Pseudo noise code in SS system

Properties

- Maximal length code 2^n-1 chips
- Using n stage shift register with a logic as a feedback
- The code has $2^n/2$ ones and $(2^n/2)-1$. Its importance?
- For biphase modulating a carrier, the residual carrier component is down by $1/(2^n-1)$
- For 1000 chips, carrier suppression is 30dB

Properties

- Autocorrelation of a code
- Mod-2 addition of a code with its phase shift result in another code
- Distribution of ones and zeros
  - $2^{n-(p+2)}$ run of p ones or zeros,
  - n ones and n-1 zeros.
  - Positions of thee run changes from code to another
- For a period less than $n*R_c$ it looks random.

PN in SSS

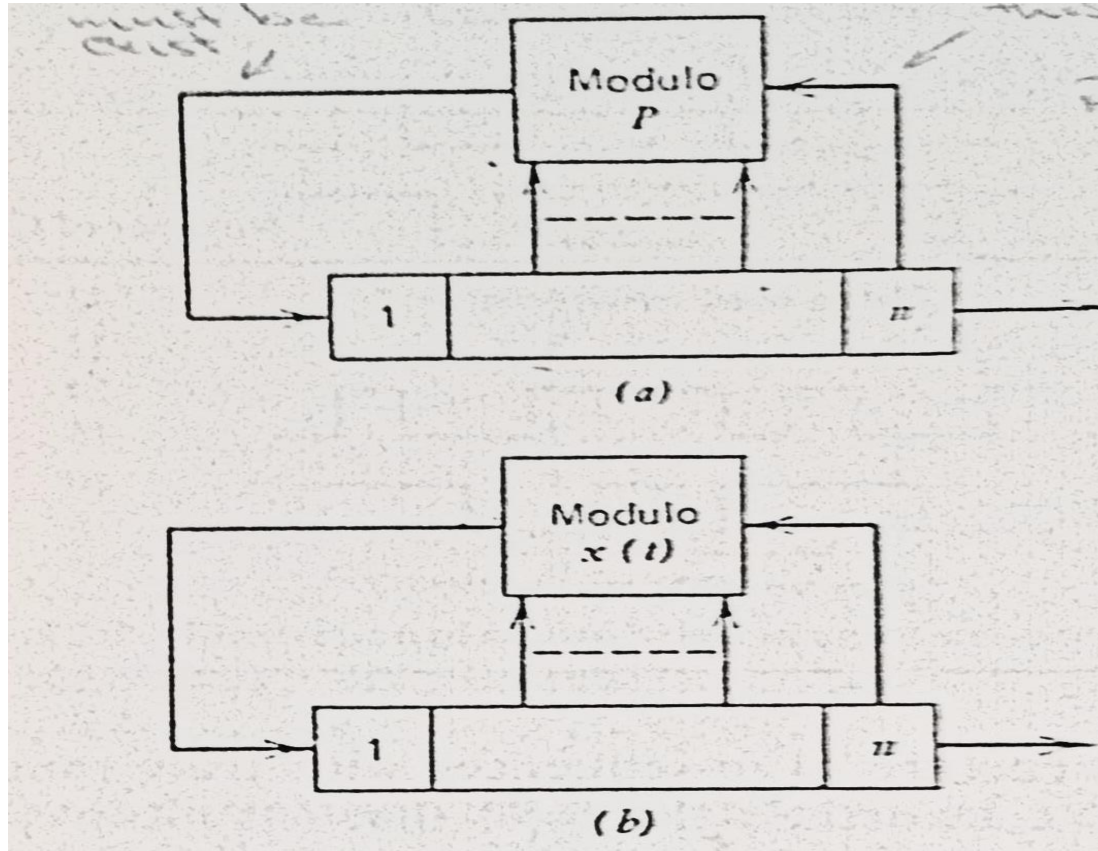
- Linear and non-linear codes
- Linear codes are usable for interference rejection, ranging and others but not for secure transmission
- If (2n+1) chips from the sequence is known, linear code is easily decipherable.
- However, we are focusing on maximal linear codes (PN sequence)

$L=P^n-1$

P is number of possible state at a stage

$L=r\,(P^n-1)$

R is the number of feedback conditions



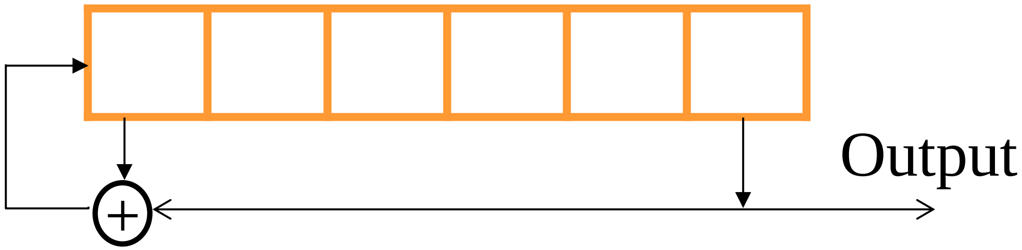
PN Sequences

PN sequences

- are periodic but appear random within one period
- are easy to generate
- are easy to re-generate and synchronize at the receiver
- have good random properties
- converge to a Gaussian process when the period tends to infinity

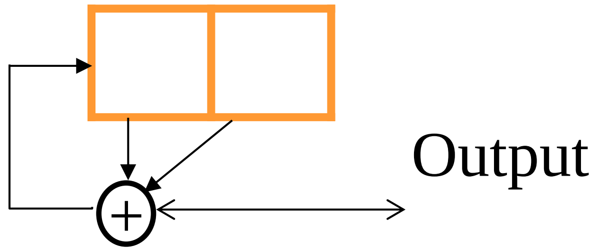
PN Sequence Generation

- Codes are periodic and generated by a shift register and XOR (mod-2)
- Maximum-length (ML) shift register sequences, *m*-stage shift register, length: $n = 2^m - 1$ bits
- All state are zeros is not allowed to occur



Generating PN Sequences

- Take m=2 =>L=3
- c_n=[1,1,0,1,1,0, . . .], usually written as bipolar c_n=[1,1,-1,1,1,-1, . . .]



m	Stages connected to modulo-2 adder
2	1,2
3	1,3
4	1,4
5	1,4
6	1,6
8	1,5,6,7

Maximal Sequence

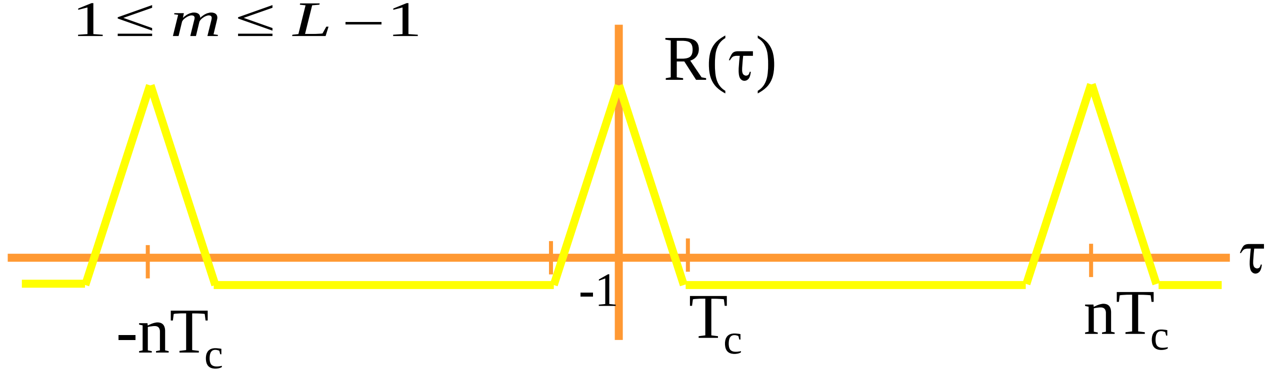
- Number of ones and zeros are equal
  - Ex: for 511 code length there is 256 ones and 255 zeros
  - Carrier suppression by a factor of $1/(2^n-1)$, so longer code is preferred.
- Distribution of ones and zeros is well defined
  - Positions of the runs is different in different code
  - Number of each run length is same
  - $2^{n-(P+2)}$ runs of length P, except one run of n ones and one run of (n-1) zeros
  - No run of n-1 ones or run of n zeros
- Mod-2 of maximal code with phase shifted version of itself results in a replica with different phase shift of the code.

- Autocorrelation is number of agreement minus disagreement over length of codes being compared.

- Autocorrelation of maximal code length is -1 at all phase shifts and vary linearly to 2^n-1 for phase shift -1 or +1 chip

$$R(m) = \sum_{n=1}^{L} c_n c_{n-m}$$

$$= \begin{cases} L & m = 0 \\ -1 & 1 \le m \le L-1 \end{cases}$$

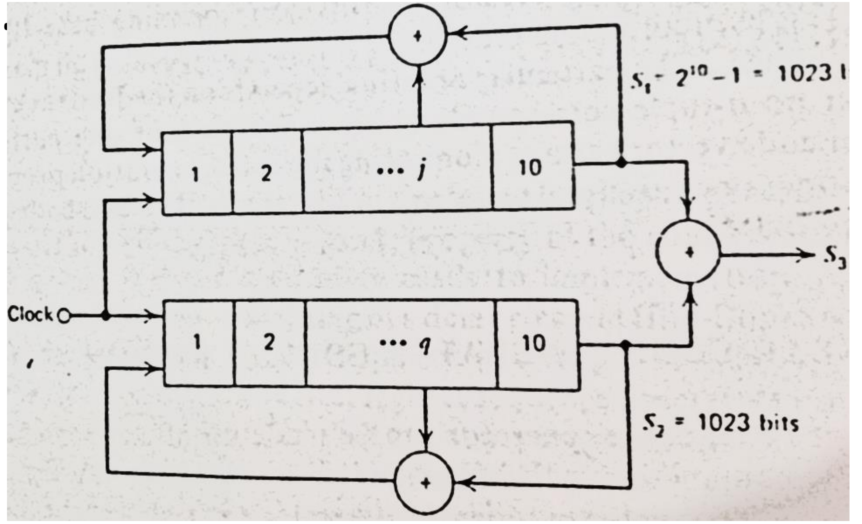


- Cross correlation measures agreement of two different codes.
  - Important when number of users have to share a frequency band

- ML sequence has combinatorial property:
  - Allows generation of any desired phase up to 2^n-1 chips
  - In GPS, all transmitters use the same code (of one week period) but with different phase (offset in time), a receiver synchronize to one signal at once.
- Two m-sequence with length L linearly added (mod-2), result in non maximal sequence of L length.
  - Different delay in the added codes

results in different code of same length:

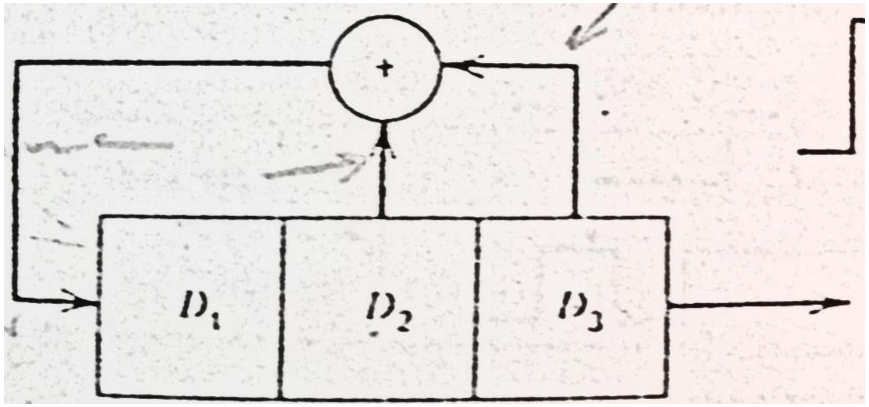
L different codes of length L can be generated



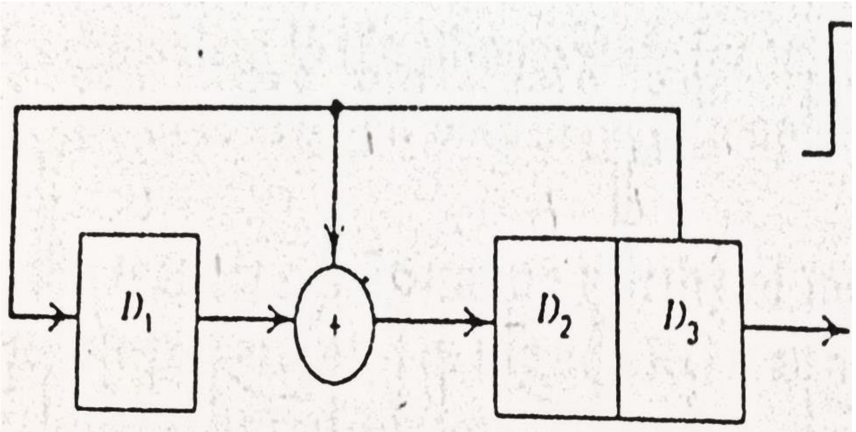
- Two m-sequence with different lengths, are linearly added (mod-2), results in non maximal sequence of length L_1*L_2. It is applied in ranging tech

Generator Configuration

- Typical seq. generator (simple type)



- Modular seq. generator
  - Delay
  - implementation



- Chip repetition rate $R_{rep}=R_c/L$
- R_{rep} determines the line spacing in the output spectrum
- Code period must exceed the length of the mission:
  - In aircraft, 8 hour code period exceed flight capability

- In DS, R_{rep} must not lie in the information band. In FH, it is equivalent to avoid to have a chip rate that falls there.
  - Otherwise, unnecessary noise will pass into the demodulator
  - Chip rate in FH usually has an integral relation to the information sent either multi chip/bit or multi bits/chip
- For ranging, if an integral no. code chips is accumulated for each mile of delay, simple count of code offset simplify the ranging

Chip Rate

- Higher chip rate for higher data rate or wider baseband bandwidth.
- Max. chip rate of a generator is determined by
  - rate of shift register stages
  - Delay in feedback

  $R_{max}=1/(T_{SR}+T_D)$

In typical SRG, $T_D=T_1+T_2+T_3+$...... depends on number of taps in feedback network, however little modification in the feedback would reduce T_D

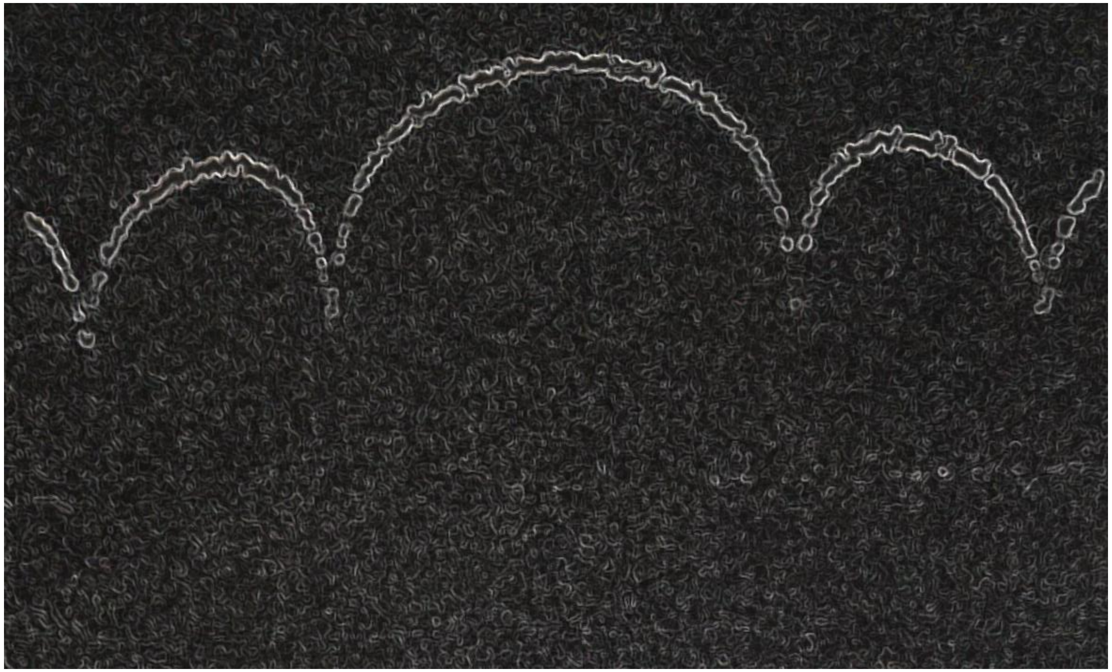
In Modular SRG, T_D is independent on number of taps

Problems with m-sequences

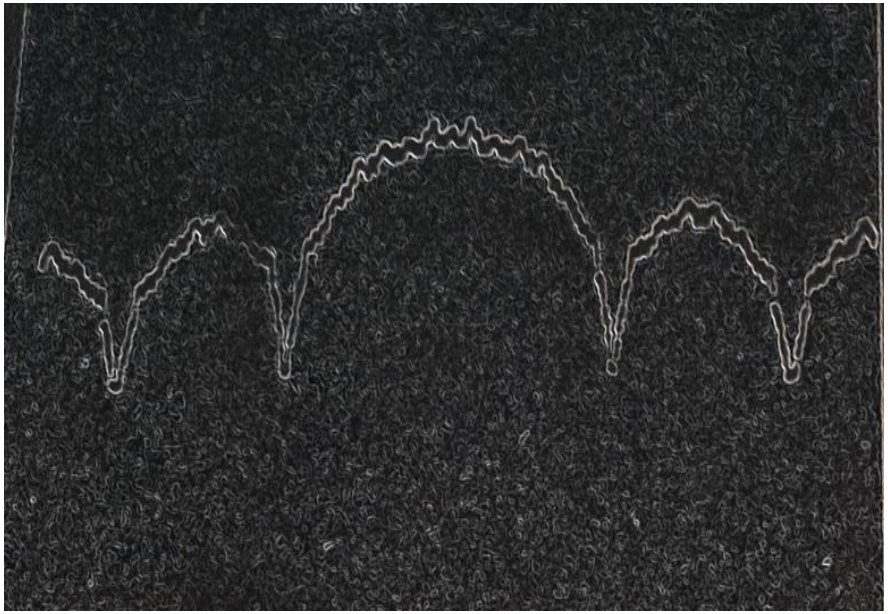
- Cross-correlations with other *m*-sequences generated by different input sequences can be quite high
- Easy to guess connection setup in 2*m* samples so not too secure

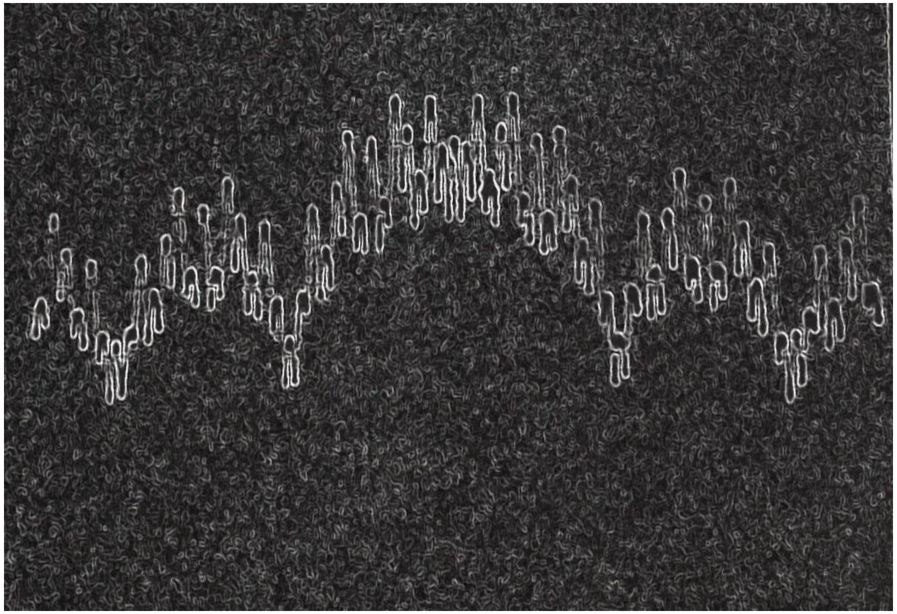
Spectrum based on Code Selection

- Generator failure can be recognized in the spectrum
- BPSK modulation by a maximal sequence of 13 stages



BPSK modulation by non Maximal code



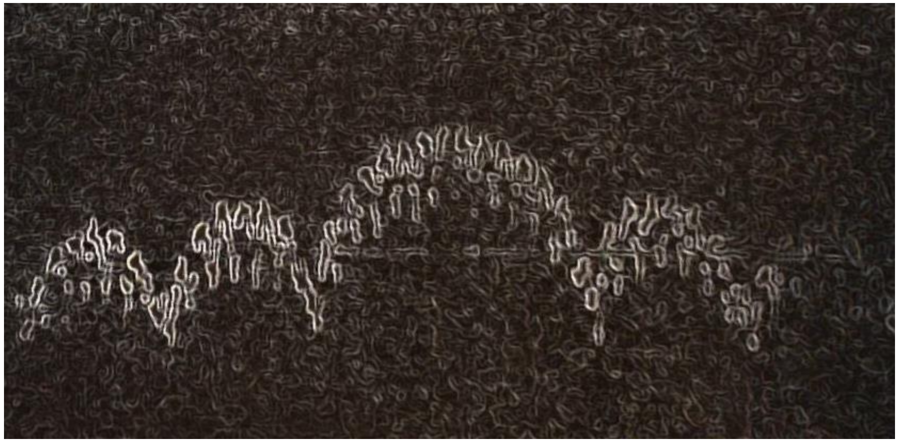


Two codes are taken from same generator

QPSK modulation by same code as in first figure. Taken from 13 and 4 stages

The symmetric around center freq. is not precise

Taken from 13 and 11 stages.
Spectrum is not symmetric

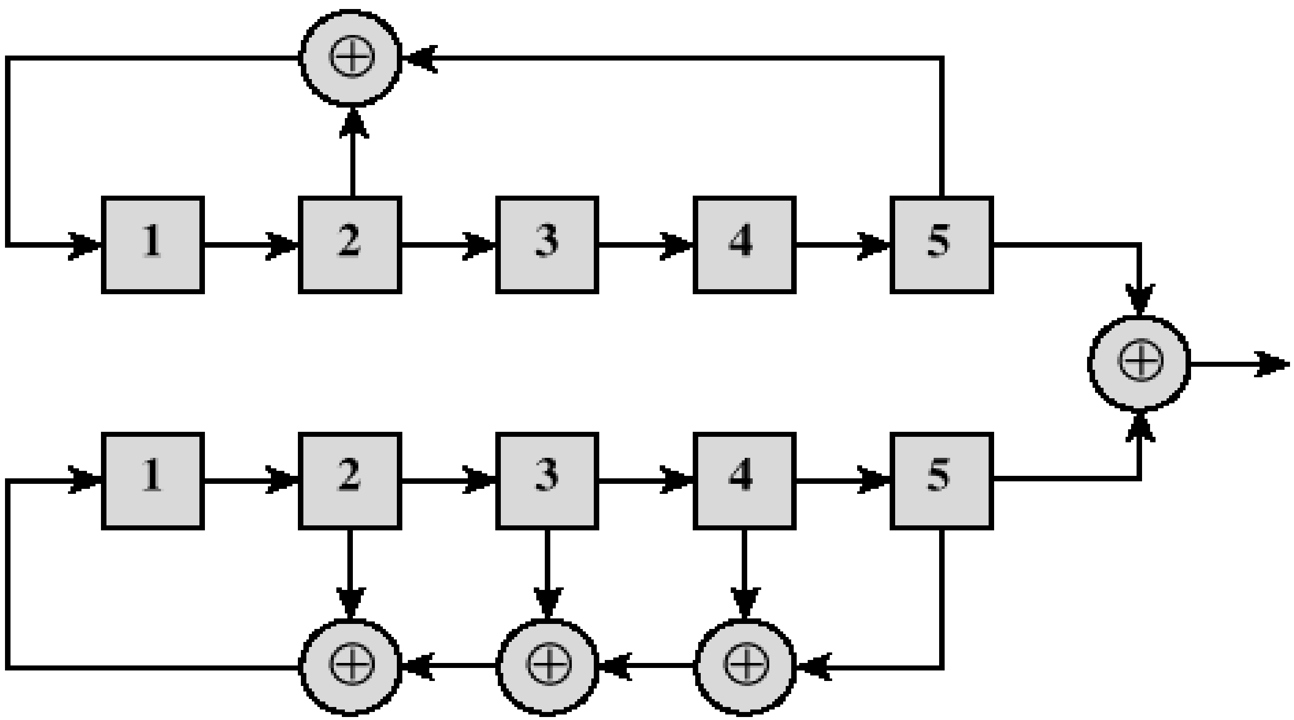


Taken from 13 and 6 stages.
Spectrum is not symmetric

Gold Sequences

- Gold sequences constructed by the XOR of two m-sequences with the same clocking
- Codes have well-defined cross correlation properties (uniform and bounded
- Only simple circuit needed to generate large number of unique codes
  - two shift registers generate the two m-sequences and these are then bitwise XORed

Shift Register Implementation



Syncopated Codes

- High rate code by multiplexing two or more slower generators
- Same as gold codes; two sequences are mod-2 added. However two separate clocks at rate R/P and phase shifted by 360/P

P number of registers used to generate the composite code, and R is the desired output rate

- If two different length sequences are used, the output sequence’s length is the multiplication. Otherwise same length sequence is generated.
- Code analysis is needed to find the correlation properties

Syncopated Codes