زانكۆی سەلاحەدین - هەولێر
Salahaddin University-Erbil

Network Analysis

Research Project
Submitted to the department of Mathematics in partial fulfillment of the requirements for the degree of BSc. In Mathematics

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Certification of the Supervisors

I certify that this report was prepared under my supervision at the Department of Mathematics / College of Education / Salahaddin University-Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

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Scientific grade: Assist. Lecturer

Data: / 3/ 2024

In view of the available recommendations, I forward this report for debate by the examining committee.

Signature: 

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Scientific grade: Assist. Professor

Chairman of the Mathematics Department

Date:

Dedication To:

- My father and mother.
- My dear supervisor.
- My brothers and sisters
- All who want to read it

Abdullatef Ali Ahmad
2024

Acknowledgement

I express my deep sense of gratitude and thanks to ALLAH the Almighty for providing me with strength, health, faith, patience, willing and self-confidence to accomplish this study.

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Abdullatef Ali Ahmad

2024

Abstract

This research project investigates network analysis through the lens of critical path calculations and its identification. Networks, characterized by interconnected nodes and edges, represent a fundamental structure for understanding relationships and flows within various systems. Critical path analysis (CPA) emerges as a crucial tool for network optimization, pinpointing the sequence of dependent tasks with the least slack time.

This project delves into the application of CPA within network analysis. We explore how critical path calculations aid in identifying the most time-sensitive elements within a network. By understanding the critical path, we can focus resources and efforts on these critical tasks, ultimately optimizing network performance and achieving project goals efficiently.

Contents

Introduction

In a large and complex project involving a number of interrelated activities, requiring a number of men, machines and materials, it is not possible for the management to make and execute an optimum schedule just by intuition based on the organizational capabilities and work experience. Managements are, thus, always on the lookout for some methods and techniques which may help in planning scheduling and controlling the project

It was only in the early l900's, that the pioneers of scientific management
started developing the scientific management techniques. Henry L. Gantt, during World War I, developed the Gantt chart for production scheduling. The Gantt chart was later modified to bar chart which was used as an important tool in both the project and production scheduling.

Chapter One

1 Background

This chapter will present some basic definitions related to our research project. (Sharma, 2009) (Sharma, 1980)

Definition 1.1: Network Logic

Some of the terms commonly used in 11etworks are defined below.

Definition 1.2: Network

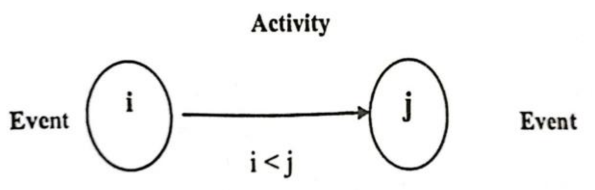
A network is a set of points called nodes, and a set of lines called branches that connect certain pairs of nodes.

Definition 1.3: Activity:

in the network activity is represented by an arrow the tail of which represents the start and the head represented the finish of the activity the length shape and direction of the arrow has no relation to the size of the activity

Definition 1.4: Event

The beginning and end points of an activity are called events or nods.



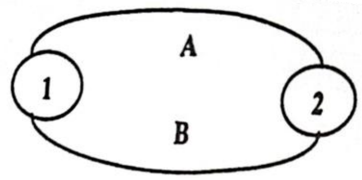
Definition 1.5: Path

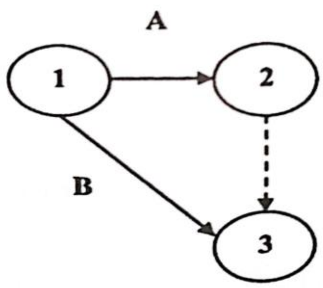
An unbroken chain of activity arrows connecting the initial event to son1e other event is called a path

Definition 1.6: Dummy

An activity which only deterlnines the dependency of one activity over the other, but does not consume any time is called a dummy activity.

Dummies are usually represented by dotted line arrows





Arrow (Network) diagram Representation (STACHO, 2021)

The rules for constructing the arrow diagram will be summarized as follows:-

Rule 1. - Each activity is represented by one and only one arrow in the network

Rule2 : Arrows should be kept straight and not curved or bent.

Rule 3 . Arrows should not cross, each other

Example (1}

Construct the arrow diagram for the activities A,B,C,…,F such that the following relationships, are satisfied

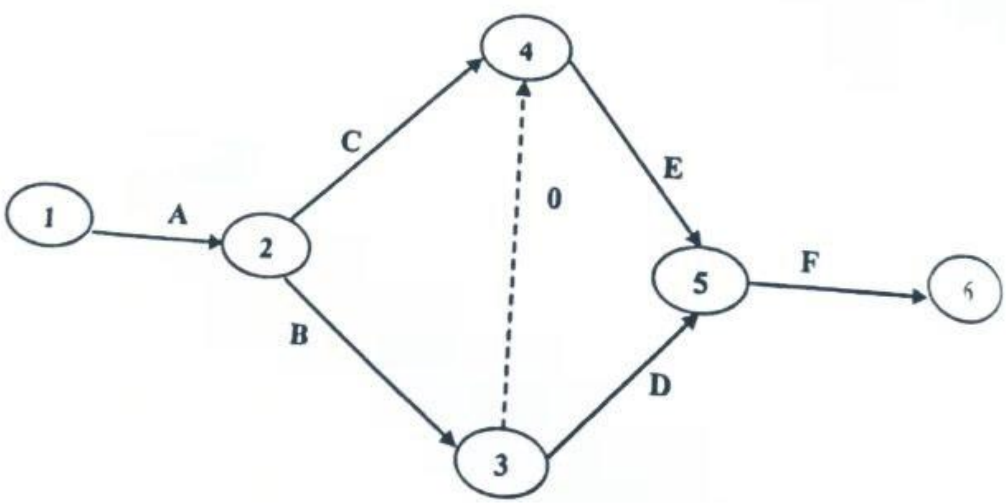
1:A the first activity of the project.

2: A precedes B , C.

3: B , precede D

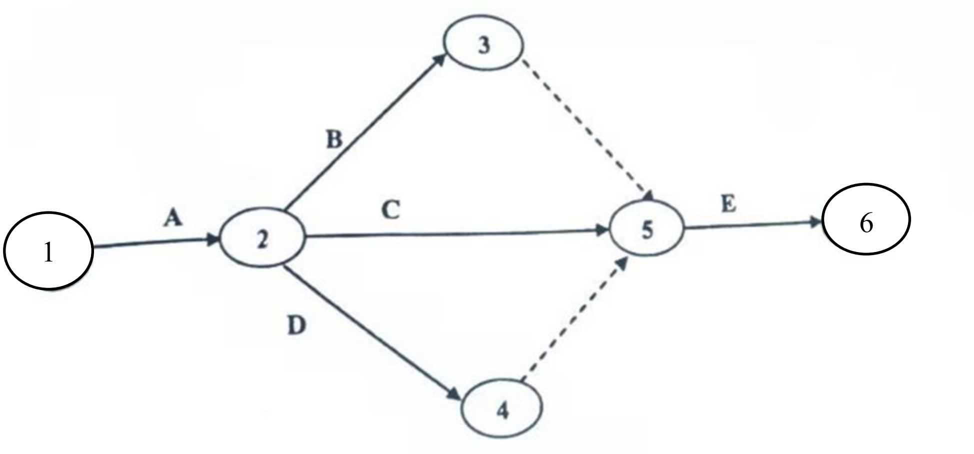
4: B , and C precede E.

5: D, and E precede F.



Example (2) Construct the arrow diagram for the activities A, B, C, D, E such that the following relationships are satisfied :-

1- A, the first activity of the project.
2- A, precedes B, C, and D.
3- B, C and D precede E.



Example (3)

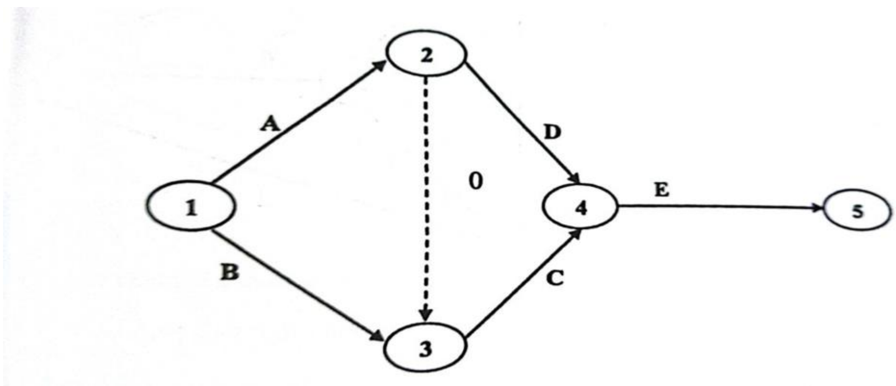
Construct the arrow diagram for the activities A, B, C ,...,F such that the following relationships are satisfied :-

1- A and B the first activities of the project.

2- A and B precede C.

3- A, precede D.

4 C, and D precede E.



Chapter Two

2.1 Critical Path Method (CPM) (STACHO, 2021) (Sharma, 2009)

An activity is said to be critical if a delay in its start will cause a delay in the completion date of the entire project. A critical path is defined as a chain of critical activities which connect the start and end events of the arrow diagram.

A critical path is the longest path through the network diagram which determines the length of the project. There can be more than one critical path if the lengths of two or more paths are the same.

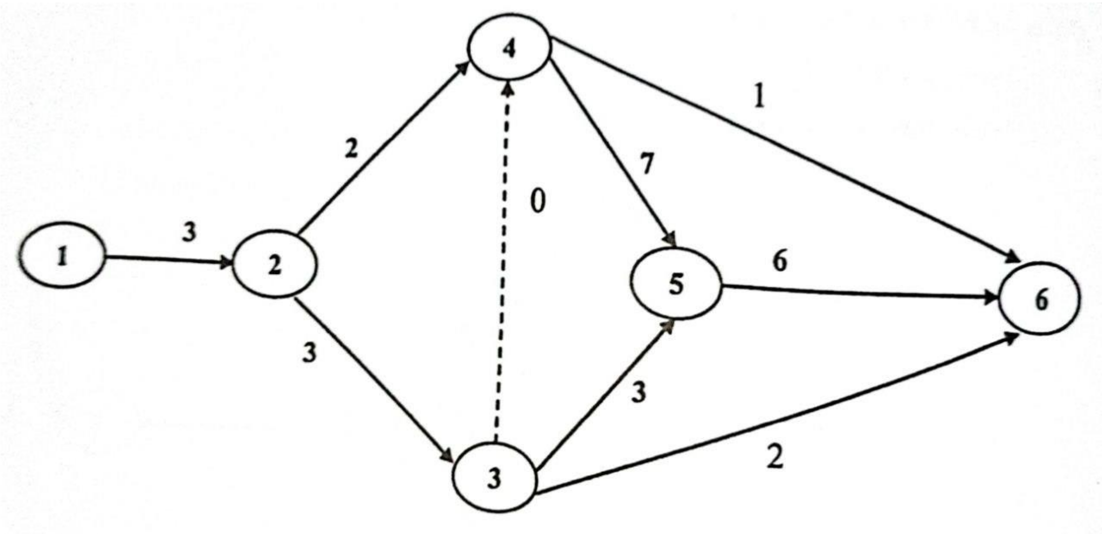
2.1.1 Critical Path Calculations

The method of determining such a path is illustrated by the following numerical example:

Example 1

Consider the network below. The time required to perform each activity is indicated

on the arrows:-



Let Es_i be the earliest start time represent the earliest occurrence time of even i

Thus, $ES_i = 0$ if i = 1

Then:

$ES_1 = 0$

The forward calculation are obtained from the formula :-

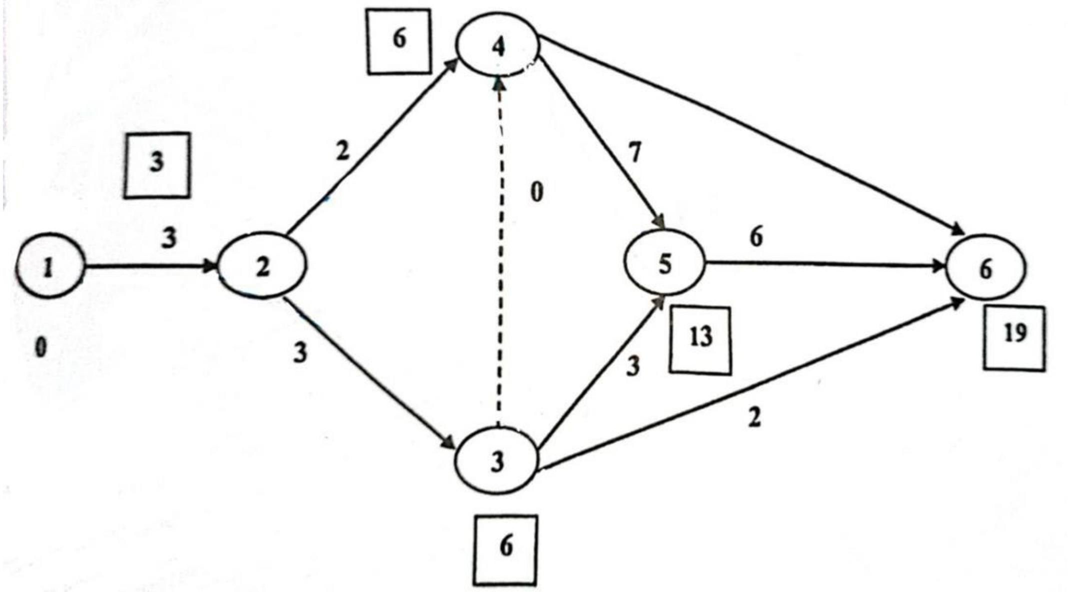
ES_j = Max (ES_i +D_{ij}) for all defined (i, j) activities.

$ES_2 = ES_1 + D_{12} = 0 + 3 = 3$ $ES_3 = ES_2 + D_{23} = 3 + 3 = 6$

ES_4=Max i 23 (ES_i +D_{i4})=Max(3+2,6+0)=Max(5,6)=6

ES_5 =Max i=34 (ES_i +D_{i5})=Max(6+3,6+7)=Max(9,13)=13

ES_6 =Max i = 3, 4, 5 (ES_i +D_{i6})=Max(6+2,6+1,13+6)=19



These numbers are shown in squares

Let LF_i be the latest finish time, if i = n then LFn = ES_n .

The backward starts from the end event, and obtained from the formula:-

LF_i = Min_j [LF_j -D_{ij}] for all defined (i, j) activities.

LF6 = ES_6 = 19

$LF_5 = LF_6 - D_{56} = 19 - 6 = 13$

LF_4 =Minj = 5, 6 (LF_j -D_{4j})

LF_4 =Min j = 5, 6 (13-7,19-1)=Min(6,18)=6

LF_3 =Min j = 4, 5, 6 (LF_j -D_{3j})

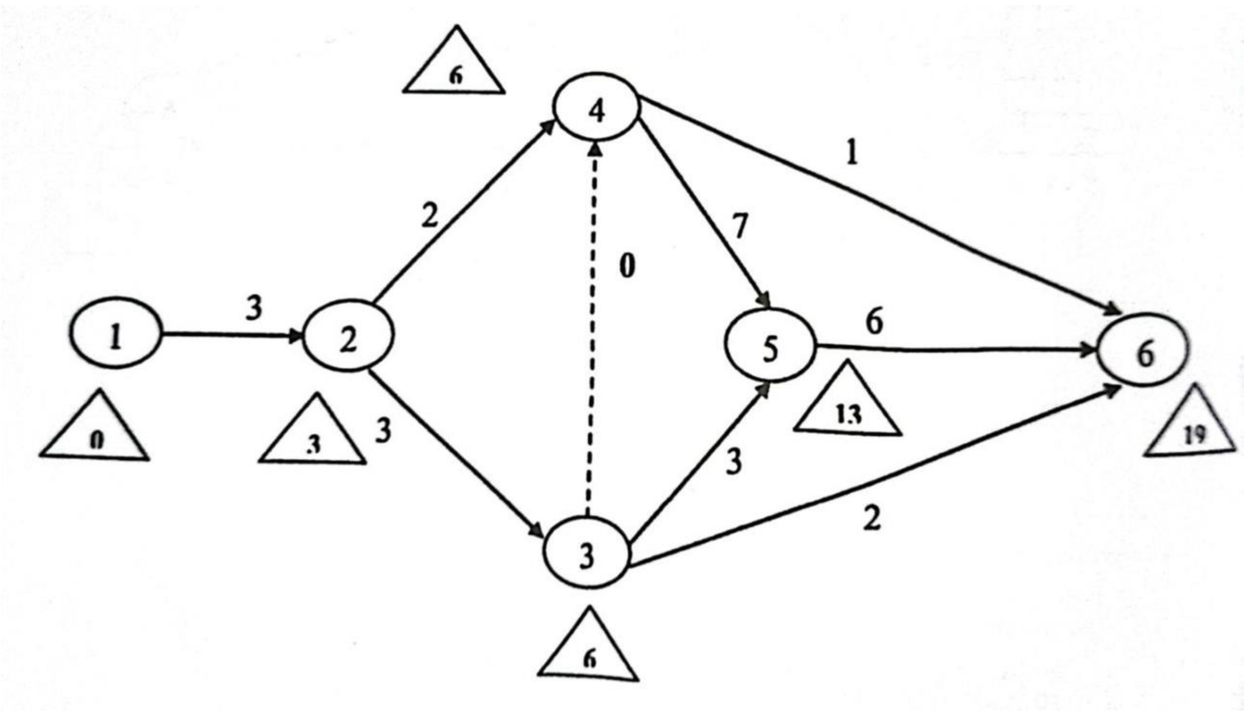
=Min i = 4 5 6 (6-0,13-3,19-2)=Min(6,10,17)=6

LF_2 =Min j = 4, 3 (LFj -D_{2j})

LF_2 =Min j = 4, 3 (6-2,6-3)=Min(4,3)=3

LF_1 = LF_2 - D_{12} = 3 - 3 = 0

In general, for any node i, the values of LF are shown in triangles



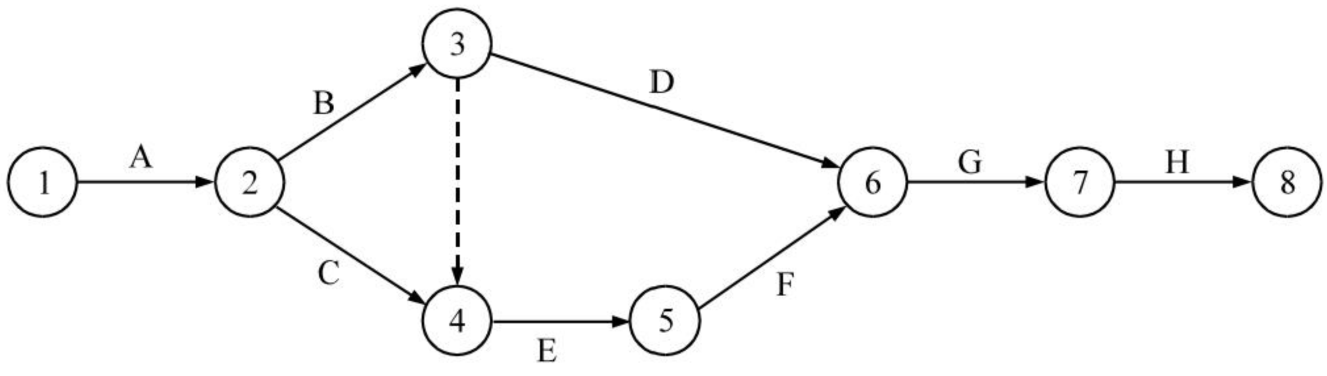
Example 2

An assembly is to be made from two paths X and Y. Both parts must be turned on a lathe Y must be polished whereas X need not be polished. The sequence of activities, together with their predecessors.

Draw a network diagram of activities for the project.

activity	Description	Predecessor Activity
A	Open work order	-
B	Get material for X	A
C	Get material for Y	A
D	Turn X on lathe	B
E	Turn Y on lathe	B, C
F	Polish Y	E
G	Assemble X and Y	D, F
H	Pack	G

Solution : The network diagram for the project is shown in:



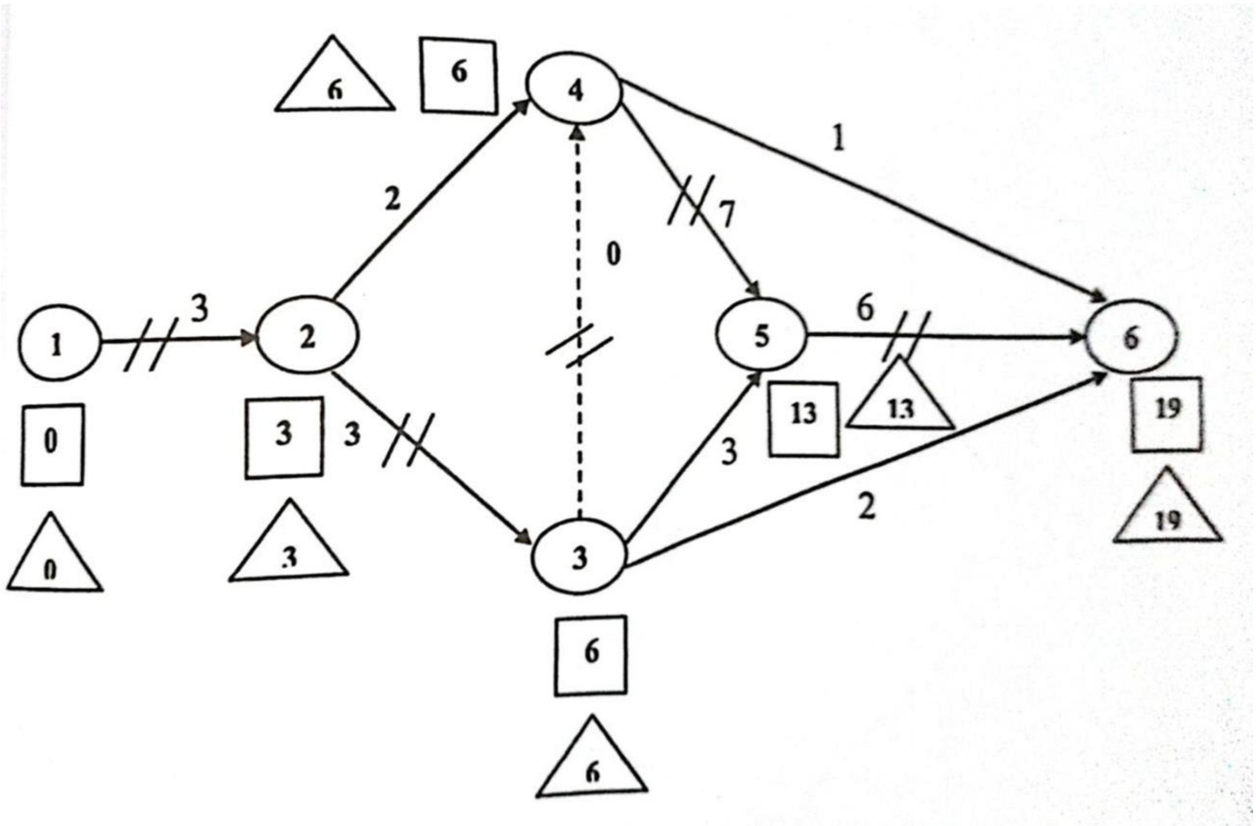
2.2 The Critical Path: (STACHO, 2021) (Sharma, 2009)

The critical path activities can now be identified by using the results of the forward and backward passes. An activity (ij) lies on the critical path if it satisfies the following three conditions:

1 - $ES_i = LF_i$

2- $ES_j = LF_j$

3- $ES_j - ES_i = LF_j - LF_i = D_{ij}$



Activities (1 - 2) , (2 - 3) , (3 - 4) (4 - 5) , and (5 - 6) define the critical path in the figure above.

Example 1

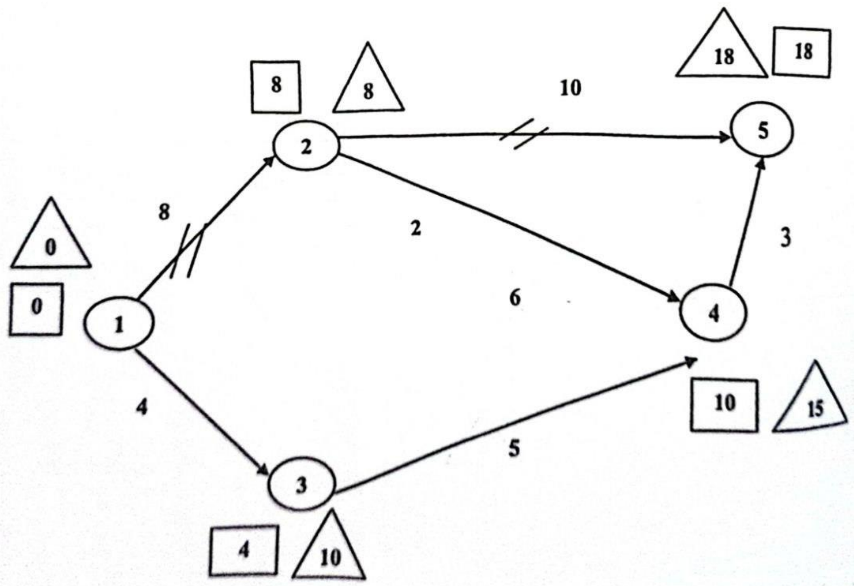
Construct the arrow diagram for the activities below. Then find:

1- The earliest star timer

2- The latest finish time.

3- The critical path.

Activity	Duration (time)
(1,2)	8
(1,3)	4
(2,4)	2
(2,5)	10
(3,4)	5
(4,5)	3



The critical path (1, 2)(2, 5) = 8 + 10 = 18 Days

Example 2

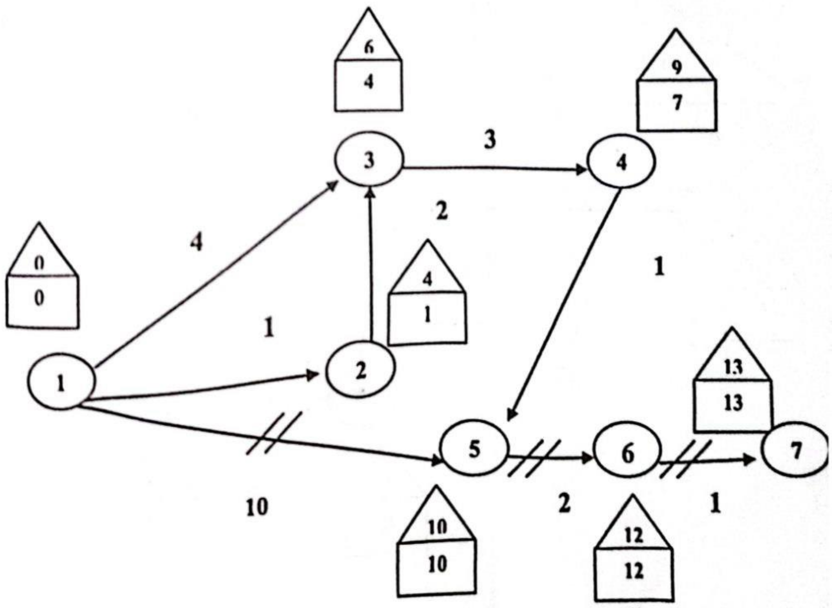
Construct the arrow diagram for the activities below. Then find :

1- The earliest start time.

2- The latest finish time.

3- The critical path.

Activity	Duration (time)
(1,2)	1
(1,3)	4
(1,5)	10
(2,3)	2
(3,4)	3
(4,5)	1
(5,6)	2
(6,7)	1



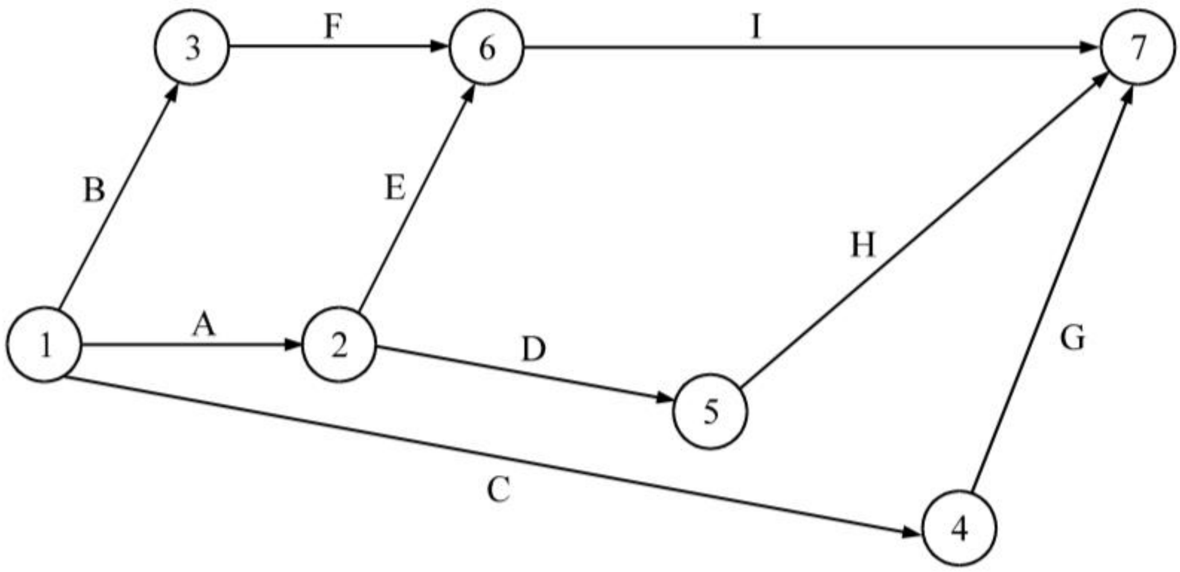
The critical path :-

(1, 5) 5), (5, 6) ,(6,)

10 + 2 + 1 = 13 days.

Example 3

The following network diagram represents activities associated with a project:



Determine the following:

(a) The earliest and latest expected completion times of each event.

(b) The critical path.

Solution

(a) The earliest and latest expected completion times for all events considering the expected completion time of each activity

Forward Pass Method

$E_1 = 0$ $\quad\quad$ $E_2 = E_1 + t_{1,2} = 0 + 7.8 = 7.8$

$E_3 = E_2 + t_{1,3} = 0 + 20 = 20$ $\quad\quad$ $E_4 = E_1 + t_{1,4} = 0 + 33 = 33$

$E_5 = E_2 + t_{2,5} = 7.8 + 18 = 25.8$

$$E_6 = Max(\, E_i + t_{i,6}) = Max(\, E_{2,6} + t_{2,6} : Max\big(\, E_{2,6} + t_{2,6}\,\big)$$

$$= Max(\, 7.8 + 20 : 20 + 9) = 29$$

$$E_7 = Max\big(\, E_i + t_{i,7}\big)$$

$$= Max(\, E_4 + t_{4,7} : E_5 + t_{5,7} : E_6 + t_{6,7} = Max(\, 33 + 9.8 : 25.8 + 8 : 29 + 4) = 42.8$$

Backward pass method

$L_7 = E_7 = 42.8$

$L_6 = L_7 - t_{6,7} = 42.8 - 4 = 38.8$

$L_5 = L_7 - t_{5,7} = 42.8 - 8 = 34.8$

$L_4 = L_7 - t_{4,7} = 42.8 - 9.8 = 33$

$L_3 = L_6 - t_{3,6} = 38.8 - 9 = 29.8$

$L_2 = Min\left(L_j - t_{2,j}\right)$

$= \ Min\left(L_5 - t_{2,5};\ L_6 - t_{2,6}\right)$

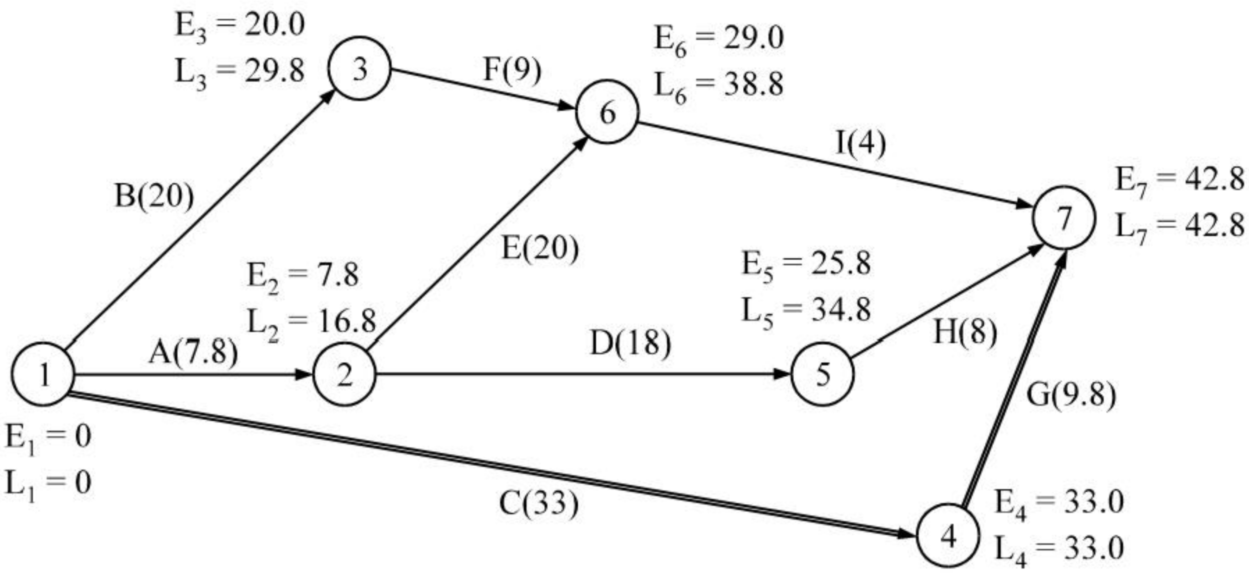
$= \ Min(34.8 - 18; 38.8 - 200) = 16.8$

$L_1 = Min\left(L_j - t_{1,j}\right)$

$= \ Min\left(L_2 - t_{1,2};\ L_3 - t_{1,3}; L_4 - t_{1,4}\right)$

$= \ Min(16.8 - 7.8; 29.8 - 20; 33 - 33) = 0$

The E-value and L-value are shown in :



(b) The critical path where E-values and L-values are the same. The critical path is: 1-4-7 and the expected completion time for the project is 42.8 weeks.

Conclusion

Our research on network analysis using critical path calculations demonstrates the effectiveness of CPA in identifying and managing network efficiency. By pinpointing the critical path, we gain valuable insights into the most time-sensitive tasks within a network. This knowledge allows for targeted resource allocation and proactive risk mitigation strategies, leading to improved project completion times and overall network performance.

This research lays the groundwork for further exploration of how critical path analysis can be integrated with advanced network analysis techniques. By incorporating factors like resource dependencies, buffer allocation, and stochastic variations, we can develop a more comprehensive understanding of network dynamics and optimize performance even in complex scenarios.

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