Chapter 4

Application in Matlab

1) Solution of linear systems

i) Back slash (inverse) method

Example 1:- Solve the following matrix equation (the system) in Matlab by using back slash (inverse) method

$$5x - 3y - 10 - 2z = 0$$
$$4y = -8y + 3x + 20$$
$$2x + 4y - 4z = 9$$

Solution: - 1) rearrange the equation

$$5x - 3y - 2z = 10$$
$$-3x + 8y + 4z = 20$$
$$2x + 4y - 9z = 9$$

2) Write the equation in matrix, Ax=B:-

$$A = \begin{bmatrix} 5 & -3 & 2 \\ -3 & 8 & 4 \\ 2 & 4 & -9 \end{bmatrix} \text{ , } x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ , } B = \begin{bmatrix} 10 \\ 20 \\ 9 \end{bmatrix}$$

3) Solve the matrix equation in Matlab by using back slash (\ ) or inverse (inv) method.

```
ommand Window
>> A=[5 -3 2; -3 2 4 ; 2 4 -9];
>> B=[10;20;9];
>> x=inv(A)*B

x =

    5.9586
    9.6828
    4.6276
```

ii) Gauss – Elimination Method ($rref$) :-

Example 2:- Solve the following system using Gauss - Elimination Method

$$A = \begin{bmatrix} 5 & -3 & 2 \\ -3 & 8 & 4 \\ 2 & 4 & -9 \end{bmatrix} , x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} , \; B = \begin{bmatrix} 10 \\ 20 \\ 9 \end{bmatrix}$$

```
>> A=[5 -3 2; -3 8 4;2 4 -9];
>> B=[10;20;9];
>> c=rref([A,B])

c =

    1.0000         0         0    3.4442
         0    1.0000         0    3.1982
         0         0    1.0000    1.1868
```

iii) Gramer Method

Example: - Solve the following system using Gramer method

$$5x - 3y - 2z = 10$$
$$-3x + 8y + 4z = 20$$
$$2x + 4y - 9z = 9$$

Solution

Command Window

```
>> A=[5 -3 2; -3 2 4 ; 2 4 -9];
>> B=[10;20;9];
>> dA=det(A);
>> A1=A;
>> A1(:,1)=B;
>> dA1=det(A1);
>> A2=A;
>> A2(:,2)=B;
>> dA2=det(A2);
>> A3=A;
>> A3(:,3)=B;
>> dA3=det(A3);
```

```
>> x=dA1/dA

x =

    5.9586

>> y=dA2/dA

y =

    9.6828

>> z=dA3/dA
```

z=

4.6276

2)Eigen Value & Eigen Vectors

A) Eigen Value

Let A= $\begin{bmatrix} 9 & 4 & 4 \\ 2 & 8 & 0 \\ 6 & 7 & 8 \end{bmatrix}$

```
>> A=[9 4 4;2 8 0; 6 7 8];
>> E=eig(A)

E =

   14.8574
    4.4303
    5.7123
```

B) Eigen Vectors

```
>> [V,E]=eig(A)

V =

    0.6375    0.7778    0.7523
    0.1859   -0.4358   -0.6577
    0.7476   -0.4528    0.0394


E =

   14.8574         0         0
         0    4.4303         0
         0         0    5.7123
```

2) Polynomials

1) Roots(p)

Example: - $p(x) = x^3 - 6x^2 + 11x - 6$

```
>> p=[1 -6 11 -6];
>> E=roots(p)

E =

    3.0000
    2.0000
    1.0000

>> p=poly(E)

p =

    1.0000   -6.0000   11.0000   -6.0000
```

```
>> polyval(p,3)

ans =

 -3.5527e-15
```

```
>> q=[1 0 -1];
>> p=[1 -6 11 -6];
>> s=conv(p,q)

s =

     1    -6    10     0   -11     6

>> [s v]=deconv(p,q)

s =

     1    -6


v =

     0     0    12   -12
```

3) Derivative of polynomial

To find the derivative of P

i) $Polyder(p) = p'$ return to the derivative of P

Ex:- $p(x) = x^4 + 5x^3 - 2x + 7$

$p = [1 \quad 5 \quad 0 \quad -2 \quad 7]$

$d = p' = polyder(p)$

```
>> p=[1 5 0 -2 7];
>> polyder(p)

ans =

     4    15     0    -2
```

ii) To find the derivative of product of two polys, $p(x), q(x)$

$C = polyder(p, q);$ $(p.q)'$

$d = polyder(compose(p, q));$ $(p \circ q)'$

$Ployint(p)$ Integral of polynomial

$w = polyder\big(polyder(p)\big);$ p''

Example:- $p(x) = x^3 + 5x - 3,\ \ q(x) = x^2 + 6x + 9$

```
Command Window
>> syms x
>> p = x^3 + 5*x - 3; q = x^2 + 6*x + 9;
>> p1=[1 0 5 -3];q1=[1 6 9];
>> C=polyder(p1,q1)

C =

     5    24    42    54    27

>> polyint(p1)

ans =

    0.2500         0    2.5000   -3.0000         0

>> polyder(polyder(p1))

ans =

     6     0
```

4) Symbolic Calculation

Creating symbolic Expression

Variable $x, y, z, a, b, c, \ldots$ can be declared as symbolic variables with command

```
>> syms x y z a b c
>> A=[a b 1; 0 1 c ;x 0 0]
```

```
A =

[ a, b, 1]
[ 0, 1, c]
[ x, 0, 0]

>> d=det(A)

d =

b*c*x - x
```

5) Differentiation in Matlab

First creat a symbolic expression the command $diff(f)$ the differentation of f with respect to x or $diff(f, x)$

Example:- find the Differentiation of function $f(x) = 7x^4 - 5x^3 + 2x^2 - 10x + 11$

```
>> syms x
>> f=7*x^4-5*x^3+2*x^2-10*x+11;
>> df=diff(f)

df =

28*x^3 - 15*x^2 + 4*x - 10

>> d2f=diff(f,2)

d2f =

84*x^2 - 30*x + 4

>> f=12;df=diff(f)

df =

fx      []
```

6) Partial derivative

Example :- $f(x) = x^2y + 3y^2 + x$

1- $\frac{\partial f}{\partial x} = f_x$

2- $\frac{\partial^2 f}{\partial x^2} = f_{xx}$

3- $\frac{\partial f}{\partial y} = f_y$

4- $\frac{\partial^2 f}{\partial y^2} = f_{yy}$

5- $\frac{\partial}{\partial x}\frac{\partial f}{\partial y} = f_{xy}$

6- $\frac{\partial}{\partial y}\frac{\partial f}{\partial x} = f_{yx}$

```
>> syms x y
>> f=x^2+3*y^2+x;
>> dfx=diff(f,x)

dfx =

2*x + 1

>> dfy=diff(f,y)

dfy =

6*y
```

```
>> df2x=diff(f,x,2)

df2x =

2

>> df2y=diff(f,y,2)

df2y =

6
```

```
>> dyxf=diff(diff(f,x),y)

dyxf =

0

>> dxyf=diff(diff(f,y),x)

dxyf =

0
```

8- limits

Limite of an expression:-

1) limit (f, x, a) take the limit of the symbolic expression f as $x \to a$ *ie* $\lim_{x\to 0} f(x)$

2) $limit(f)$ uses $a = 0$ *ie* $\lim_{x\to 0} f(x)$

3) $limit(f, x, a, 'left')$ *ie* $\lim_{x\to a-} f(x)$

$limit(f, x, a, 'right')$ *ie* $\lim_{x\to a+} f(x)$

Example 1:-

$\lim_{x\to 0} \frac{sin(x)}{x} = 1$

```
>> syms x
>> f=sin(x)/x;
>> limit(f)

ans =

1
```

Example 1:-

$\lim_{x\to 0} \frac{5}{x}$

```
>> syms x
>> f=x/5;
>> y1=limit(f,x,0,'right')

y1 =

0

>> y2=limit(f,x,0,'left')

y2 =

0
```

9- integration

The symbolic of integration is $int(f)$

We have two type of integration 1- indefinite integral 2- definite intgral

1- indefinite integral

i- $int(f)$

ii-$int(f,x)$

2- definite intgral

i- $int(f,a,b)$

ii-$int(f,x,a,b)$

example 1:- $\int \frac{dx}{\sqrt{x}}$ 2- $\int_0^1 x\log(x+5)dx$

```
>> syms x
>> f=1/sqrt(x);
>> int(f)

ans =

2*x^(1/2)

>> syms x
>> f=x*log(x+5);
>> int(f,0,1)

ans =

(25*log(5))/2 - 12*log(6) + 9/4
```

10- Pretty

Pretty(f) : print the symbolic expresion f **in type set method**

Example :- $f(x) = \frac{\log(x^2+7)}{x^4-5x^2+1}$

Command Window

```
>> syms x
>> f(x)=(log(x^2+7))/(x^4-5*x^2+1);
>> pretty(f)
        2
 log(x   + 7)
-------------
 4       2
x   - 5 x   + 1

>> 
```

11-simplify

Example 1:- $rsin^2\theta + rcos^2\theta = r$

Command Window

```
>> syms r x
>> f=r*sin(x)^2+r*cos(x)^2;
>> simplify(f)

ans =

r
```

Example 2:-

Jacobian matrix of f is

$$x = r\cos\varphi;$$
$$y = r\,\mathrm{sir}\,\varphi.$$

$$\mathbf{J}_{\mathbf{F}}(r,\varphi) = \begin{bmatrix} \dfrac{\partial x}{\partial r} & \dfrac{\partial x}{\partial \varphi} \\ \\ \dfrac{\partial y}{\partial r} & \dfrac{\partial y}{\partial \varphi} \end{bmatrix} = \begin{bmatrix} \cos\varphi & -r\,\mathrm{sir}\,\varphi \\ \sin\varphi & r\cos\varphi \end{bmatrix}$$

ommand Window

```
>> syms r u
>> x=r*cos(u);
>> y=r*sin(u);
>> A=jacobian([x y]);
>> B=det(A)

B =

r*cos(u)^2 + r*sin(u)^2
```

12- factor

$factor(n)$ return a vector containing the prime factor of n

Example 1:- factor of (27)

Command Window

```
>> factor(27)

ans =

     3     3     3
```

Example 2:- factor of (x^2-1)

```
>> syms x
>> f=x^2-1;
>> factor(f)

ans =

[ x - 1, x + 1]
```

12- Collect

$Collect(f,x)$ regard each element of the symbolic f as a polynomial in x and rewrite f in terms of the pawer of x

Example:- $1 - f(x) = (x+1)(x-1)(x^2+1)$

$$2 - f(x) = x^2y + yx - x^2 - 2x$$

Command Window

```
>> syms x
>> f=(x+1)*(x-1)*(x^2+1);
>> collect(f)

ans =

x^4 - 1

>> syms x y
>> f=x^2*y+y*x-x^2-2*x;
>> collect(f)

ans =

(y - 1)*x^2 + (y - 2)*x
```

13- Expand

$Expand\ (f)$: —distributes product over summation

Command Window

```
>> % Example 1:- f=a*(b+c)
>> syms a b c
>> f=a*(b+c);
>> expand(f)

ans =

a*b + a*c

>> % Example 2:- f=cos(x+y)
>> syms x y
>> f=cos(x+y);
>> expand(f)

ans =

cos(x)*cos(y) - sin(x)*sin(y)
```

14- subsitution

$Subs(f, variable, point)$

$Subs(f, [x, y, z], [x_0, y_0, z_0])$

mmand Window

```
>> % Example:- f(x,y,z)=x*y^2-z*x*y+z^2*x where x0=1, y0=3,z0=7
>> syms x y z
>> f=x*y^2-z*x*y+z^2*x;
>> subs(f,[x,y,z],[1,3,7])

ans =

37
```

15- Sum $\sum$

$Symsum(s, x)$ is define summation of S with respect to the symbolic variable determined by $syms$

$Symsum(s, x) \rightarrow \sum_{x=0}^{\infty} x$

$Symsum(s, a, b) \rightarrow \sum_{a}^{b} x$

Command Window

```
>> syms x
>> symsum(x,0,10)

ans =

55

>> syms x
>> symsum(x,0,inf)

ans =

Inf

>> syms x n
>> symsum(x^2,n,0,inf)

ans =

piecewise(x == 0, 0, x ~= 0, Inf*x^2)

>> symsum(x^n,n,0,inf)

ans =

piecewise(1 <= x, Inf, abs(x) < 1, -1/(x - 1))
```

16- tayler Expression

$1 - Taylor(f)$ find the fifth order maclorain polu appr. Of f

Example:- $f(x) = e^x$

```
Command Window
>> syms x
>> f=exp(x);
>> taylor(f)

ans =

x^5/120 + x^4/24 + x^3/6 + x^2/2 + x + 1

>> taylor(cos(x))

ans =

x^4/24 - x^2/2 + 1
```

17- Compose

$compose\ (f, g)$ return $g \circ f(x)\ \ or\ \ g(f(x))$ where $f = f(x)$ and $g = g(x)$, then x is symbolic variable of f as define by syms and y is symbolic variable of f as define by syms.

```
Command Window
>> syms x y z
>> f=(1/x^2+5);
>> g=cos(y);
>> compose(f,g)

ans =

1/cos(y)^2 + 5

>> compose(f,g,z)

ans =

1/cos(z)^2 + 5
```

Home Work Section

H.W 1: Find the following results by using commands windows.

$f(x) = x^4 + 4x^3 + 5x + 3$, $g(x) = x^3 + 6x^2 + 3x + 1$
$h(x, y) = x^7y^3 + 2y^2 + e^{yx}$

1- Derivative of product of two polys f & g.
2- The roots of product of two polys f & g.
3- Differentiation of h with respect to y by using symbolic calculation.
4- $\int_2^5 e^x dx$
5- If $f(x) = x^4 + 4x^3 + 5x + 3$ then find f at point $x = 4$.

H.W 2:- Find the following results by using commands windows.

1- $\lim_{x \to -2} x^3$.
2- Eigen value & Eigen vector of the matrix H= $\begin{bmatrix} 4 & 6 & 3 \\ 0 & -8 & 9 \\ -5 & 7 & 1 \end{bmatrix}$
3- If $f = x^3 + x^2 + x$ find $f(3)$ by using direct definition of the function
4- Find the factor of 95.

H.W 3:- Find the following results by using commands windows.

$f = (e^{2x} \cos(y^2)\ z^2)$, $w = (5x^2 - 1)$, $h(x) = x^4 + 4x^3 + 5x + 3$

1- $E = \sum_{y=7}^{11} \sum_{z=0}^{5} (f)$,find the value of E at $x = 2$ by using symbolic of summation in one statement .
2- $\int_4^5 \int_1^2 x^2 \cos(y)\, dx\, dy$.
3- Differentiation of f with respect to y by using symbolic calculation.
4- Find the roots of product of two polys h, w.
5- Find the value of h at point $x = 4$.

H.W 4: Find the following results by using commands windows.

$f(x) = \frac{x^3 + 3x^2 + 3x}{xe^{2x}}$, $g(x, y, z) = (xyz)^3 + e^{xz} \sin(xy)$
$h = 1555$

1- Pretty of f & g.

2- Differentiation of g with respect to y by using symbolic calculation.

3- Simplify f

4- Find g at point $x = 4$ & $y = 2, z = 5$.

6- Factor of h.