies, that takes into account the finiteness of the speed of light. Relativistic inertial (or gravitational) mass has been determined. The relativistic force of inertia, the value of which is invariant to the Lorentz transformation, has been determined. A generalized form of the law of universal gravitation is presented, such that the gravitating masses in it contain corrective relativistic multipliers.

On the basic of these new definitions, a method similar to that of classical mechanics is used to calculate a relativistic orbit of circulation. A new formula for the angle of displacement of perihelion has been derived, which takes into account both the value of the focal parameter and the eccentricity of the orbit, and which for the planet Mercury gives the correct numerical result.

Keywords: relativistic mass, relativistic force of inertia, relativistic orbit, perihelion displacement, Mercury.

Introduction

All known theories of gravity come from field concepts that assume the presence of some massive bodies, or distributed masses of matter, creating a force-attracting field in the space around them. The quantities that characterize the field-gravitational, or metric coefficients, satisfy a certain system of differential equations. The description of the field consists in the construction of a family of geodetic lines in it, along which third-party test bodies placed in this field are obliged to move. At the same time, it is believed, that the masses of these test bodies themselves are so small that they do not introduce any distortion into the original field created by other masses. In particular, this approach to the problem of two bodies with point masses M and m, in which the mass m is much lesser than the mass of M, leads to the fact that the center of inertial reference frame is combined with a point with mass M, and the value of m does not affect the shape of its trajectory. In other words, in the very original formulation of the problem is already laid violation (or, one might say neglect) of the third law of Newton.

In the orbit of the planet Mercury closest to the sun, in 1859 the observation (made by French astronomer Le Verrier), an anomalous shift in the classical elliptical orbit towards the movement of Mercury was noticed. This displacement is approximately $0.1037'' \approx 5*10^{-7}$ rad per revolution. At the same time, for 100 earth years, an angle of about $43''$ runs up. At the end of 19th century, the German researcher Paul Gerber, and then Albert Einstein in a 1915 paper "Explanation of Mercury perihelion movement in general theory of relativity" gave the following calculation formula, of the mentioned displacement, for the system of two-point bodies, of which one serves as an attractive inertia center,

$$\delta\varphi = \frac{6\pi GM}{C^2(1-e^2)a} \qquad \text{(A)}$$

In the case of the Sun-Mercury system, the value of additional angel calculated by this formula is almost exactly the same as the observed. However, is it permissible in such a system to not take into account the original mass of Mercury in comparison with the mass of the sun? After all, their ratio is $M_{mer}/M_{sun} \approx 1.6*10^{-7}$, that is, the value of the same order of smallness as the desired displacement! Still, it is not logical to try to calculate a certain small value neglecting in the condition of the problem the value of the same order of smallness.

In our opinion, to overcome this obvious discrepancy between the accepted idealization in the formulation of the problem and the expected result of its solution, can only be a new relativistic theory of the two-body problem, which does not ignore Newton's third law but necessarily applies it in the same way as is it applied in a similar classical version. The development of such a theory and derivation of a new formula of the angle of displacement of perihelion this paper is devoted.

1. Relativistic mass and the force of inertia

In the first paper of A. Einstein on the theory of relativity published in 1905 entitled "on the electrodynamics of moving bodies", relativistic longitudinal and transverse masses were introduced:

$$\text{Longitudinal mass} = \frac{m}{\sqrt{\left(1-\frac{v^2}{c^2}\right)^3}}\ , \tag{1}$$

$$\text{Transversal mass} = \frac{m}{1-\frac{v^2}{c^2}}\ , \tag{2}$$

Where v- is the value of the velocity of the body attributed to some inertial reference frame; c- is the speed of light in a vacuum, m- is the mass of the resting body.

Thus, in the case of a rectilinear motion of a body, it has an effective mass given by the expression (1), and in the case of a uniform rotation along the circle, its effective mass is given by expression (2). If the body moves along an arbitrary spatial trajectory, then the formulas (1), (2) are naturally combined by the following single formula:

$$\mu = m\sqrt{\frac{1-\frac{v^2}{c^2}sin^2\ \widehat{va}}{\left(1-\frac{v^2}{c^2}\right)^3}}\ , \tag{3}$$

Where the symbol $\widehat{va}$-denotes the angle formed by the velocity and acceleration vectors. Accordingly, Newton's second law (or the force of inertia) is generalized to motion, taking into account the finiteness of the speed of the light, as follow:

$$\mu a = F \tag{4}$$

In this case, a direct calculation can show that the value

$$I^2 = \frac{1-\frac{v^2}{c^2}sin^2\widehat{va}}{\left(1-\frac{v^2}{c^2}\right)^3}a^2 \tag{5}$$

-is the essence of the kinematic invariant with respect to Lorentz coordinate transformations. And, therefore, the magnitude of the force of inertia, determined by formula (4), is also Lorentz invariant! So, the definition of force (4) meets the following basic requirements:

(1) F^2- The invariant of the Lorentz transformations

(2) The direction of the force vector F coincides with the direction of the vector acceleration a
(3) If $c \to \infty$ (or, if $v \ll$c), F transits into the Newtonian force.
Nevertheless, it is possible to give an even more general definition for the force of inertia that satisfies the same requirements. Indeed, consider two similar trajectories described by the radius-vectors $r(t)$ and $r_1(t) = kr(t)$ with an arbitrary constant k. Then the velocities and accelerations of points sliding along such trajectories will also be similar:

$$v_1(t) = kv(t), \quad a_1(t) = ka(t) \tag{6}$$

According to formula (5), we have such a Lorentz invariant

$$I_1^2 = \frac{1-\frac{v_1^2}{c^2}sin^2\widehat{v_1a_1}}{\left(1-\frac{v_1^2}{c^2}\right)^3}a_1^2\ ,$$

Which taking into account (6) can be written

$$I_1^2 = \frac{1-k^2\frac{v_1^2}{c^2}sin^2\widehat{va}}{\left(1-k^2\frac{v^2}{c^2}\right)^3}k^2a^2$$

It follows that the expression

$$\frac{I_1^2}{k^2} = \frac{1-k^2\frac{v^2}{c^2}sin^2\widehat{va}}{\left(1-k^2\frac{v^2}{c^2}\right)^3}\ a^2$$

Is also an invariant of the Lorentz transformation for any, value other than zero constant k^2. Therefore, the invariant force of inertia should generally be defined as follows:

$$m\sqrt{\frac{1-k^2\frac{v^2}{c^2}sin^2\widehat{va}}{\left(1-k^2\frac{v^2}{c^2}\right)^3}}\ a\ = F \tag{7}$$

Where the value of an arbitrary constant k is determined by the condition of a specific problem.
Accordingly, the generalized inertial relativistic mass is given by the expression

$$\mu\ = m\sqrt{\frac{1-k^2\frac{v^2}{c^2}sin^2\widehat{va}}{\left(1-k^2\frac{v^2}{c^2}\right)^3}} \tag{8}$$

2. The law of universal gravitation in relativistic form

When writing the law of universal gravitation, which takes into account the finiteness of the speed of light, we intend to transfer the fundamental principles of the equivalence of inertial and gravitational masses of classical mechanics and to keep the very form of Newton's law unshakable. At the same time, all the beauty and harmony of the classics are preserved. So, we take as postulate: two-point masses are attracted according to the law

$$F = -G\frac{\mu_1\mu_2}{r^3}r \tag{9}$$

Here: $G = 6{,}672 \cdot 10^{-11}\left[\frac{H\cdot M^2}{kg^2}\right]$ – is the gravitational constant of the vacuum; r – is the distance between these material points; μ_1 *and* μ_2 – gravitational (the same –inertial) between given material point masses presented by the formulas

$$\mu_1 = {}^{m_1}\sqrt{\frac{1-k_1^2\frac{v_1^2}{c^2}\sin^2\widehat{v_1 a_1}}{\left(1-k_1^2\frac{v_1^2}{c^2}\right)^3}} \tag{10}$$

$$\mu_2 = {}^{m_2}\sqrt{\frac{1-k_2^2\frac{v_2^2}{c^2}\sin^2\widehat{v_2 a_2}}{\left(1-k_2^2\frac{v_2^2}{c^2}\right)^3}} \tag{11}$$

Let us now derive the equation of motion of two isolated gravitationally bound material points. By virtue of the law of inertia and Newton's third law, we have

$$\mu_1\, a_1 = F\,, \mu_2 a_2 = -F, \tag{12}$$

Where $r_1\,, r_2$– are the radius-vectors of our interacting point bodies, relative to some inertial reference frame; $v_1\,, v_2$ – their velocities, a_1, a_2 – their accelerations.
From the equation (12) follows the linear dependance of acceleration vectors

$$\mu_1 a_1 + \mu_2 a_2 = 0 \tag{13}$$

On the line segment $[m_1, m_2]$ given by our material points, there must necessarily be one fixed point as the center of its rotation. And this point cannot be anything other than a center of mass known from the classics. The radius-vector of the center of mass of a system of two material points is given by the formula

$$R = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} \tag{14}$$

And is characterized by

$$\boldsymbol{V} = \dot{\boldsymbol{R}} = 0.$$

In this case, it is easiest to assume that $R = 0$, that is, to place the center of our inertial reference frame in the center of mass.
Further, differentiating the equation (14) twice, we get

$$m_1 v_1 + m_2 v_2 = 0 \quad (15)$$

$$m_1 \dot{\boldsymbol{a}}_1 + m_2 \dot{\boldsymbol{a}}_2 = 0 \quad (16)$$

The equations (13) and (16) must be identical! This is achieved by properly selecting the arbitrary constants k_1 and k_2 in the formulas for relativistic masses (10) and (11). We introduce vectors

$$r = r_1 - r_2, \quad v = v_1 - v_2, \quad a = a_1 - a_2 \quad (17)$$

Then from the equations (14), (15), (16) (provided $R = 0$) follow equality

$$r_1 = \frac{m_2}{m_1+m_2} r \quad (18)$$

$$\boldsymbol{r}_2 = \frac{m_1}{m_1+m_2} \boldsymbol{r} \quad (19)$$

$$v_1 = \frac{m_2}{m_1+m_2} v \quad (20)$$

$$\boldsymbol{v}_2 = \frac{m_1}{m_1+m_2} \boldsymbol{v} \quad (21)$$

$$a_1 = \frac{m_2}{m_1+m_2} a \quad (22)$$

$$\boldsymbol{a}_2 = \frac{m_1}{m_1+m_2} \boldsymbol{a} \quad (23)$$

Hence, we conclude: in order to make the equations (13) and (16) coincide, the arbitrary constants k_1 and k_2 in formulas (10),(11) should be defined as follows

$$k_1 = 1 + m_1/m_2 \,, \;\; k_2 = 1 + m_2/m_1 \quad (24)$$

Now the expressions (10), (11) for relativistic masses take the form of

$$\mu_1 = m_1 \gamma \,, \mu_2 = m_2 \gamma \quad (25)$$

Where, $$\gamma = \sqrt{\frac{1 - \frac{v^2}{c^2} sin^2 \widehat{va}}{\left(1 - \frac{v^2}{c^2}\right)^3}} \quad (26)$$

The equation of motion is obtained by equating the force of inertia (12) with the attracting force (9). We have

$$\mu_1 a_1 = -G\frac{\mu_1\mu_2}{r^3}r\ .$$

Introducing here the expressions of acceleration (22) and relativistic mass (25) we come to this form of equation of motion in the problem for two bodies.

$$a = -G(m_1 + m_2)\gamma\frac{r}{r^3} \qquad (27)$$

3. Relativistic Orbit

Denoting

$$M = m_1 + m_2$$

And rewrite the equation of motion again

$$a = -GM\gamma\frac{r}{r^3} \qquad (28)$$

This equation specifies some flat motion occurring with the central acceleration vector, and therefore with the constant sectoral velocity and with a constant magnitude of the angular momentum of the linear velocity vector. Indeed, by virtue of identity

$$[\boldsymbol{r}, \boldsymbol{a}] = \frac{d}{dt}[\boldsymbol{r}, \boldsymbol{v}] = 0$$

We have

$$\boldsymbol{l} = [\boldsymbol{r}, \boldsymbol{v}] = const. \qquad (29)$$

Let's assume that the radius-vector r is the function of the angular coordinate φ and the latter is the function of time t. That is

$$r = r(\varphi), \varphi = \varphi(t).$$

Then,

$$v = \frac{dr}{dt} = \frac{dr}{d\varphi}\frac{d\varphi}{dt}$$

The value

$$\omega = \frac{d\varphi}{dt} \qquad (30)$$

Is the angular velocity of flat motion. With this value, the vector of linear velocity and the vector of angular velocity are written as follows:

$$v = \omega\frac{dr}{d\varphi}\ , \qquad (31)$$

$$l = [r, v] = \omega\left[r, \frac{dr}{d\varphi}\right] \qquad (32)$$

Its easy to notice that in the case of flat motion, we have the following equations

$$\left(\frac{dr}{d\varphi}\right)^2 = \left(\frac{dr}{d\varphi}\right)^2 + r^2 \,,$$

$$\left\|\left[r, \frac{dr}{d\varphi}\right]\right\| = r^2$$

Therefore, the values of velocity and momentum are given by the equations

$$v^2 = \omega^2\left(\left(\frac{dr}{d\varphi}\right)^2 + r^2\right), \qquad (33)$$

$$l = \omega r^2 \qquad (34)$$

Since, in our problem of two bodies $l = const.$, the last equation represents one of the first integrals of the equation of motion (28). Introducing it in (33) we get

$$v^2 = l^2\left(\left(\frac{1}{r^2}\frac{dr}{d\varphi}\right)^2 + \frac{1}{r}\right) \qquad (35)$$

The multiplication $v^2 sin^2\widehat{va}$ is also expressed in terms of the magnitude of the moment l and the magnitude of the distance r . Indeed,

$$l = |[r, v]| = rv\ sin\widehat{rv}$$

But due to the centrality of the acceleration vector a,

$$sin^2\widehat{va} = sin^2\widehat{rv}$$

Hence,

$$v^2 sin^2\widehat{va} = \frac{l^2}{r^2} \qquad (36)$$

Formulas (35) , (36) suggest the usefulness of introducing instead of the distance function $r(\varphi)$ a function to the inverse of it:

$$\lambda(\varphi) = \frac{1}{r(\varphi)} \qquad (37)$$

Now formulas (35), (36) take the following form

$$v^2 = l^2\left(\left(\frac{d\lambda}{d\varphi}\right)^2 + \lambda^2\right), \qquad (38)$$

$$v^2 sin^2\widehat{va} = l^2\lambda^2 \qquad (39)$$

The coefficient γ responsible for the for the relativistic correction in equation of motion (28) and taking into account the last equality, is written as follows

$$\gamma = \sqrt{\frac{1 - \frac{l^2}{c^2}\lambda^2}{\left(1 - \frac{v^2}{c^2}\right)^3}}\,, \qquad (40)$$

Or, as a result of decomposition according to the parameter ${}^{1}/{}_{c^2}$

$$\gamma = 1 + \frac{1}{2c^2}(3v^2 - l^2\lambda^2) + O\left(\frac{1}{c^4}\right) \qquad (41)$$

Or, by adding (38),

$$\gamma = 1 + \frac{l^2}{2c^2}\left(3\left(\frac{d\lambda}{d\varphi}\right)^2 + 2\lambda^2\right) + O\left(\frac{1}{c^4}\right) \qquad (42)$$

Back to our equation of motion (28). It allows for the derivation of another first integral, as well as the equations for the orbital function $\lambda(\varphi)$. Multiply both parts of it scalarly by the velocity vector v. We have

$$va = -GM\gamma\frac{1}{r^3}vr \qquad (43)$$

Or, by virtue of identities

$$va = \frac{1}{2}\frac{dv^2}{dt}, \quad vr = \frac{1}{2}\frac{dr^2}{dt},$$

$$\frac{1}{2}\frac{dv^2}{dt} = -GM\gamma\frac{1}{r^3}\frac{1}{2}\frac{dr^2}{dt}$$

Or,

$$dv^2 = 2GM\gamma d\frac{1}{r} \qquad (44)$$

And, finally, taking into account the replacement (37),

$$\frac{dv^2}{d\lambda} = 2GM\gamma \qquad (45)$$

From this equation, another first integral of motion is derived, to find which it is necessary to separate the variables in (45) and calculate the two integrals that are "taken". We will not reproduce these generally speaking obvious actions, but we will obtain a second-order equation for finding the orbital function $\lambda(\varphi)$, the knowledge of which allows us to calculate all other characteristic dependencies and parameters of the problem.

Differentiating expression (38) by the variable λ and substituting the calculated derivative in to the left side of equation (45) results in exact equation describing the motion of one point body (e.g., a planet) around another point body (e.g., the Sun) That's the equation

$$\frac{d^2\lambda}{d\varphi^2} + \lambda = \frac{\gamma}{h}(!) \quad , \qquad (46)$$

Where,

$$h = \frac{l^2}{GM} \qquad (47)$$

If we use decomposition (42) for the coefficient γ, we get a shortened form of a trajectory equation. Namely

$$\frac{d^2\lambda}{d\varphi^2} + \lambda = \frac{1}{h} + \frac{l^2}{2hc^2}\left(3\left(\frac{d\lambda}{d\varphi}\right)^2 + 2\lambda^2\right) + O\left(\frac{1}{c^4}\right) \qquad (48)$$

This equation acquires a more compact and easy-to-study form if we introduce a dimensionless small parameter and a dimensionless trajectory function. Let

$$\varepsilon = \frac{l^2}{2h^2c^2} = \frac{GM}{2hc^2}, \qquad \xi = h\lambda. \tag{49}$$

Then

$$\frac{d^2\xi}{d\varphi^2} + \xi = 1 + \varepsilon\left(3\left(\frac{d\xi}{d\varphi}\right)^2 + 2\xi^2\right) + O(\varepsilon^2) \tag{50}$$

In accordance with the structure of the right-hand side, we look for a solution to this equation in the form of

$$\xi = \xi_0 + \varepsilon\xi_1 + O(\varepsilon^2) \tag{51}$$

By introducing this expression into equation (50) and equating in it the terms with the same degrees of the small parameter ε for the zero and first iteration we obtain the following system of equations

$$\frac{d^2\xi_0}{d\varphi^2} + \xi_0 = 1 \tag{52}$$

$$\frac{d^2\xi_1}{d\varphi^2} + \xi_1 = 3\left(\frac{d\xi_0}{d\varphi}\right)^2 + 2\xi_0^2 \tag{53}$$

The solution of the first of these equations gives the main part of the trajectory in the form of classical elliptical orbit.

$$\xi_0 = 1 + e\,cos\varphi \tag{54}$$

In which the large axis is oriented along the polar axis, and the integration constant e has the meaning of the eccentricity of the ellipse.
Substituting (54) to the right side of (53) we get the following equation that defines the first iteration

$$\frac{d^2\xi_1}{d\varphi^2} + \xi_1 = \frac{4+5e^2}{2} + 4\,ecos\varphi - \frac{e^2}{2}cos2\varphi \tag{55}$$

Here it is sufficient to write down in the right-hand side only a particular solution. It looks like this

$$\xi_1 = \frac{4+5e^2}{2} + 2\,e\,\varphi sin\varphi + \frac{e^2}{6}cos2\varphi \tag{56}$$

So, we have the above function of the trajectory of movement of one of two interacting material points in the polar coordinate system, the center of which is located in the other:

$$\xi = 1 + e\,cos\varphi + \varepsilon\left(\frac{4+5e^2}{2} + 2\,e\varphi sin\varphi + \frac{e^2}{6}cos2\varphi\right)O(\varepsilon^2) \tag{57}$$

If an explicit expression for the function of the distance between the points is of interest, it looks like this

$$\frac{r}{h} = \frac{1}{\xi} = \frac{1}{\xi_0} - \varepsilon\frac{\xi_1}{\xi_0^2} + O(\varepsilon^2)\,,$$

Where the found iterations should be substituted (54) and (56).

4. Perihelion Displacement

As you know, the period of revolution in the classical elliptical orbit is determined by the formula

$$T = \sqrt{\frac{h^3}{GM}} \int_0^{2\pi} \frac{d\varphi}{\xi_0^2} = \sqrt{\frac{h^3}{GM}} \frac{2\pi}{(1-e^2)^{3/2}} \qquad (58)$$

Where $\xi_0 = 1 + e\, cos\varphi$

Let's see now at what additional angle the radius-vector of the planet rotating in our relativistic orbit will turn during this time.
Let's write down the first integral of motion (34) in this form

$$l = \frac{1}{\lambda^2}\frac{d\varphi}{dt}$$

Taking into account (47), (49)

$$dt = \sqrt{\frac{h^3}{GM}} \frac{d\varphi}{\xi^2} \qquad (59)$$

We found the above orbital function (φ) in the form of decomposition by a small parameter ε:

$$\xi = \xi_0 + \varepsilon \xi_1 + O(\varepsilon^2)\ .$$

Substitute this decomposition in equation (59). Then

$$dt = \sqrt{\frac{h^3}{GM}} \left(\frac{1}{\xi_0^3} - 2\varepsilon \frac{\xi_1}{\xi_0^3} + O(\varepsilon^2)\right) d\varphi \qquad (60)$$

We integrate the left-hand side of the equation from 0 to T, and the right-hand side from 0 to $2\pi + \Delta\varphi$. We have

$$T = \sqrt{\frac{h^3}{GM}} \left(\int_0^{2\pi+\Delta\varphi} \frac{d\varphi}{\xi_0^2} - 2\varepsilon \int_0^{2\pi+\Delta\varphi} \frac{\xi_1}{\xi_0^3} d\varphi\right) + O(\varepsilon^2) =$$

$$= \sqrt{\frac{h^3}{GM}} \left[\int_0^{2\pi} \frac{d\varphi}{\xi_0^2} + \int_{2\pi}^{2\pi+\Delta\varphi} \frac{d\varphi}{\xi_0^2} - 2\varepsilon \left(\int_0^{2\pi} \frac{\xi_1}{\xi_0^3} d\varphi + \int_{2\pi}^{2\pi+\varphi} \frac{\xi_1}{\xi_1^3} d\varphi\right)\right] + O(\varepsilon^2)$$

Holding here only the first-order terms of the small angle $\Delta\varphi$ and the small parameter ε, we get

$$T = \sqrt{\frac{h^3}{GM}}\left(\int_0^{2\pi}\frac{d\varphi}{\xi_0^2} + \frac{\Delta\varphi}{\xi_0^2 2\pi} - 2\varepsilon\int_0^{2\pi}\frac{\xi_1}{\xi_0^3}d\varphi\right) + O(\varepsilon^2) \quad (61)$$

Let’s take in this formula for the T time of one classical revolution (58). Then the following formula follows from it for the additional angle of rotation of perihelion due to relativistic effect. Here it is

$$\Delta\varphi = 2\varepsilon(1+e)^2\int_0^{2\pi}\frac{\xi_1}{\xi_0^3}d\varphi + O(\varepsilon^2) \quad (62)$$

Iterations ξ_0, ξ_1 are given by formulas (54), (56), that allow this integral to be calculated in explicit form. Indeed, to do this, it is enough to use the following three table integrals.

$$\int_0^{2\pi}\frac{d\varphi}{\xi_0^3} = \frac{(2+e^2)\pi}{(1-e^2)^{5/2}}$$

$$\int_0^{2\pi}\frac{e\varphi\, sin\varphi}{\xi_0^3}d\varphi = \frac{\pi}{(1+e)^2} - \frac{\pi}{(1-e^2)^{3/2}} \leq 0$$

$$\int_0^{2\pi}\frac{e^2 cos2\varphi}{\xi_0^3}d\varphi = \frac{3\pi e^4}{(1-e^2)^{5/2}}$$

Substitution of which in (62) results in the formula

$$\Delta\varphi = 4\pi\varepsilon\left[1 + \frac{1+4{,}5e^2+1{,}5e^4}{(1-e)^2\sqrt{1-e^2}}\right] \quad (63)$$

or

$$\Delta\varphi = \frac{2\pi GM}{hc^2}\left[1 + \frac{1+4{,}5e^2+1{,}5e^4}{(1-e)^2\sqrt{1-e^2}}\right] \quad (64)$$

The eccentricity of Mercury is 0.206. Introducing this value to formula (64), we get the magnitude of Mercury’s angle of displacement in one revolution:

$$\Delta\varphi \cong 0.978\frac{6\pi GM}{hc^2} \quad (65)$$

In terms of the Earth’s century, during which Mercury makes 415 revolutions, this angle is approximately 42,3" and almost coincides with the really observed displacement of the major axis of Mercury.

5. CONCLUSIONS

In this paper a new formula (64) for calculating the angle of displacement of the large axis of the elliptical orbit during the time T of the classical period of the planet's revolution has been obtained. Namely, this angle is the result of modern astronomical observations, and namely, this angle was first recorded by the French astronomer Urban Leverrier for the planet Mercury in 1859. In other words, it expresses the angle between two neighboring extremes, but does not take into account the temporal dependence of the planet's motion along its trajectory.

Thus, comparing formulas (A), (64) we can say that although they are both due to the consequence of relativistic effect (i.e., accounting for the finiteness of the speed of light), only formula (64) can be tested experimentally! And the fact that they both give the same correct results for the Sun-Mercury system is a pure accident that give rise to an erroneous belief in formula (A) for decades.

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