زانكۆی سەلاحەدین - هەولێر
Salahaddin University-Erbil

QUADRATIC INTERPOLATION METHOD

Research Project

Submitted to the department of Mathematics in partial fulfillment of the requirements for the degree of BSc. In Mathematics

By

Sabiha Walid Ibrahim

Supervised by:

Dr. Maher A. Nawkhass

April – 2024

Certification of the Supervisors

I certify that this report was prepared under my supervision at the Department of Mathematics / College of Education / Salahaddin University-Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

Signature:

Supervisor: Maher A. Nawkhass

Scientific grade: Assist. Lecturer

Data: / 4/ 2024

In view of the available recommendations, I forward this report for debate by the examining committee.

Signature:

Name: Dr. Rashad Rashed Haji

Scientific grade: Assist. Professor

Chairman of the Mathematics Department

Date:

Dedication To

- *My father and mother*
- *My dear supervisor*
- *My brothers and sisters*
- *All who want to read it*

Sabiha Walid Ibrahim

2024

Acknowledgement

I express my deep sense of gratitude and thanks to ALLAH the Almighty for providing me with strength, health, faith, patience, willing and self-confidence to accomplish this study.

My sincere thanks and appreciation are extended to the presidency of Salahaddin University, especially the deanery of the College of Education for their facilities to carry out my research work.

I would like to give special thanks to my supervisor “Dr. Maher A. Nawkhass” for his constant and valuable guidance and encouragement during my research work. His attention, support and timely suggestions were useful.

My deepest thanks go to Assist. professor “Dr. Rashad Rashed Haji” the head of Mathematic Department of the College of Education, especially the library staff of the College of Education.

Finally, I would like to state my heartily tanks to my family especially my father and my mother as they brought me up to the stage. Also, thanks to all of those who taught me even one word that helped me in my study.

Sabiha Walid ibrahim

2024

Abstract

This research project delves into the concept of quadratic interpolation, a technique employed to approximate unknown values within a dataset. It outlines the method's core principles, highlighting the construction of a second-degree polynomial through three known data points. This approach enables the estimation of a function's behavior at intermediate locations, proving valuable in diverse fields like data analysis, numerical methods, and computer graphics. The abstract concludes by acknowledging the inherent approximation nature of quadratic interpolation and the potential for inaccuracies, particularly with complex or non-smooth functions. It emphasizes the need for alternative methods with higher polynomial degrees when higher precision is crucial.

Contents

Introduction

The quadratic interpolation method uses the function values only; hence it is useful to find the minimizing step (λ^*) of functions f(X) for which the partial derivatives with respect to the variables xi are not available or difficult to compute [5.2, 5.5]. This method finds the minimizing step length λ^* in three stages. In the first stage the S-vector is normalized so that a step length of $\lambda = 1$ is acceptable. In the second stage the function $f(\lambda)$ is approximated by a quadratic function $h(\lambda)$ and the minimum $\tilde{\lambda}^*$, of $h(\lambda)$ is found. If $\tilde{\lambda}^*$ is not sufficiently close to the true minimum λ^*, a third stage is used. In this stage a new quadratic function (refit) $h'(\lambda) = a' + b'\lambda + c'\lambda^2$ is used to approximate $f(\lambda)$, and a new value of $\tilde{\lambda}^*$ is found. This procedure is continued until a $\tilde{\lambda}^*$ that is sufficiently close to λ^* is found.

Chapter One

1 Background

This chapter will present some basic definitions and theorems related to our research project. (Mishra and Ram, 2019, Gen and Cheng, 1999)

Definition 1.1. Unimodal Function

A unimodal function is a function that has a single maximum or minimum within a given interval. In the context of quadratic interpolation search, the algorithm assumes that the function being optimized is unimodal.

Definition 1.2. Parabola A parabol

is a U-shaped curve that can be represented by a quadratic equation. In quadratic interpolation search, the algorithm fits a parabola to three points in the interval to estimate the location of the maximum or minimum.

Definition 1.3. Quadratic Polynomial

A quadratic polynomial is a polynomial of degree two. It can be written in the form $f(x) = ax^2 + bx + c$, where a, b, and c are coefficients. In quadratic interpolation search, a quadratic polynomial is fitted to three points to approximate the behavior of the function being optimized.

Definition 1.4. Vertex of a Parabola

The vertex of a parabola is the point where the parabola reaches its maximum or minimum value. In quadratic interpolation search, the vertex of the parabola is used as the next test point to refine the search interval.

Definition 1.5 Numerical Optimization

Numerical optimization is the process of finding the maximum or minimum of a function within a given domain. Quadratic interpolation search is an optimization technique used to efficiently locate the maximum or minimum of a function.

Definition 1.6. Binary Search

Binary search is a search algorithm that works by repeatedly dividing the search interval in half. It is often used to find a specific value in a sorted list. Quadratic interpolation search is an alternative to binary search that is specifically designed for finding the maximum or minimum of a function

Definition 1.7. Golden Section Search

Golden section search is a search algorithm that iteratively narrows down the search interval based on the golden ratio. It is commonly used to find the minimum of a unimodal function. Quadratic interpolation search can be seen as an alternative approach to golden section search for finding the maximum or minimum of a function.

Theorem 1.8. If a unimodal function f(x) is continuous on the interval [a, b], and three distinct points (a, f(a)), (c, f(c)), and (b, f(b)) are chosen such that $a < c < b$, then there exists a unique quadratic polynomial p(x) such that $p(a) = f(a)$, $p(c) = f(c), and\ p(b) = f(b)$.

This theorem establishes the existence and uniqueness of the quadratic polynomial used in quadratic interpolation search. It assures that given three distinct points, a quadratic polynomial can be constructed that passes through these points.

Remark 1.9. Quadratic interpolation search assumes that the function being optimized is unimodal. If the function is not unimodal, meaning it has multiple local maxima or minima within the interval, the algorithm might not converge to the global maximum or minimum.

It is essential to note that quadratic interpolation search is most effective when the function being optimized is unimodal. If the function has multiple peaks or valleys, the algorithm may converge to a local maximum or minimum rather than the global one.

Remark 1.10. Quadratic interpolation search requires the function to be smooth within the interval. Functions that are highly oscillatory or have discontinuities may not yield reliable results with this algorithm .

The algorithm assumes that the function being optimized is smooth. If the function has rapid oscillations or contains discontinuities within the interval, the fitting of the quadratic polynomial and the estimation of the maximum or minimum may not be accurate.

Remark 1.11. Quadratic interpolation search is an iterative process that converges to the maximum or minimum of the function within a given precision. The convergence rate depends on the initial interval and the behavior of the function The algorithm iteratively refines the search interval to converge to the maximum or minimum. The rate of convergence depends on factors such as the initial interval and the behavior of the function.

A well-chosen initial interval and a smooth, well-behaved function can lead to faster convergence.

Remark 1.12. Quadratic interpolation search is generally more efficient than binary search or golden section search for finding the maximum or minimum of a unimodal function. However, it requires evaluating the function at three points in each iteration, which may be computationally expensive.

Compared to binary search or golden section search, quadratic interpolation search typically requires fewer iterations to converge to the maximum or minimum. However, it involves evaluating the function at three points in each iteration, which can be computationally expensive if the function evaluation is time-consuming.

Theorem 1.13. In the absence of round-off errors, quadratic interpolation search converges quadratically, meaning that the error decreases quadratically with each iteration.

Under ideal conditions without any round-off errors, quadratic interpolation search exhibits quadratic convergence. This means that the error between the estimated maximum or minimum and the true maximum or minimum decreases quadratically with each iteration, leading to faster convergence.

Remark 1.14. Quadratic interpolation search is sensitive to the initial interval and the choice of points within the interval. Poorly chosen initial intervals or points can lead to slow convergence or incorrect results.

The effectiveness of quadratic interpolation search depends on the initial interval and the choice of points within the interval. If the initial interval is too broad or the points are not well-distributed, the algorithm may require more iterations to converge or might not yield accurate results.

It is important to consider these theorems and remarks when implementing and applying quadratic interpolation search, as they provide insights into its assumptions, limitations, and expected behavior.

Chapter Two

In this chapter we will present the procedure (Algorithm) and related figure with several examples Find the Minimizing Function. (Chong et al., 2023, Eiselt et al., 2019)

2.1 Algorithm to Find the Minimizing Function

Stage 1. In this stage,† the S vector is normalized as follows: Find $\Delta = \max_i |s_i|$, i

where si is the ith component of S and divide each component of S by . Another method of normalization is to find Δ= $(s12 + s2 + \cdots + sn2)1/2$ and divide each component of S by .

Stage 2. Let

$$h(\lambda) = a + b\lambda + c\lambda^2 \tag{2.2}$$

function be the quadratic used for approximating the function f (λ). It is worth noting at this point that a quadratic is the lowest-order polynomial for which a finite minimum can exist. The necessary condition for the minimum of h(λ) is that

$$\frac{dh}{d\lambda} = b + 2c\lambda = 0$$

Nonlinear Programming I: One-Dimensional Minimization Methods that is,

$$\tilde{\lambda}^* = -\frac{b}{2c} \tag{2.3}$$

The sufficiency condition for the minimum of h(λ) is that

$$\left.\frac{d^2h}{d\lambda^2}\right|_{\tilde{\lambda}^*} > 0$$

That is,

$$c > 0 \tag{2.4}$$

To evaluate the constants a,b, and c in Eq. (2.2), we need to evaluate the function f(λ) at three points. Let λ = A,λ = B, and λ = C be the points at which the function f (λ) is evaluated and let f_A , f_B , and f_C be the corresponding function values, that is,

$$f_A = a + bA + cA^2$$
$$f_B = a + bB + cB^2$$
$$f_C = a + bC + cC^2 \qquad (2.5)$$

The solution of Eqs. (2.5) gives

$$a = \frac{f_A BC(C-B) + f_B CA(A-C) + f_C AB(B-A)}{(A-B)(B-C)(C-A)} \qquad (2.6)$$

$$b = \frac{f_A(B^2-c^2) + f_B(C^2-A^2) + f_C(A^2-B^2)}{(A-B)(B-C)(C-A)} \qquad (2.7)$$

$$c = \frac{-f_A(B-C) + f_B(C-A) + f_C(A-B)}{(A-B)(B-C)(C-A)} \qquad (2.8)$$

From Eqs. (2.3), (2.7), and (2.8), the minimum of h(λ) can be obtained as

$$\tilde{\lambda}^* = \frac{-b}{2c} = \frac{f_A(B^2-c^2) + f_B(C^2-A^2) + f_C(A^2-B^2)}{2[f_A(B-C) + f_B(C-A) + f_C(A-B)]} \qquad (2.9)$$

provided that c, as given by Eq. (2.8), is positive.

To start with, for simplicity, the points A , B, and C can be chosen as 0, t, and 2t, respectively, where t is a preselected trial step length. By this procedure, we can save one function evaluation since f_A = f (λ = 0) is generally known from the previous iteration (of a multivariable search). For this case, Eqs. (2.6) to (2.9) reduce to

$$a = f_A \qquad (2.10)$$

$$b = \frac{4f_B - 3f_A - f_C}{2t} \qquad (2.11)$$

$$c = \frac{f_C + f_A - 2f_B}{2t^2} \qquad (2.12)$$

$$\tilde{\lambda}^* = \frac{4\,f_B - 3\,f_A - f_C}{4f_B - 2f_C - 2f_A}t \tag{2.13}$$

Provided that

$$c = \frac{f_C + f_A - 2f_B}{2t^2} > 0 \tag{2.14}$$

The inequality (2.14) can be satisfied if

$$\frac{f_A + f_C}{2} > f_B \tag{2.15}$$

(i.e., the function value $\boldsymbol{f_B}$ should be smaller than the average value of $f_A$and f_C). This can be satisfied if f_B lies below the line joining $f_A$and $f_C$as shown in Fig 2.1 The following procedure can be used not only to satisfy the inequality (2.15) but also to ensure that the minimum $\tilde{\lambda}^*$ lies in the interval $0 < \tilde{\lambda}^* < 2t$.

1. Assuming that $\boldsymbol{f_A}$ = f (λ = 0) and the initial step size t0 are known, evaluate the function f at λ = t_0 and obtain $\boldsymbol{f_1}$ = f (λ = t_0). The possible outcomes are shown in Fig 2.2.

2. If $f_1 > f_A$ is realized (Fig. 2.2c), set $f_C = f_1$ and evaluate the function f at λ = $t_0/2$ and $\tilde{\lambda}^*$ using Eq. (2.13) with t = $t_0/2$.

3. If $f_1 \le f_A$ is realized (Fig 2.2 a or b), set $f_B = f_1$, and evaluate the function f at λ = $2t_0$to find F_2= f(λ = $2t_0$). This may result in any one of the situations shown in Fig. 5.14.

4. If f_2 turns out to be greater than f_1 (Fig b or c), set $f_C = f_2$ and compute $\tilde{\lambda}^*$ according to Eq. (2.13) with t = t_0.

5. If f_2 turns out to be smaller than f_1, set new $f_1 = f_2$ and $t_0 = 2t_0$, and repeat steps 2 to 4 until we are able to find $\tilde{\lambda}^*$.

Stage 3. The $\tilde{\lambda}^*$ found in stage 2 is the minimum of the approximating quadratic $h(\lambda)$ and we have to make sure that this $\tilde{\lambda}^*$ is sufficiently close to the true minimum λ^* of $f(\lambda)$ before taking $\lambda^* \simeq \tilde{\lambda}^*$. Several tests are possible to ascertain this. One possible test is to compare $f(\tilde{\lambda}^*)$with $h(\tilde{\lambda}^*)$ and consider $\tilde{\lambda}^*$ a sufficiently good approximation

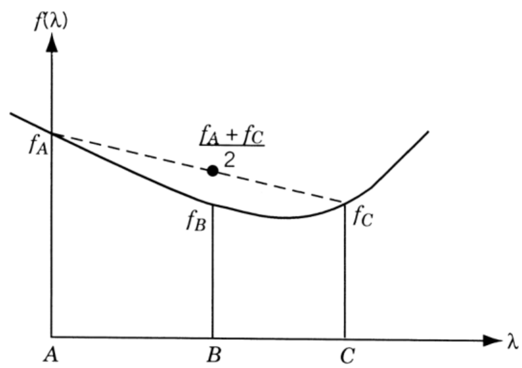

Figure 2.1 f_B smaller than $(f_A + f_C)/2$.

Nonlinear Programming I: One-Dimensional Minimization Methods

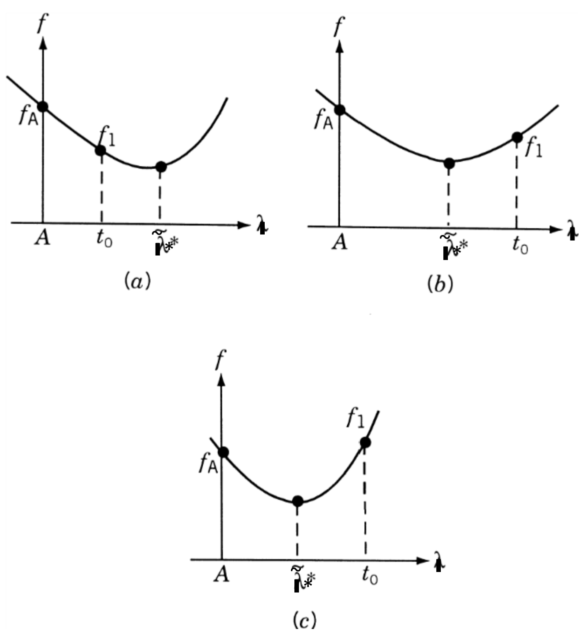

Figure 2.2 Possible outcomes when the function is evaluated at $\lambda = t_0$: (a) $f_1 < f_A$ and $t_0 < \tilde{\lambda}^*$;(b) $f_1 < f_A$ and $t_0 > \tilde{\lambda}^*$; (c) $f_1 > f_A$ and $t_0 > \tilde{\lambda}^*$.

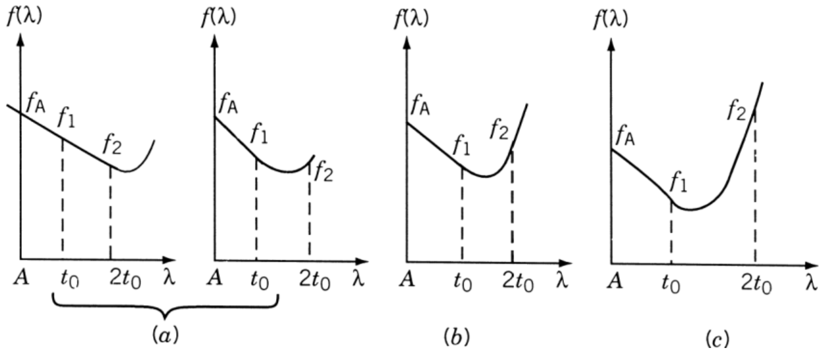

Figure 2.3 Possible outcomes when function is evaluated at $\lambda = t_0$ and $2t_0$: (a) $f_2 < f_1$ and $f_2 < f_A$; (b) $f_2 < f_A$ and $f_2 > f_1$; (c) $f_2 > f_A$ and $f_2 > f_1$.

if they differ not more than by a small amount. This criterion can be stated as

$$\left|\frac{h(\tilde{\lambda}^*) - f(\tilde{\lambda}^*)}{f(\tilde{\lambda}^*)}\right| \leq \varepsilon_1 \qquad (2.16)$$

Another possible test is to examine whether *df/d*λ is close to zero at $\tilde{\lambda}^*$. Since the derivatives of f are not used in this method, we can use a finite-difference formula for *df/d*λ and use the criterion

$$\left|\frac{f(\tilde{\lambda}^* + \Delta\tilde{\lambda}^*) - (\tilde{\lambda}^* + \Delta\tilde{\lambda}^*)}{2\Delta\tilde{\lambda}^*}\right| \leq \varepsilon_2 \qquad (2.17)$$

to stop the procedure. In Eqs. (2.16) and (2.17), ε_1 and ε_2 are small numbers to be specified depending on the accuracy desired.

If the convergence criteria stated in Eqs. (2.16) and (2.17) are not satisfied, a new quadratic function $h'(\lambda) = a' + b'\lambda + c'\lambda 2$

is used to approximate the function f(λ). To evaluate the constants a′, b′, and c′, the three best function values of the current $f_A = f(\lambda = 0)$, $f_B = f(\lambda = t_0)$,$f_C = f(\lambda = 2t_0)$, and $\tilde{f}$ = f(λ = $\tilde{\lambda}^*$) are to be used. This process of trying to fit another polynomial to obtain a better approximation to $\tilde{\lambda}^*$ is known as *refitting* the polynomial.

For refitting the quadratic, we consider all possible situations and select the best three points of the present A ,B ,C, and $\tilde{\lambda}^*$. There are four possibilities, as shown in Fig. 2.4. The best three points to be used in refitting in each case are given in Table . A new value of $\tilde{\lambda}^*$ is computed by using the general formula, Eq. (2.9). If this $\tilde{\lambda}^*$ also does not satisfy the convergence criteria stated in Eqs. (2.16) and (2.17), a new quadratic has to be refitted according to the scheme outlined in Table .

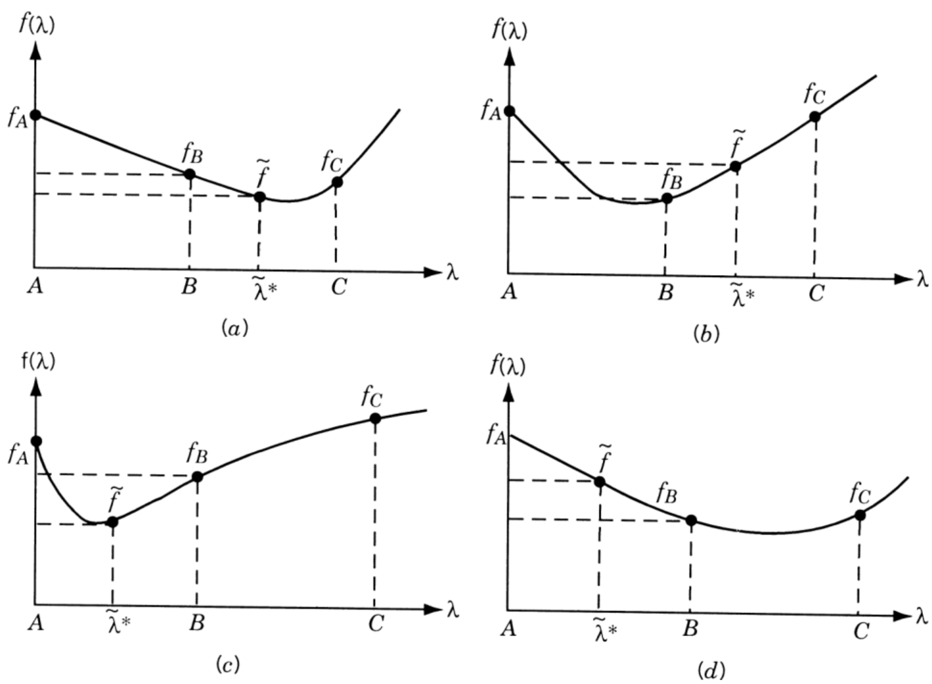

Figure 2.4 Various possibilities for refitting.

Case	characteristics	New point for refitting	
		new	old
1	$\tilde{\lambda}^* > B$ $\tilde{f} > f_B$	A B C Neglect old A	B $\tilde{\lambda}^*$ C
2	$\tilde{\lambda}^* > B$ $\tilde{f} < f_B$	A B C Neglect old c	A B $\tilde{\lambda}^*$
3	$\tilde{\lambda}^* < B$ $\tilde{f} < f_B$	A B C Neglect old c	A $\tilde{\lambda}^*$ B
4	$\tilde{\lambda}^* < B$ $\tilde{f} > f_B$	A B C Neglect old c	$\tilde{\lambda}^*$ B C

Example Find the minimum of $f = \lambda^5 - 5\lambda^3 - 20\lambda + 5$.

SOLUTION Since this is not a multivariable optimization problem, we can proceed directly to stage 2. Let the initial step size be taken as $t_0 = 0.5$ and $A = 0$.

Iteration 1

$$f_A = f(\lambda = 0) = 5$$

$$f_1 = f(\lambda = t_0) = 0.03125 - 5(0.125) - 20(0.5) + 5 = -5.59375$$

Since $f_1 < f_A$, we set $f_B = f_1 = -5.59375$, and find that

$$f_2 = f(\lambda = 2t_0 = 1.0) = -19.0$$

As $f_2 < f_1$, we set new $t_0 = 1$ and $f_1 = -19.0$. Again we find that $f_1 < f_A$ and hence set $f_B = f_1 = -19.0$, and find that $f_2 = f(\lambda = 2t_0 = 2) = -43$. Since $f_2 < f_1$, we again set $t_0 = 2$ and $f_1 = -43$. As this $f_1 < f_A$, set $f_B = f_1 = -43$ and evaluate $f_2 = f(\lambda = 2t_0 = 4) = 629$. This time $f_2 > f_1$ and hence we set $f_C = f_2 = 629$ and compute $\tilde{\lambda}^*$ from Eq. (2.13) as

$$\tilde{\lambda}^* = \frac{4(-43)-3(5)-629}{4(-43)-2(629)-2(5)}(2) = \frac{1632}{1440} = 1.13$$

Convergence test: Since $A = 0$, $f_A = 5$, $B = 2$, $f_B = -43$, $C = 4$, and $f_C = 629$, the values of a, b, and c can be found to be

$$a = 5,\ b = -204,\ c = 90$$

and

$$h(\tilde{\lambda}^*) = h(1.135) = 5 - 204(1.135) + 90(1.135)^2 = -110.9$$

Since

$$\tilde{f} = f(\tilde{\lambda}^*) = (1.135)^5 - 5(1.135)^3 - 20(1.135) + 5.0 = -23.127$$

we have

$$\left|\frac{h(\tilde{\lambda}^*) - f(\tilde{\lambda}^*)}{f(\tilde{\lambda}^*)}\right| = \left|\frac{-116.5 + 23.127}{-23.127}\right| = 3.8$$

As this quantity is very large, convergence is not achieved and hence we have to use *refitting*.

Iteration 2

Since $\tilde{\lambda}^* < B$ and $\tilde{f} > f_B$, we take the new values of A,B, and C as

$A = 1.135, \quad f_A = -23.12$
$B = 2.0, \quad f_B = -43.0$
$C = 4.0, \quad f_C = 629.0$

and compute new , using Eq. (2.9), as

$$\tilde{\lambda}^* = \frac{(-23.127)(4.0-16.0)+(-43.0)(16.0-1.29)+(629.0)(1.29-4.0)}{2[(-23.127)(2.0-4.0)(-43.0)(4.0-1.135)+(629.0)(1.135-2.0)]} = 1.661$$

Convergence test: To test the convergence, we compute the coefficients of the quadratic as

$$a = 288.0, b = -417.0, c = 125.3$$

As $h(\tilde{\lambda}^*) = h(1.661) = 288.0 - 417.0(1.661) + 125.3(1.661)^2 = -59.7$

$\tilde{f} = f(\tilde{\lambda}^*) = 12.8 - 5(4.59) - 20(1.661) + 5.0 = -38.37$

we obtain

$$\left|\frac{h(\tilde{\lambda}^*) - f(\tilde{\lambda}^*)}{f(\tilde{\lambda}^*)}\right| = \left|\frac{-59.70 + 38.37}{-38.37}\right| = 0.556$$

Since this quantity is not sufficiently small, we need to proceed to the next refit.

Example: The function to be minimized is f(x) = x^2 - x and is illustrated in Figure E5. la. Three points bracketing the minimum (- 1.7, -0.1, 1.5) are used to start the search for the minimum of f(x); we use equally spaced points here but that is not a requirement of the method.

Solution

$$x_1 = -1.7 \qquad x_2 = -0.1 \qquad x_3 = 1.5$$

$$f(x_1)=4.59 \qquad f(x_2)=0.11 \qquad f(x_3)=0.75$$

$$\Delta x = 1.6$$

Two different formulas for quadratic interpolation can be compared: Equation (5.8), the finite difference method, and Equation (5.12).

$$\tilde{x}^* = x_2 - \frac{\Delta x[f(x_3)-f(x_1)]}{2[f(x_3)-2f(x_2)+f(x_1)]}$$

$$= -0.1 - \frac{1.6(0.75-4.59)}{2(0.75-2(0.11)+4.59)} = 0.50$$

$$\tilde{x}^* = \frac{1}{2}\frac{[x_2{}^2 - x_3{}^2]f(x_1) + [x_3{}^2 - x_1{}^2]f(x_2) + [x_1{}^2 - x_2{}^2]f(x_3)}{(x_2 - x_3)f(x_1) + (x_3 - x_1)f(x_2) + (x_1 - x_2)f(x_3)}$$

$$= \frac{1}{2}\frac{[(-0.5)^2-(1.5)^2](4.59)+[(1.5)^2-(-1.7)^2](0.11)+[(-1.7)^2-(-0.1)^2](0.75)}{[(-0.1)-(1.5)](4.59)+[(1.5)-(-1.7)](0.11)+[(-1.7)-(-0.1)](0.75)}$$

$$=0.50$$

Note that a solution on the first iteration seems to be remarkable, but keep in mind that the function is quadratic so that quadratic interpolation should be good even if approximate formulas are used for derivatives.

Conclusion

Quadratic interpolation serves as a valuable tool for estimating unknown values within a set of known data points. By fitting a second-degree polynomial curve, or parabola, through three data points, it allows us to approximate the function's behavior at intermediate locations. This method proves particularly useful in various fields, including data analysis, numerical methods, and computer graphics, where data might not be readily available at all points of interest.

As a key takeaway, it's important to remember that quadratic interpolation offers an approximation, not an exact solution. While it provides a reliable means for estimating intermediate values, it's crucial to acknowledge its limitations and potential inaccuracies, especially when dealing with complex or non-smooth functions. For situations demanding higher precision, alternative interpolation methods with higher polynomial degrees might be necessary.

References

CHONG, E. K., LU, W.-S. & ZAK, S. H. 2023. *An Introduction to Optimization: With Applications to Machine Learning*, John Wiley & Sons.

EISELT, H., SANDBLOM, C.-L., EISELT, H. & SANDBLOM, C.-L. 2019. Applications of nonlinear programming. *Nonlinear Optimization: Methods and Applications*, 77-121.

GEN, M. & CHENG, R. 1999. *Genetic algorithms and engineering optimization*, John Wiley & Sons.

MISHRA, S. K. & RAM, B. 2019. *Introduction to unconstrained optimization with R*, Springer Nature.

پوختە

ئەم پرۆژە توێژینەوەیە قووڵ دەبێتەوە لە چەمکی ئینتەرپۆلاشیۆنی چوارگۆشەیی، کە تەکنیکێکە بۆ نزیککردنەوەی بەها نەناسراوەکان لەناو کۆمەڵە داتایەکدا بەکاردەهێنرێت. بنەما سەرەکییەکانی شێوازەکە دەخاتە ڕوو، تیشک دەخاتە سەر دروستکردنی فرەژمارەیەکی پلە دوو لە ڕێگەی سێ خاڵە داتا ناسراوەکانەوە. ئەم ڕێبازە ڕێگە خۆش دەکات بۆ خەملاندنی هەڵسوکەوتی فەنکشنێک لە شوێنە نێوانەکاندا، کە بەنرخی لە بوارە جۆراوجۆرەکانی وەک شیکاری داتا، شێوازە ژمارەییەکان و گرافیکی کۆمپیوتەردا دەسەلمێنێت. پوختە بە داننان بە سروشتی نزیکبوونەوەی سروشتی ئینتەرپۆلاسیۆنی چوارگۆشەیی و ئەگەری نادروستی، بە تایبەتی لەگەڵ ئەرکە ئاڵۆزەکان یان نانەرمەکاندا کۆتایی دێت. جەخت لەسەر پێویستی شێوازی بەدیل دەکاتەوە کە پلەی فرەژمارەیی بەرزتریان هەبێت کاتێک وردبینی بەرزتر زۆر گرنگە.

زانكۆی سەلاحەدین - هەولێر
Salahaddin University-Erbil

چەمکی ئینتەرپۆلاشیۆنی چوارگۆشەیی

پرۆژەی دەرچوونە

پێشکەش بە بەشی (ماتماتیک) کراوە، وەک بەشێک لەپێداویستەکانی بەدەستهێنانی بڕوانامەی بەکالۆریۆس لە زانستی (ماتماتیک)

ئامادەکراوە لەلایەن:
صبیحە ولید ابراهیم

بەسەرپەرشتی:
ماهر علی ناوخاص

نيسان– ٢٠٢٤