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 *Salahaddin University-Erbil*

*College of Engineering*

*Software and Informatics Eng. Department*

**Sample of Questions (Questions Bank)**

**Discrete Mathematics**

**Academic year 2023-2024**

**Year of Study**: First Year Students

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**Level of Course**: Undergraduate Degree

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Q1:Define the following Terms:

Existential Quantifier, Irreflexive Relation, Complement of the set, Propositional Function, Field, Group, Ring, Context Free Grammar.

Q2: Which of these sentences are propositions? What are the truth values of those that are propositions?

a) Boston is the capital of Massachusetts.

 b) Miami is the capital of Florida.

 c) 2 + 3 = 5.

 d) 5 + 7 = 10.

 e) *x* + 2 = 11.

Q3: What is the negation of each of these propositions?

a) Mei has an MP3 player.

b) There is no pollution in New Jersey.

c) 2 + 1 = 3.

d) The summer in Maine is hot and sunny.

Q4: Let *p* and *q* be the propositions

 *p* : It is below freezing.

 *q* : It is snowing.

Write these propositions using *p* and *q* and logical connectives (including negations).

a) It is below freezing and snowing.

b) It is below freezing but not snowing.

c) It is not below freezing and it is not snowing.

d) It is either snowing or below freezing (or both).

e) If it is below freezing, it is also snowing.

f ) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.

g) That it is below freezing is necessary and sufficient for it to be snowing.

Q5: Let *p* and *q* be the propositions

 *p* :You drive over 65 miles per hour.

 *q* :You get a speeding ticket.

Write these propositions using *p* and *q* and logical connectives (including negations).

a) You do not drive over 65 miles per hour.

b) You drive over 65 miles per hour, but you do not get a speeding ticket.

c) You will get a speeding ticket if you drive over 65 miles per hour.

d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.

e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

f ) You get a speeding ticket, but you do not drive over 65 miles per hour.

g)Whenever you get a speeding ticket, you are driving over 65 miles per hour.

Q6: Determine whether each of these conditional statements is true or false.

a) If 1 + 1 = 2, then 2 + 2 = 5.

b) If 1 + 1 = 3, then 2 + 2 = 4.

c) If 1 + 1 = 3, then 2 + 2 = 5.

d)If monkeys can fly, then 1 + 1 = 3.

Q7: Construct a truth table for each of these compound propositions.

a) *p* ∧￢*p*

b) *p* ∨￢*p*

c) *(p* ∨￢*q)* → *q*

d) *(p* ∨ *q)* → *(p* ∧ *q)*

e) *(p* → *q)* ↔ *(*￢*q* →￢*p)*

f )*(p* → *q)* → *(q* → *p)*

Q8: Construct a truth table for each of these compound propositions.

a) *p* →￢*q*

b) ￢*p* ↔ *q*

c) *(p* → *q)* ∨ *(*￢*p* → *q)*

d) *(p* → *q)* ∧ *(*￢*p* → *q)*

e) *(p* ↔ *q)* ∨ *(*￢*p* ↔ *q)*

f ) *(*￢*p* ↔￢*q)* ↔ *(p* ↔ *q)*

Q9: Construct a truth table for each of these compound propositions.

a) *p* → *(*￢*q* ∨ *r)*

b) ￢*p* → *(q* → *r)*

c) *(p* → *q)* ∨ *(*￢*p* → *r)*

d) *(p* → *q)* ∧ *(*￢*p* → *r)*

e) *(p* ↔ *q)* ∨ *(*￢*q* ↔ *r)*

f ) *(*￢*p* ↔￢*q)* ↔ *(q* ↔ *r)*

Q10: Construct a truth table for *((p* → *q)* → *r)* → *s*.

Q11: Construct a truth table for *(p* ↔ *q)* ↔ *(r* ↔ *s)*.

Q12: Prove without Truth Table that:

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Q13:Prove without Truth Table that:

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Q14: Prove without Truth Table that:

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Q15:Prove without Truth Table that:

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Q16: Show that *(p* → *q)* ∧ *(q* → *r)* → *(p* → *r)* is a tautology.

Q17: Suppose that *A,B*, and *C* are sets such that *A* ⊆ *B* and *B* ⊆ *C*. Show that *A* ⊆ *C*.

Q18: Find the power set of each of these sets, where *a* and *b* are distinct elements.

a) {*a*}.

b) {*a, b*}.

c) {∅*,* {∅}}.

Q19: Let *A* = {*a, b, c, d*} and *B* = {*y, z*}. Find

a) *A* × *B*.

b) *B* × *A*.

Q20: Let *A* = {1*,* 2*,* 3*,* 4*,* 5} and *B* = {0*,* 3*,* 6}. Find

a) *A* ∪ *B*.

b) *A* ∩ *B*.

c) *A* − *B*.

d) *B* − *A*.

Q21: Show that if *A* and *B* are sets, then:

*A* − *B* = *A* ∩ *Bc*.

Q22: Show that if *A* and *B* are sets, then:

*(A* ∩ *B)* ∪ *(A* ∩ *Bc)* = *A*.

Q23: Show that if *A* is a subset of a universal set *U*, then

a) *A* ⊕ *A* = ∅.

b) *A*⊕∅ = *A*.

Q24: Show that if *A* is a subset of a universal set *U*, then

a) *A* ⊕ *U* = *Ac*.

b) *A* ⊕ *Ac* = *U*.

Q25: Let *A* = {0*,* 2*,* 4*,* 6*,* 8*,* 10}, *B* = {0*,* 1*,* 2*,* 3*,* 4*,* 5*,* 6}, and *C* = {4*,* 5*,* 6*,* 7*,* 8*,* 9*,* 10}. Find

a) *A* ∩ *B* ∩ *C*.

b) *A* ∪ *B* ∪ *C*.

c) *(A* ∪ *B)* ∩ *C*.

d) *(A* ∩ *B)* ∪ *C*.

Q26:Let,and.Find:

a)

b)

Q27:Let,,,,,,.Find:

a)

b)

Q28:Let,,,.Find:

a)

b)

Q29: Show that if *A, B* and *C* are sets, then:

 

Q30: Show that if A is a set and Bi is an index set, then:

 

Q31: Consider these relations on the set of integers:

*R*1 = {*(a, b)* | *a* ≤ *b*}*,*

*R*2 = {*(a, b)* | *a > b*}*,*

*R*3 = {*(a, b)* | *a* = *b* or *a* = −*b*}*,*

*R*4 = {*(a, b)* | *a* = *b*}*,*

*R*5 = {*(a, b)* | *a* = *b* + 1}*,*

*R*6 = {*(a, b)* | *a* + *b* ≤ 3}*.*

Which of these relations contain each of the pairs *(*1*,* 1*)*, *(*1*,* 2*)*, *(*2*,* 1*)*, *(*1*,*−1*)*, and *(*2*,* 2*)*?

Q32: Consider the following relations on {1*,* 2*,* 3*,* 4}:

*R*1 = {*(*1*,* 1*), (*1*,* 2*), (*2*,* 1*), (*2*,* 2*), (*3*,* 4*), (*4*,* 1*), (*4*,* 4*)*}*,*

*R*2 = {*(*1*,* 1*), (*1*,* 2*), (*2*,* 1*)*}*,*

*R*3 = {*(*1*,* 1*), (*1*,* 2*), (*1*,* 4*), (*2*,* 1*), (*2*,* 2*), (*3*,* 3*), (*4*,* 1*), (*4*,* 4*)*}*,*

*R*4 = {*(*2*,* 1*), (*3*,* 1*), (*3*,* 2*), (*4*,* 1*), (*4*,* 2*), (*4*,* 3*)*}*,*

*R*5 = {*(*1*,* 1*), (*1*,* 2*), (*1*,* 3*), (*1*,* 4*), (*2*,* 2*), (*2*,* 3*), (*2*,* 4*), (*3*,* 3*), (*3*,* 4*), (*4*,* 4*)*}*,*

*R*6 = {*(*3*,* 4*)*}*.*

Which of these relations are reflexive?

Q33: For each of these relations on the set {1*,* 2*,* 3*,* 4}, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

a) {*(*2*,* 2*), (*2*,* 3*), (*2*,* 4*), (*3*,* 2*), (*3*,* 3*), (*3*,* 4*)*}

b) {*(*1*,* 1*), (*1*,* 2*), (*2*,* 1*), (*2*,* 2*), (*3*,* 3*), (*4*,* 4*)*}

c) {*(*2*,* 4*), (*4*,* 2*)*}

d) {*(*1*,* 2*), (*2*,* 3*), (*3*,* 4*)*}

e) {*(*1*,* 1*), (*2*,* 2*), (*3*,* 3*), (*4*,* 4*)*}

f ) {*(*1*,* 3*), (*1*,* 4*), (*2*,* 3*), (*2*,* 4*), (*3*,* 1*), (*3*,* 4*)*}

Q34: Determine whether the relation *R* on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where *(x, y)* ∈ *R* if and only if

a) *xy*.

b) *xy* ≥ 1.

c) *x* = *y* + 1 or *x* = *y* − 1.

d) *x* = *y*2.

e) *x* ≥ *y*2.

Q35: Represent each of these relations on {1*,* 2*,* 3} with a matrix (with the elements of this set listed in increasing order).

a) {*(*1*,* 1*), (*1*,* 2*), (*1*,* 3*)*}

b) {*(*1*,* 2*), (*2*,* 1*), (*2*,* 2*), (*3*,* 3*)*}

c) {*(*1*,* 1*), (*1*,* 2*), (*1*,* 3*), (*2*,* 2*), (*2*,* 3*), (*3*,* 3*)*}

d) {*(*1*,* 3*), (*3*,* 1*)*}

Q36: Which of these relations on {0*,* 1*,* 2*,* 3} are equivalence relations?

a) {*(*0*,* 0*), (*1*,* 1*), (*2*,* 2*), (*3*,* 3*)*}

b) {*(*0*,* 0*), (*0*,* 2*), (*2*,* 0*), (*2*,* 2*), (*2*,* 3*), (*3*,* 2*), (*3*,* 3*)*}

c) {*(*0*,* 0*), (*1*,* 1*), (*1*,* 2*), (*2*,* 1*), (*2*,* 2*), (*3*,* 3*)*}

d) {*(*0*,* 0*), (*1*,* 1*), (*1*,* 3*), (*2*,* 2*), (*2*,* 3*), (*3*,* 1*), (*3*,* 2*),(*3*,* 3*)*}

e) {*(*0*,* 0*), (*0*,* 1*), (*0*,* 2*), (*1*,* 0*), (*1*,* 1*), (*1*,* 2*), (*2*,* 0*),(*2*,* 2*), (*3*,* 3*)*}

Q37: Show that if R is a relation from set A to set B, then:

a) Dom (R) = Ran (R-1).

b) Ran (R) = Dom (R-1).

Q38: Show that if R is a relation over the set A, then:

 

Q39: If R, S and T are relations over the set A, then:

 

Q40: If R, S and T are relations over the set A, then:

 

Q41: If R, S and T are relations over the set A, then:

 

Q42: If R, S and T are relations over the set A, then:

 If RS, thenand

Q43:Let A= {1,2,3,4,5}, R1 and R2 are Relations defined on A as follows:

 R1=

 R2=

 Show that these relations are Equivalence Relations or not.

Q44:Let,,and,R={(x,y)},S={(y,z)}, then Find RS.

Q45: List the ordered pairs in the relations on {1*,* 2*,* 3} corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

a)

b) 

c) 

Q46: Determine whether the relations represented by the matrices below are reflexive, irreflexive, symmetric, antisymmetric, and/or transitive.

a)

b) 

c) 

Q47: Represent each of these relations on {1*,* 2*,* 3*,* 4} with a matrix (with the elements of this set listed in increasing order).

a) {*(*1*,* 2*), (*1*,* 3*), (*1*,* 4*), (*2*,* 3*), (*2*,* 4*), (*3*,* 4*)*}

b) {*(*1*,* 1*), (*1*,* 4*), (*2*,* 2*), (*3*,* 3*), (*4*,* 1*)*}

c) {*(*1*,* 2*), (*1*,* 3*), (*1*,* 4*), (*2*,* 1*), (*2*,* 3*), (*2*,* 4*), (*3*,* 1*), (*3*,* 2*)*,*(*3*,* 4*), (*4*,* 1*), (*4*,* 2*), (*4*,* 3*)*}

d) {*(*2*,* 4*), (*3*,* 1*), (*3*,* 2*), (*3*,* 4*)*}

Q48: List the ordered pairs in the relations on {1*,* 2*,* 3*,* 4} corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

a)

b) 

c) 

Q49: Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph.

c

b

a

f

e

d

Q50: Determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed graph.

b

a

d

c

Q51: Draw all subgraphs of this graph.

b

a

d

c

Q52: From the Graph below:

c

a

b

a) Find the Order and the Size of above Graph.

b) Find Degree of the vertices a, b, c.

Q53: Use an adjacency matrix to represent the given graph.

b

a

c

d

Q54: Use an adjacency matrix to represent the given graph.

b

a

d

c

Q55: Draw a graph represented by the given adjacency matrix.

 

Q56: Draw a graph represented by the given adjacency matrix.

 

Q57: Draw a graph represented by the given adjacency matrix.

 

Q58: Show that (Z4, +) Commutative Group or not.

Q59: Show that (Z,) Group or not by defineas:

 

Q60: Let and , then Show that (G,+) Group or not.

Q61: Give the Language corresponding to the Grammar, where {S, A}, {a, b}.

P: 

 

 

And then give the Type of the Grammar.

Q62: Give the Language corresponding to the Grammar, where {S, A}, {a, b}.

P: 

 

 

And then give the Type of the Grammar.

Q63: Give the Language corresponding to the Grammar, where {S}, {0, 1}.

P: 

 

And then give the Type of the Grammar.

Q64: Give the Language corresponding to the Grammar, where {S, N, A}, {0, 1}.

P: 

 

 

 

 

 

 

 

 

And then give the Type of the Grammar.

Q65: Give the Language corresponding to the Grammar, where {S}, {0, 1}.

P: 

 

And then give the Type of the Grammar.

Q66: Let A= {a, b}, S= {q0, q1, q2}, where F is:

 

And  is an Initial State, try to draw the Graph corresponding to the Machine.

Q67: Let A= {a, b}, S= {q0, q1, q2}, Z= {x, y, z}, , where F is:

, where g is:

And  is an Initial State, if “abaab” is the input string, then:

Find the output series and the state series.

Q68: Let A= {a, b}, S= {q0, q1, q2}, where F is:

 

And  is an Initial State, try to create the Table corresponding to the Machine.

Q69: Let A= {0, 1}, S= {q0, q1. q2, q3}, T= {q0},where F is:

 

And  is an Initial State, test the string (10011001) to know the machine accepts it or not.

Q70: Draw the state diagram for the finite-state machine with this state table.

|  |  |  |
| --- | --- | --- |
| States | Input Symbols | Input Symbols |
| f | G |
| A | b | A | B |
| S0 | S1 | S0 | 0 | 1 |
| S1 | S0 | S2 | 0 | 1 |
| S2 | S1 | S1 | 0 | 0 |

Q71: Draw the state diagram for the finite-state machine with this state table.

|  |  |  |
| --- | --- | --- |
| States | Input Symbols | Input Symbols |
| f | g |
| A | b | A | B |
| S0 | S1 | S0 | 0 | 0 |
| S1 | S2 | S0 | 1 | 1 |
| S2 | S0 | S3 | 0 | 1 |
| S3 | S1 | S2 | 1 | 0 |

Q72: Draw the state diagram for the finite-state machine with this state table.

|  |  |  |
| --- | --- | --- |
| States | Input Symbols | Input Symbols |
| f | g |
| A | b | A | B |
| S0 | S0 | S4 | 1 | 1 |
| S1 | S0 | S3 | 0 | 1 |
| S2 | S0 | S2 | 0 | 0 |
| S3 | S1 | S1 | 1 | 1 |
| S4 | S1 | S0 | 1 | 0 |

Q73: Find the Inverse of 3 mod 10 using Chinese Remainder Theorem (CRT).

Q74: Find the Inverse of 5 mod 23 using Euclidean Algorithm.

Q75: Find the Inverse of 7 mod 65 using Chinese Remainder Theorem (CRT).

Q76: Find the Inverse of 11 mod 26 using Euclidean Algorithm.

Q77: Find the Value of using Fast Exponentiation Algorithm.

Q78: Find the Value of Using Euler Function.

Q79: Find the Value of Using Euler Function.

Q80: What is the value of the Prefix expression 

Q81: What is the value of the Postfix expression 

Q82: Depending on the Following Data (18, 100, 6, 5, 50, 12, 14, 70, 45, 120). Answer the following points:

a) Draw the Diagram of Ordered Binary Tree.

b) Use Postorder, Preorder and Inorder to Traversal the Tree.

Q83: Depending on the Following Data (18, 100, 6, 5, 50, 12, 14, 70, 45, 120). Answer the following points:

a) Draw the Diagram of Ordered Binary Tree.

b) Delete the Digit 50 from the Tree, then Rebuilt the Tree after deleting the Digit.