Question Bank

Q1: Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution function $\sim N(\mu, \sigma^2)$, Find:

1. MLE estimator for σ^2.
2. The MSE for two estimators defined as:
   $S^2 = \sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{n-1}$; $\hat{\sigma}^2 = \sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{n}$ then compare the two estimators according to MSEs.

a- Construct the confidence interval for μ , the mean of a random variable $X \sim N(\mu, \sigma^2)$ when the variance σ^2 is unknown and the size of the random sample is small.

b- if a R. S. of size n=10 taken from a population distributed as normal gives $\bar{X}$ $164.9, S = 4.157$ find 95% C. I. for:

i) μ when σ^2 is unknown. ii) for σ^2 when μ is unknown.

c- a random variable $X \sim N(\mu, \sigma^2)$. Construct a 95 percent confidence interval for σ^2 when the mean μ is unkown.

Q2: If a random variable $X \sim B(n, P))$ and $\hat{P} = X/n$, then:

1. Is $\hat{P}$ a Minimum Variance Unbiased and consistent estimator for P ?
2. Find the efficiency for $\hat{P}$.

Q3: Let $X_1, X_2, .., X_n$ be a random sample from a distribution function defined as:

$$f(x, \theta) = \frac{2X}{\theta^2} \qquad 0 < x < \theta$$

1) Find an estimator for θ.
2) If an estimator for θ is defined as $\acute{\theta} = \frac{4}{5} * \bar{X}$, Can the Cramer Roa Lower Bound be applied to $\hat{\theta}$? Explain your answer.

$f(x) = \frac{cX^2}{9}$ $for \quad 0 < x < 3 \ and\ zero\ else,$ where c is constant. Define $Y = X^3$, find the probability distribution function for Y.

b- If F has F-distribution $F(4,2)$, find a and b so that $pr\{F \leq a\} = 0.05$,

and $pr\{F \leq b\} = 0.95$.

Q4: **a-** Let X_1, X_2 *be* two stochastically independent random variables distributed as Binomial where $X_1 \sim B(10, \frac{1}{2})$ and $X_2 \sim B(5, \frac{1}{2})$, define $Y = X_1 - X_2 + 5$, find the distribution function for Y , by using the m.g.f technique.

Q5- Let the random variable X have the p.d.f given by:

$$f(x) = 1 \quad for\ 0 < x < 1, \quad and\ zero\ elsewhere$$

Assume that X_1 , X_2 are stochastically independent random variables distributed as $f(x)$. Define $Y_1 = X_1 + X_2$, $Y_2 = X_1 - X_2$

1. find the distribution function for Y_1 .
2. Are Y_1 , Y_2 independent?

Q6: Assume that X_1, X_2, X_3 are identically independent random variables from uniform distribution having the p.d.f. $f(x) = 1$, $0 < x < 1$ and zero elsewhere.

1- Show that there is a unique median m of the distribution with $\mathrm{F(m)} = 1/2$;

2- Find the value for the median of X.

Q7: If X_1, X_2, X_3 are iid random variables from a distribution function defined as:

$\mathrm{f(x)} = 2\mathrm{x} \quad 0 < x < 1, \quad zero\ elsewher$.

1- Drive the p.d.f. for $\mathrm{Min}\{X_i\}$,

2- Find $\Pr\{ \mathrm{Min}\{X_i\} > 0.5\} \}$

Q8: Let two iid random variables with distribution function defined as:

$f(x) = 2(1-x) \qquad 0 < x < 1 \quad and\ zero\ else\ where$

1. Find probability that $\{\max\{X\} > 2 * Min\{X\}\}$
2. If $\mathrm{f(x)} = e^x \quad 0 < x < \infty$; then show that Z= $(\frac{X_1}{X_2})$ has an F- distribution .

Q9: a. Let $\boldsymbol{X}$ have the pdf $\boldsymbol{f(x) = (1/2)^x}$, $\boldsymbol{x = 1, 2, 3, ..,}$ $\boldsymbol{zero\ elsewhere}$. Find the probability distribution function for $\boldsymbol{Y = X^3}$.

b. Assume that $\boldsymbol{X_1}$, $\boldsymbol{X_2}$ are two stochastically independent binomial random variables with parameters $\boldsymbol{m, p\ and\ n, p}$ respectively. Show that $\boldsymbol{Y = X_1 + X_2}$ has binomial distribution with parameters $\boldsymbol{r, p}$ where $\boldsymbol{r = n + m}$.

Q10- a. Consider two stochastically independent random variables $\boldsymbol{X}$ and $\boldsymbol{Y}$ that are distributed as $X \sim N(0,1)$ *and* $Y \sim \chi^2(r)$ such that $f(y) = \frac{1}{\Gamma(\frac{r}{2})2^{\frac{r}{2}}} y^{(\frac{r}{2}-1)} e^{-\frac{y}{2}}$, **for** $y > 0$, **and zero elsewhere**. Define $T = \frac{X}{\sqrt{Y/r}}$, obtain the pdf for T.

b. If F has an F distribution with parameters $r_1 = 4$ and $r_2 = 2$, find a and b so that $pr\{F \le a\} = 0.05$, and $pr\{F \le b\} = 0.95$.

Q11: a- **Let X have the pdf $f(x) = (1/2)^x$, $x = 1, 2, 3, ..,$ *zero elsewhere*. Find the probability distribution function for $Y = X^3$.**

b. **Assume that X_1, X_2 are two stochastically independent binomial random variables with parameters m, p *and* n, p respectively. Show that $Y = X_1 + X_2$ has binomial distribution with parameters r, p where $r = n + m$.**

Q12) a. **Consider two stochastically independent random variables X and Y that are distributed as $X \sim N(0,1)$ *and* $Y \sim \chi^2(r)$ such that $f(y) = \frac{1}{\Gamma(\frac{r}{2})2^{\frac{r}{2}}} y^{(\frac{r}{2}-1)} e^{-\frac{y}{2}}$, for $y > 0$, and zero elsewhere. Define $T = \frac{X}{\sqrt{Y/r}}$, obtain the pdf for T.**

b. **If F has an F distribution with parameters $r_1 = 4$ and $r_2 = 2$, find a and b so that $pr\{F \le a\} = 0.05$, and $pr\{F \le b\} = 0.95$.**

Q13: **Define a random $Y = \frac{1}{2} * (X_1 - X_2)$ where X_1 and X_2 are stochastically independent random variables having the joint p.d.£. $f(x_1, x_2) = \frac{1}{4} e^{(-\frac{x_1 + x_2}{2})}$, for: $0 < x_1 < \infty$, $0 < x_2 < \infty$. Find**

1. **The distribution function for Y.**
2. **Probability that $\{Y > 1\}$.**

Q14) **Let X_1, X_2, X_3 be independent random variables taken from a uniform distribution having p.d.f. $f(x) = 1$, $0 < x < 1$ zero elsewhere.**

1. **Derive the distribution function for $Min\{X_i\}$;**

2. **Determine that prob{ $Min\{X_i\} > 0.3\}$;**
3. **If $Z = (Max\{X_i\} - Min\{X_i\})$ is the rang for $\{X_i\}$ then find the pdf for Z.**
4.

Q15: Define a random $Y = \frac{1}{2} * (X_1 - X_2)$ where X_1 and X_2 are stochastically independent random variables having the joint p.d.£. $f(x_1,x_2) = \frac{1}{4}\, e^{(-\frac{x_1+x_2}{2})}$, for: $0 < x_1 < \infty$, $0 < x_2 < \infty$. Find

3. The distribution function for Y.
4. Probability that { $Y > 1$ }.

Q15: Let X_1, X_2, X_3 be independent random variables taken from a uniform distribution having p.d.f. $f(x) = 1, \quad 0 < x < 1$ zero elsewhere.

5. Derive the distribution function for $Min\{X_i\}$;
6. Determine that prob{ $Min\{X_i\} > 0.3$};
7. If $Z = (Max\{X_i\} - Min\{X_i\})$ is the rang for $\{X_i\}$ then find the pdf for Z.